MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.952: The Early Universe — March 2, 2009
Prof. Alan Guth

PROBLEM SET 3

DUE DATE: Wedneday, March 11, 2009, at 5 pm.

ANNOUNCEMENT: There will be no class on Monday, March 9, as I will be flying back from a conference in Grand Cayman.

PROBLEM 1: DISTANCE TO A GALAXY AT $z = 6.96$ *(10 points)*

In the previous problem set, you calculated the time of emission for the light arriving now from the most distant galaxy with a spectroscopically measured redshift, which is $z = 6.96$. In this problem you are to calculate the distance to the galaxy, using each of the three measures of distance in common use: proper distance, luminosity distance, and angular size distance. Again you should use the WMAP 5-year recommended parameters, with the relativistic matter density based on the COBE 1999 temperature of 2.725 K:

Parameter	**WMAP 5-Year Recommended Fit**
$\boldsymbol{H_0}$	70.5 ± 1.3 km·s^{-1}·Mpc^{-1}
Baryonic matter $\boldsymbol{\Omega_b}$	0.0456 ± 0.0015
Dark matter $\boldsymbol{\Omega_{\rm dm}}$	0.228 ± 0.013
Vacuum energy $\boldsymbol{\Omega_\Lambda}$	0.726 ± 0.015
Relativistic matter $\boldsymbol{\Omega_R}$	8.4×10^{-5}

PROBLEM 2: VELOCITY OF DISTANT GALAXIES *(10 points)*

Consider first a flat universe filled with nonrelativistic matter ($\Omega_M = 1$, $\Omega_\Lambda = \Omega_R = 0$), which is known as the Einstein-de Sitter model. For this model, calculate the recession velocity v as a function of redshift z, where recession velocity v is defined as the present rate of change of the proper distance. At what value of z is the recession velocity $v/c = 1$? In such a universe, would it be possible to observe redshifts larger than this value?

For the realistic model described by the parameters in Problem 1, calculate the velocity of recession v of the galaxy at $z = 6.96$, expressed as a fraction of the speed of light. You should find that the fraction v/c is bigger than one. Calculate, to at least three significant figures, the value of z for which $v/c = 1$.

PROBLEM 3: ANGULAR DIAMETER DISTANCE *(10 points)*

A peculiar feature of the angular diameter distance is that it is not monotonic. It reaches a maximum value as a function of z, and then becomes smaller for larger z. Intuitively, as one looks further out into space one is also looking backwards in time, so a specific object can look bigger because it is being seen in a smaller universe. For the Einstein-de Sitter model, at what value of z does the angular diameter distance achieve its maximum? In terms of the present value H_0 of the Hubble expansion rate, what is the maximum value of the angular diameter distance in this model? For the realistic model described by the parameters given in Problem 1, what is the maximum angular diameter distance, and for what redshift z is it achieved?

PROBLEM 4: SAHA EQUATION *(10 points)*

Set up your computer to numerically solve the Saha equation. For the parameters given in Problem 1, for what temperature T is the equilibrium ionization fraction X equal to one half? To get an idea of how sensitive this result is to the variation of parameters, calculate the temperature that would give 50% equilibrium ionization (i.e., $X = \frac{1}{2}$) (a) if the mass of the electron were twice as large as its real value, or half as large, with the binding energy fixed; (b) if the binding energy of hydrogen were twice as large, or half as large, as its physical value; (c) if the number density of baryons were 10 times as large, or 10 times smaller, than the WMAP 5-year recommended value.