8.962 Problem Set 2 Solutions

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1. [5 pts] Show that the number density of dust measured by an observer whose 4-velocity is $\vec{U}$ is given by $n = -\vec{N} \cdot \vec{U}$, where $\vec{N}$ is the matter current 4-vector.

   Solution: Consider the frame in which the observer is at rest. In this frame, we have

$$\vec{U} \doteq (1, 0, 0, 0) \tag{1}$$
$$\vec{N} \doteq \left(n, N^1, N^2, N^3\right), \tag{2}$$

   where n is the number density the observer measures (which is not, in general, the number density of the dust in its rest frame). In this frame, we have

$$n = -\vec{N} \cdot \vec{U}, \tag{3}$$

   but the dot-product is a Lorentz scalar, so this will be true in any frame.

2. [5 pts] Take the limit of the continuity equation for $|\mathbf{v}| \ll 1$ to get $\partial n / \partial t + \partial \left(n v^i\right) / \partial x^i = 0$. We have $\vec{N} = n_0 \vec{U}$, where n_0 is the number density of matter in its rest frame. To first order in the velocity, we have

$$\vec{U} \doteq \left(1 + \mathcal{O}\left(v^2\right), \mathbf{v} + \mathcal{O}\left(v^2\right)\right). \tag{4}$$

   This implies that

$$\partial_\mu N^\mu \approx \frac{\partial n}{\partial t} + \frac{\partial \left(n v^i\right)}{\partial x^i} = 0, \tag{5}$$

   where we have replaced n_0 by n because they are equal to first order in v.

3. In an inertial frame $\mathcal{O}$, calculate the components of the stress-energy tensors of the following systems:

   (a) [5 pts] A group of particles all moving with the same 3-velocity $\mathbf{v} = \beta \vec{e}_x$ as seen in $\mathcal{O}$. Let the rest-mass density of these particles be ρ_0, as measured in their own rest frame. Assume a sufficiently high density of particles to enable treating them as a continuum.

   Solution: We have

$$T^{\mu\nu} = \rho_0 u^\mu u^\nu, \tag{6}$$

where $\vec{u}$ is the four-velocity of the particles in $\mathcal{O}$. The components of $\vec{u}$ are

$$\vec{u} \doteq \left(\frac{1}{\sqrt{1-\beta^2}}, \frac{\beta}{\sqrt{1-\beta^2}}, 0, 0 \right), \tag{7}$$

so T, expressed as a matrix, is

$$T \doteq \rho_0 \begin{pmatrix} \frac{1}{1-\beta^2} & \frac{\beta}{1-\beta^2} & 0 & 0 \\ \frac{\beta}{1-\beta^2} & \frac{\beta^2}{1-\beta^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

(b) [10 pts] A ring of N similar particles of rest mass m rotating counterclockwise in the $x-y$ plane about the origin of $\mathcal{O}$, at a radius a from this point, with an angular velocity ω. The ring is a torus of circular cross-section $\delta a \ll a$, within which the particles are uniformly distributed with a high enough density for the continuum approximation to apply. Do not include the stress-energy of whatever forces keep them in orbit. Part of this calculation will relate ρ_0 of part (a) to N, a, δa, and ω.

Solution: A point at angle θ with respect to the x-axis on the ring moves with four-velocity

$$\vec{u}(\theta) \doteq \left(\frac{1}{\sqrt{1-a^2\omega^2}}, -\frac{a\omega\sin(\theta)}{\sqrt{1-a^2\omega^2}}, \frac{a\omega\cos(\theta)}{\sqrt{1-a^2\omega^2}}, 0 \right). \tag{9}$$

The number density in frame $\mathcal{O}$ is

$$n = \frac{N}{2\pi^2 a \delta a^2}. \tag{10}$$

We know that this is the zero component of the number flux four-vector, which is $n_0\vec{u}$, where n_0 is the number density in the rest frame of the ring. Therefore we have

$$n_0 = \frac{n}{u^0} = \frac{N\sqrt{1-a^2\omega^2}}{2\pi^2 a\delta a^2} \tag{11}$$

As before, we have $T^{\mu\nu} = \rho_0 u^\mu u^\nu = m n_0 u^\mu u^\nu$, so, in matrix form,

$$\begin{aligned} T(\theta) = & \frac{mN}{2\pi^2 a \delta a^2 \sqrt{1-a^2\omega^2}} \\ & \times \begin{pmatrix} 1 & -a\omega\sin\theta & a\omega\cos\theta & 0 \\ -a\omega\sin\theta & a^2\omega^2\sin^2\theta & -a^2\omega^2\sin\theta\cos\theta & 0 \\ a\omega\cos\theta & -a^2\omega^2\sin\theta\cos\theta & a^2\omega^2\cos^2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{12}$$

on the ring, and $T = 0$ elsewhere.

