MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.962: General Relativity | February 12, 2018
Prof. Alan Guth

Lecture 1: Wed 02/07/2018*
REVIEW OF SPECIAL RELATIVITY I

1.1) COURSE INFORMATION:

Staff:

	Name	**Room**	**Email**
Lecturer:	Alan Guth	6-322	guth@ctp.mit.edu
Recitations:	Nikhil Raghuram		nikhilr@mit.edu
Teaching Assistant:	Reginald Caginalp		caginalp@mit.edu

Office hours:

Day	**Name**	**Time**	**Room**
Monday:	Alan Guth	5:30–6:30 pm	Room 6-322
Tuesday:	Reggie Caginalp	3:00–4:00 pm	8-320
Wednesday	Nikhil Raghuram	5:00–6:00 pm	8-320

Class Times:

Lecture: Monday and Wednesday, 11:05 – 12:25 pm, Room 56-154
Recitations: Friday, 11:05 – 11:55 am, Room 26-328
Monday, 4:05 – 4:55 pm, Room 26-328

Website: http://web.mit.edu/8.962/www

Currently the site includes Problem Set 1 (due Thursday, February 15, 2018, 5:00 pm, at the Physics Department homework boxes), and also the Complete Lecture Notes from Spring 2017, by Andrew Turner.

Problem Sets:

The course will be graded entirely on the problem sets. There will be no quizzes and no final exam. There will usually be one problem set per week, due Thursdays at 5:00 pm.

No problem set grades will be dropped, since the problem sets will often contain some new material that should not be skipped. To make up for this policy, however, I will be fairly generous with extensions. If you are having a busy week and it is difficult for you to finish the problem set in time, please email me, cc'd to Reggie, letting me know what the situation is, and how much extra time you expect to need.

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* Adapted from lecture notes typed by Andrew Turner in 2017.

Each problem set will be worth a different number of points, and your final numerical grade will be your total score, expressed as a percent of the maximum possible. Thus, problem sets with more points will count more toward your final grade.

You are encouraged to work on the problem sets in groups, discussing the problems, the methods of attack, and the answers. With the right mix of students, this can be an effective way to learn, and can make the problem sets more fun. Nonetheless, it is important for the fairness of the grading system, and for the pedagogical value of the problem sets, that each student write up the solutions independently.

In working the problem sets, you should feel free to consult any publications or web documents. If you consult such documents, you should still, of course, write up the solution in your own words. It is strictly off limits to use solutions written by other 8.962 students, either current or past, or to use solutions that were circulated in earlier years in 8.962.

A homework problem which appears to be copied from another student, from a solution circulated in a previous term, or copied more or less verbatim from some other source (without rewriting in your own words) will be given a reduced grade, possibly a zero. Except in blatant cases, however, students will be given a warning the first time this happens, and will be given an opportunity to redo the relevant solutions. Since the borderline between collaboration and copying is a fine line, and since I want to encourage collaboration, there is nothing that you can do on the homework — in this course — that will lead to an interview with the Committee on Discipline. (Remember, however, that you should not assume that this policy holds in other classes; different professors have different points of view on these issues.)

Textbooks:

The official textbook is

Spacetime and Geometry, by Sean Carroll (Addison Wesley, 2004).

Other very useful books include

Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, by Steven Weinberg (John Wiley & Sons, 1972).

General Relativity, by Robert M. Wald (University of Chicago Press, 1984).

Note that both Carroll's and Weinberg's textbooks grew out of courses given at MIT.

Prerequisites:

Officially, the prerequisites are 18.03 (Differential Equations), 18.06 (Linear Algebra), and 8.07 (Electromagnetism II). Realistically, what I think is really needed is a knowledge of the basics of special relativity, and multivariable calculus (i.e., partial derivatives, divergences, curls, and gradients). Most important of all is the absence of fear of long equations. Differential geometry will be developed as the class progresses, and is not a prerequisite.

1.2) WHY GENERAL RELATIVITY:

General relativity is beautiful, fascinating, and important for a number of reasons:

(1) General relativity is probably the greatest triumph of theoretical reasoning in all of science. When Einstein invented general relativity, he was aware that simultaneity is not well-defined in special relativity, and so Newton's law of gravity is no longer well-defined, since it describes forces that act on one body that are determined by the positions of other bodies at the same time. So Einstein realized that Newton's law of gravity would need to be replaced, but all he knew was that the new theory should be consistent with special relativity, and should reduce to Newton's law in the appropriate limit. In 10 years he developed the theory of general relativity, and the amazing thing is that it has been tested for over a century, with typical accuracies of order 1%, and it has always been found to work.

