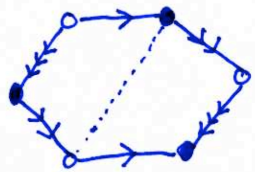
Summary of the band structure: the result is more memorable than the derivation!

- Honeycomb lattice ⇒ 2 site unit cell ⇒ 2 component wavefunction ("spinor") for each k.

- Brillouin zone



the • and ∘ points are "ramified".

- The point group C_{3v} contains $2\pi/3$ rotations and reflections in lines like ⋰. (Actually C_{6v}, left intact only by $k=0$)

The generic momentum breaks this to nothing; momenta at the boundary break it to just a reflection; The corners leave C_{3v} intact. N.B.: non-abelian; acts on the spinor irreducibly.

- At the corners the zones touch; one has for small deviations a conical E vs. δk.

Problem: What sort of lattices + internal structure give cones?: How about "colored" cones? (multi-)

14.3.9 The 0-field effect may be difficult to realize. Weak fields !? 14.20

14.3.10 For the conventional QHE, the implication is that instead of the naive quantization $\sigma_{xy} = \frac{4e^2}{h} N$ (note: 2 spins, 2 cones)

one has $\frac{2e^2}{h}, \frac{6e^2}{h}, \frac{10e^2}{h}, \ldots$

↑ from 0 mode

This has been observed.

14.3.11 Because the effective mass is small, the splitting between Landau levels ("gap") is very large. One observes the QHE at room temperature.

14.3.12 Superconductivity? (cf: Buckyballs)

14.3.13 There are many other interesting and promising aspects of graphene!

Of theoretical interest: $\alpha_{eff.} \propto \frac{1}{c_{eff.}} \gg \alpha$!

It is spellbinding to think that so many profound implications could come from a pencil and an adhesive tape

Graphene dreams

For many years it was believed that carbon nanotubes would create a revolution in nano-electronics because of their microscopic dimensions and very low electrical resistance. These hopes, however, have not yet come to fruition because of various difficulties. These include producing nanotubes with well-defined sizes, the high resistance at the connections between nanotubes and the metal contacts that connect them to circuits, and the difficulty of integrating nanotubes into electronic devices on a mass-production scale.

Walt de Heer argues that with graphene we will be able to avoid all of these problems. Using electron-beam lithography it is possible to pattern graphene into electron waveguides, and to control its electronic properties by applying external voltages using electronic gates. Furthermore, unlike 1D nanotubes, graphene is a continuous medium and hence the heating associated with high resistance at electrical contacts is minimized. This kind of heating is essentially the limiting factor for the miniaturization of silicon microchips, so graphene is especially interesting for the electronics industry. Perhaps even more remarkably, graphene offers the prospect of carving whole processors out of a single sheet.

Graphene research is still in its infancy and we wait to see what marvels it will produce in both fundamental science and technological applications. It is spellbinding to think that so many profound implications could come from a pencil and an adhesive tape. Indeed, the new field of graphene science illustrates well the remark of Ludwig Wittgenstein: "The aspects of things that are most important to us are hidden because of their simplicity and familiarity."

More about: Graphene

C Berger *et al.* Ultrathin epitaxial graphite: 2D electron gas properties and a route toward graphene-based nanoelectronics *J. Phys. Chem.* **108** 19912

V P Gusynin and S G Sharapov 2005 Unconventional integer quantum Hall effect in graphene *Phys. Rev. Lett.* **95** 146801

K S Novoselov *et al.* 2004 Electric field effect in atomically thin carbon films *Science* **306** 666–669

K S Novoselov *et al.* 2005 Two-dimensional gas of massless Dirac fermions in graphene *Nature* **438** 197–200

K S Novoselov *et al.* 2006 Unconventional quantum Hall effect and Berry's phase of 2π in bilayer graphene *Nature Physics* **2** 177–180

N M R Peres *et al.* 2006 Electronic properties of disordered two-dimensional carbon *Phys. Rev.* B **73** 125411

Y Zhang *et al.* 2005 Experimental observation of quantum Hall effect and Berry's phase in graphene *Nature* **438** 201–204

Correction: There is a trivial but unfortunate slip in algebra at the bottom of page 14.13. In the concluding equality, $E_2/4\pi$ should read $E_2/2\pi$. This gives quantization appropriate to the <u>integer</u> QHE (an important conclusion!)