(c) [5 pts] Two such rings of particles, one rotating clockwise and the other counter-clockwise, at the same radius a. The particles do not collide or otherwise interact in any way.

Solution: By linearity, we can simply sum the stress-energy tensors for the two rings. One is exactly the solution we found above, and the other has $\omega \to -\omega$. Adding the two, we find that the energy-flux terms vanish; in matrix form, we get:

$$T(\theta) = \frac{mN}{\pi^2 a\delta a^2\sqrt{1-a^2\omega^2}} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a^2\omega^2\sin^2\theta & -a^2\omega^2\sin\theta\cos\theta & 0 \\ 0 & -a^2\omega^2\sin\theta\cos\theta & a^2\omega^2\cos^2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

on the ring, and $T = 0$ (as before) elsewhere.

4. Use the identity $\partial_\nu T^{\mu\nu} = 0$ to prove the following results for a bounded system (i.e., a system for which $T^{\mu\nu} = 0$ beyond some bounded region of space):

(a) [3 pts] In some inertial frame, $\partial_t \int T^{0\alpha} d^3x = 0$. This expresses conservation of energy and momentum.

Solution: Call our spatial volume of integration V. We choose V large enough that ∂V is in the region of space where $T^{\mu\nu} = 0$. That way, when we apply Gauss' Law, integrals of T over the boundary vanish. We have

$$\partial_t \int_V T^{0\alpha} d^3x = \int_V \partial_t T^{0\alpha} d^3x = -\int_V \partial_i T^{i\alpha} d^3x. \tag{14}$$

In this form, we can apply Gauss' Law (the divergence theorem), to obtain

$$\partial_t \int_V T^{0\alpha} d^3x = -\int_{\partial V} T^{i\alpha} n_i d^2x = 0, \tag{15}$$

where the integral vanishes because T is zero on the boundary of V.

(b) [6 pts] $\partial_t^2 \int T^{00} x^i x^j d^3x = 2\int T^{ij} d^3x$. This result is a version of the virial theorem; it will come in quite handy when we derive the quadrupole formula for gravitational radiation.

Solution: Applying the stress-energy conservation identity, we have

$$\begin{aligned} \partial_t^2 \int_V T^{00} x^i x^j d^3x &= -\partial_t \int_V \left(\partial_k T^{k0}\right) x^i x^j d^3x \\ &= \int_V \left(\partial_k \partial_l T^{kl}\right) x^i x^j d^3x. \end{aligned} \tag{16}$$

But, because T vanishes on ∂V, we know that

$$\int_V \partial_k \left[\left(\partial_l T^{kl} \right) x^i x^j \right] d^3x = 0$$
$$= \int_V \left(\partial_k \partial_l T^{kl} \right) x^i x^j d^3x + \int_V \left(\partial_l T^{kl} \right) \left(\delta^i{}_k x^j + x^i \delta^j{}_k \right) d^3x, \quad (17)$$

so we have

$$\partial_t^2 \int_V T^{00} x^i x^j d^3x = - \int_V \left(\partial_l T^{kl} \right) \left(\delta^i{}_k x^j + x^i \delta^j{}_k \right) d^3x. \quad (18)$$

That was a little opaque. Essentially, we just integrated by parts in 3-D by exploiting Gauss' Law.

Integrating by parts again on equation (18) gives

$$\partial_t^2 \int_V T^{00} x^i x^j d^3x = \int_V T^{kl} \left(\delta^i{}_k \delta^j{}_l + \delta^i{}_l \delta^j{}_k \right) d^3x = 2 \int_V T^{ij} d^3x, \quad (19)$$

which is what we were asked to show.

(c) [6 pts] $\partial_t^2 \int T^{00} (x^i x_i)^2 d^3x = 4 \int T^i{}_i x^j x_j d^3x + 8 \int T^{ij} x_i x_j d^3x$. No pithy wisdom for this one.