(2) When I was a graduate student in the 1960's and 70's, general relativity was considered a specialized subfield of physics, rather separate from the mainstream. Now, however, general relativity has become a key part of physics research:

(a) Particle theory (i.e., fundamental physics).

Historically, it was easy to leave gravity out phenomenologically, because it is a factor of about 10^{38} weaker than the other forces. No one has ever detected the gravitational interaction of two elementary particles. So the standard model of particle physics ignored gravity completely. This was acceptable, since gravity is so weak, and it was also necessary, since the standard model is a renormalizable quantum field theory, while a quantum field theory based on general relativity fails to be renormalizable. However, gravity is an integral part of many attempts to extend the standard model of particle physics, such as string theory, the AdS/CFT correspondence, holography, etc.

(b) Cosmology — when I was a graduate student, cosmology was also viewed as a specialized subfield of physics, rather inactive and separated from the mainstream. But with the discovery and precise measurements of the cosmic microwave background, the progress in understanding big bang nucleosynthesis, the accumulation of evidence for dark matter, and the introduction of cosmic inflation, cosmology is now a thriving and active area of research. And of course the language of cosmology is general relativity.

(c) Astrophysics — the physics of black holes and their role in galactic evolution, and the new field of gravitational wave astronomy, have made general relativity an essential part of astrophysics.

1.3) Review of Special Relativity

The basic assumptions of special relativity are:

1) **All laws of physics, including the statement that light travels at a fixed speed c, hold in any inertial coordinate system.**
2) **Any coordinate system that is moving at fixed velocity with respect to an inertial coordinate system is also inertial.**

If an inertial coordinate system is defined as a coordinate system for which particles with no forces on them move at fixed velocities, then the second assumption above is really a tautology. The statements that all laws of physics hold in any inertial frame, and that any frame that moves at fixed velocity with respect to an inertial frame is also an inertial frame are often called *Galilean relativity*. This form of relativity was important to Galileo's world view, because he used it to argue that even though the Earth is moving around the Sun at very high speed, we would not feel the motion.

The statement that the speed of light should always be c might seem somewhat counterintuitive. If we were to jump into a spaceship that could travel at $\frac{1}{2}c$ and chase a light ray, it seems intuitive that we would see the light ray recede from our spaceship at only $\frac{1}{2}c$. Einstein discovered, however, that we can avoid the seeming contradictions by modifying our assumptions about how the observations of one observer are related to those of another observer.

In particular, Einstein discovered that assumptions (1) and (2) above are consistent provided that we take into account three kinematic consequences of special relativity:

(1) Time dilation: A clock moving relative to an inertial frame will "appear" to run slowly by a factor of

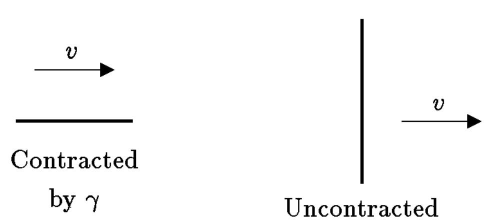
$$\boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}} \ . \tag{1.1}$$

Here, the word "appear" does not mean what one observer sees with his or her eyes; finding what one observer actually sees requires taking into account the light travel time from the point of emission to the observer's eyes. The word "appear" here means that we have already taken out the effect of this light travel time; we imagine that all of space is filled with observers at rest in the same inertial frame, and that these observers have synchronized their clocks in this frame. The synchronization can be carried out, for example, by the observer at the origin sending out a start signal at noon EST on February 7, 2018. Each local observer has measured her distance from the origin, and when the signal arrives, she sets her clock to noon plus the calculated light travel time. The word "appear" is then used to describe observations that are always made locally by members of this family of observers, and later collected to piece together the full picture of what happened.

To see how time dilation follows from the postulates of special relativity, we can consider the thought experiment of a light clock. Consider a clock that, in its rest frame, consists of two stationary parallel mirrors with a measuring rod between them, and a light beam bouncing back and forth between them, just next to the measuring rod. By observing the time it takes the light beam to make one full transit, we can use this setup as a clock. Now suppose that the clock is moving at constant velocity v in a direction perpendicular to the measuring rod. In this frame, the light pulse moves diagonally from one mirror to the other and back. This path is longer than the path in the rest frame of the clock, and so the clock runs slower as viewed from this frame. A simple calculation shows that the moving light-clock runs slowly by precisely the factor γ. We infer that *all* clocks in uniform motion must run slowly by this factor, because the same laws of physics hold in all inertial frames. If, for example, light-clocks appeared to run slowly when they are at rest in a moving inertial frame, but pendulum clocks did not, then the moving frame would not be equivalent to the original stationary frame.

This argument uses the assumption that the speed of light is the same for all observers, as well as the assumption that the separation between the two mirrors is the same in both frames. The latter assumption will be discussed with the Lorentz–Fitzgerald contraction in the next paragraph.

(2) Lorentz–Fitzgerald Contraction: Any rod moving along its length at speed v relative to an inertial frame will "appear" contracted by a factor of γ. There is no contraction along directions perpendicular to the direction of motion.



To see how this effect follows from the original assumptions, we can again consider a thought experiment with a light clock. This time, however, we ask how the speed of the clock is changed if the clock moves at speed v in the direction along the length of the measuring rod. We know that the clock must run slowly by a factor of γ, since we have argued that *all* clocks in uniform motion run slowly by a factor of γ.

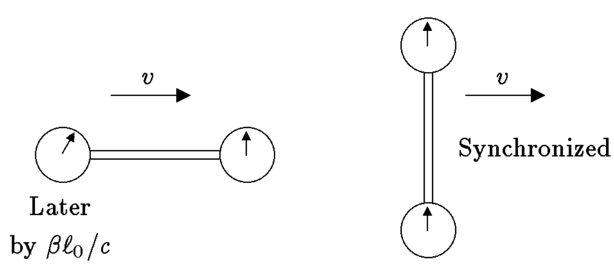
Knowing that the speed of light is fixed at c, however, we can trace the light rays and calculate the speed of the clock directly. If we carried out this calculation, we would find that the clock slows by a factor of γ if and only if its length contracts by exactly a factor of γ.

If a rod moves perpendicular to its length, however, there is no length contraction. This result is required by consistency. Imagine two rods which point in the same direction, and which both have the same rest length. Rod 1 is at rest in our reference frame, and Rod 2 is moving at constant velocity in a direction perpendicular to the length. We can arrange that the two rods pass each other at $t = 0$ in our frame, and we can even arrange that the two centers coincide as they pass. Now suppose that the length of the moving rod, Rod 2, does not appear to be the same as the length of the stationary rod. Suppose, for example, that the moving rod was shorter. In that case the ends of the moving rod will coincide at $t = 0$ with interior points of the stationary rod. If the moving rod had knives mounted at its ends, the knives would cut off the ends of the stationary rod, Rod 1. This cutting would have a permanent effect, so all observers would have to agree that it happened. But if the moving rod cuts the stationary rod, then we can ask how this event would look from the point of view of the rest frame of Rod 2. If all inertial frames are equivalent, observers in the rest frame of Rod 2 would have to see Rod 1 as shorter, and if it had knives at its ends, Rod 2 would have its ends cut off. Analogous contradictions arise if one assumes that moving rods become longer, so the only consistent solution is that motion perpendicular to the length of a rod has no effect on its length.

(3) Relativity of simultaneity: If two clocks that are synchronized in their rest frame are viewed in a frame where they are moving along their line of separation at speed v relative to an inertial frame, the trailing clock will lag by an amount

$$\Delta t = v\ell_0/c^2 = \beta\ell_0/c \; , \tag{1.2}$$

where ℓ_0 is the rest frame separation of the clocks, and $\beta \equiv v/c$.



On Problem Set 1 you will figure out for yourselves how to derive this effect by two simple thought experiments. Both thought experiments involve a train, with a clock at each end, moving on a straight line at relativistic speeds. In one thought experiment, one imagines that the clocks are synchronized by a light signal sent from one end to the other, and in the other thought experiment one imagines that the clocks are synchronized when both are together at one end of the train, and then one clock is carried very slowly to the other end. In both cases, you will show that if the clocks are synchronized by a procedure appropriate for the frame of the moving train, they will appear in the stationary frame to be out of synchronization by exactly the amount shown in Eq. (1.2).

Although it is less talked about than time dilation or Lorentz-Fitzgerald contraction, the relativity of simultaneity is absolutely essential for the consistency of special relativity. If you and I are moving at a uniform velocity relative to each other, your clocks will appear to me to be running slowly, and your rulers will appear contracted. At the same time, however, *my* clocks will appear to you to be running slowly, and *my* rulers will appear to you to contracted. The apparent contradictions in these statements are resolved by the issue of simultaneity. Since you and I will have different ideas about what simultaneity means, the observations described in this paragraph will not actually contradict each other.