Solution: Generalizing from part (b), we know that

$$\partial_t^2 \int_V T^{00} f(\mathbf{x}) d^3x = \int_V T^{ij} \left(\partial_i \partial_j f(\mathbf{x}) \right) d^3x. \quad (20)$$

So, we just need to figure out what $\partial_i \partial_j \left(x^k x_k \right)^2$ reduces to:

$$\begin{aligned} \partial_i \partial_j \left(x^k x_k \right)^2 &= 2\partial_i \left(x^k x_k \left[\delta^k{}_j x_k + \delta_{kj} x^k \right] \right) \\ &= 4\partial_i \left(x^k x_k x_j \right) \\ &= 4\delta_{ij} x^k x_k + 4x_j \left(\delta^k{}_i x_k + \delta_{ki} x^k \right) \\ &= 4\delta_{ij} x^k x_k + 8x_i x_j. \end{aligned} \quad (21)$$

Inserting into our general formula, equation (20), we find

$$\partial_t^2 \int T^{00} (x^i x_i)^2 d^3x = 4 \int_V T^i{}_i x^j x_j d^3x + 8 \int_V T^{ij} x_i x_j d^3x, \quad (22)$$

which is what we were asked to show.

5. The vector potential $\vec{A} \doteq (A^0, \mathbf{A})$ generates the electromagnetic field tensor via

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \; .$$

(a) [5 pts] Show that the electric and magnetic fields in a specific Lorentz frame are given by

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \\ \mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla A^0 \ . \end{aligned}$$

Here, ∇ is taken to be the normal gradient operator in Euclidean space.

Solution: Writing F as a matrix, we have

$$F_{\mu\nu} \doteq \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0. \end{pmatrix} \tag{23}$$

We can write this more compactly as

$$F_{0i} = -E_i \qquad F_{i0} = E_i \qquad F_{ij} = \epsilon_{ijk} B_k. \tag{24}$$

We see (note that $A_0 = -A^0$)

$$F_{0i} = \partial_0 A_i + \partial_i A^0, \tag{25}$$

which implies that

$$\mathbf{E} = -\nabla A^0 - \partial_t \mathbf{A}. \tag{26}$$

Also,

$$F_{ij} = \partial_i A_j - \partial_j A_i = \epsilon_{ijk} B_k \tag{27}$$

is invariant under cyclic permutation of indices (e.g. $i \to j$, $j \to k$ and $k \to i$). When $i = 1$, $j = 2$, $k = 3$ we find that

$$B_z = \partial_x A_y - \partial_y A_x. \tag{28}$$

Cyclic permutations give all the components of

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{29}$$

(b) [5 pts] Show that Maxwell's equations hold if and only if

$$\partial_\mu \partial^\mu A^\alpha - \partial^\alpha \partial_\mu A^\mu = -4\pi J^\alpha \ .$$

Solution: Maxwell's equations consist of two "source-free" equations given by

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \tag{30}$$
$$\nabla \cdot \mathbf{B} = 0, \tag{31}$$

and two "sourced" equations given by

$$\nabla \cdot \mathbf{E} = \rho \tag{32}$$

$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J}. \tag{33}$$

(Never mind the units.) The beauty of the potential formulation is that the source-free equations are automatically satisfied because of the identities $\nabla \times \nabla A_0 \equiv 0$ and $\nabla \cdot \nabla \times \mathbf{A} \equiv 0$. That leaves only the sourced equations to check.

The sourced equations can be written compactly as

$$\partial_\nu F^{\mu\nu} = J^\mu, \tag{34}$$

where the four-current, $J^\mu \doteq (\rho, \mathbf{J})$. Let's check this. For $\mu = 0$, we have

$$\partial_i F^{0i} = \nabla \cdot \mathbf{E} = \rho. \tag{35}$$

For $\mu = i$ we have

$$\partial_t F^{i0} + \partial_j F^{ij} = \nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J}, \tag{36}$$

so it checks. But,

$$\partial_\nu F^{\mu\nu} = \partial^\mu \partial_\nu A^\nu - \partial_\nu \partial^\nu A^\mu = J^\mu, \tag{37}$$

which is exactly what we were asked to prove.

(c) [5 pts] Show that a gauge transformation of the form

$$A_\mu^{\text{new}} = A_\mu^{\text{old}} + \partial_\mu \phi$$

leaves the field tensor unchanged.

Solution: Because partial derivatives commute, we have

$$F_{\mu\nu}^{\text{new}} = F_{\mu\nu}^{\text{old}} + \partial_\mu \partial_\nu \phi - \partial_\nu \partial_\mu \phi = F_{\mu\nu}^{\text{old}}. \tag{38}$$

(d) [5 pts] Show that one can adjust the gauge so that

$$\partial_\mu A^\mu = 0\ .$$

Show that Maxwell's equations take on a particularly simple form with this gauge choice. Use the operator $\Box \equiv \partial_\mu \partial^\mu$ to simplify your result.

Solution: Suppose we are given an $\vec{A}$ which does not satisfy the gauge condition. Then we seek a gauge transformation generator, ϕ, which produces

$$\partial_\mu A'^\mu = \partial_\mu \left(A^\mu + \partial^\mu \phi \right) = 0. \tag{39}$$

Evidently, this ϕ satisfies

$$\partial_\mu \partial^\mu \phi \equiv \Box \phi = -\partial_\mu A^\mu. \tag{40}$$

But this is just the wave equation and (ignoring some subtleties of boundary conditions and topology) we know that we can always find unique solutions to the wave equation.

If $\partial_\mu A^\mu = 0$, then one term in equation (37) drops out, leaving

$$-\Box A^\mu = J^\mu. \tag{41}$$

6. A coordinate system for a uniformly accelerating observer

   (Note: A fairly large amount of background definitions prior to formulating the question itself. You might want to just scan this material initially, then come back to it as you work through the parts of this problem.)

   A former 8.962 student is now an astronaut. She moves through space with acceleration g in the x direction. In other words, her 4-acceleration $\vec{a} = d\vec{u}/d\tau$ (where τ is time as measured on the astronaut's own clock) only has spatial components in the x direction, and is normalized such that

   $$\sqrt{\vec{a}\cdot\vec{a}} = g \ .$$

   This astronaut assigns coordinates $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ as follows:

   First, she defines spatial coordinates to be $(\bar{x}, \bar{y}, \bar{z})$, and sets the time coordinate $\bar{t}$ to be her own proper time. She defines her position to be $(\bar{x} = g^{-1}, \bar{y} = 0, \bar{z} = 0)$ (not a unique choice, but a convenient one). Note that she remains *fixed* with respect to these coordinates — that's the point of coordinates for an accelerated observer!

   Second, at $\bar{t} = 0$, the astronaut chooses $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ to coincide with the Euclidean coordinates (t, x, y, z) of the inertial reference frame that momentarily coincides with her motion. In other words, though the astronaut is not inertial, there is an inertial frame that, at $\bar{t} = 0$, is momentarily at rest with respect to her. This is the frame used to assign $(\bar{x}, \bar{y}, \bar{z})$ at $\bar{t} = 0$. The clocks of that frame are set such that they are synchronized with her clock at that moment.

   Observers who remain at fixed values of the spatial coordinates $(\bar{x}, \bar{y}, \bar{z})$ are called coordinate-stationary observers (CSOs). Note that the CSOs are also accelerated observers, though not necessarily accelerating at the same rate as the astronaut. The astronaut requires the CSO worldlines to be orthogonal to hypersurfaces $\bar{t} =$ constant. She also requires that for each $\bar{t}$ there exists some inertial frame, momentarily at rest with respect to the astronaut, in which all events with $\bar{t} =$ constant are simultaneous. The accelerated motion of the astronaut can thus be described as movement through a sequence of inertial frames which momentarily coincide with her motion.

   It is easy to see that $\bar{y} = y$ and $\bar{z} = z$; henceforth we drop these coordinates from the problem.

(a) [5 pts] What is the 4-velocity $\vec{u}$ of the astronaut, as a function of $\bar{t}$, as measured by CSOs in the initial inertial frame [the frame that uses coordinates (t, x, y, z)]? (Hint: by considering the conditions on $\vec{u} \cdot \vec{u}$, $\vec{u} \cdot \vec{a}$, and $\vec{a} \cdot \vec{a}$, you should be able to find simple forms for u^t and u^x.) After you have worked out $\vec{u}$, compute $\vec{a}$.

Solution: We know that $u^2 = -1$, $a^2 = g^2$, and $\vec{u} \cdot \vec{a} = 0$. Writing these in terms of components of $\vec{u}$, we find

$$(u^x)^2 = \left(u^t\right)^2 - 1 \tag{42}$$

$$\left(\frac{du^x}{d\tau}\right)^2 = \left(\frac{du^t}{d\tau}\right)^2 + g^2 \tag{43}$$

$$u^t \frac{du^t}{d\tau} = u^x \frac{du^x}{d\tau}. \tag{44}$$

These equations reduce to

$$\frac{du^x}{d\tau} = g\sqrt{1 + (u^x)^2}, \tag{45}$$

which has the solution with initial condition $u^x(0) = 0$ (initially the accelerated axes agree with (t, x), so the motion is purely in the time direction)

$$u^x = \sinh(g\tau) = \sinh(g\bar{t}), \tag{46}$$

which implies that

$$u^t = \cosh(g\bar{t}). \tag{47}$$

Given these results, we have

$$a^t = g\sinh(g\bar{t}) \qquad a^x = g\cosh(g\bar{t}). \tag{48}$$

(b) [5 pts] Integrate up this 4-velocity to find the position $[T(\bar{t}), X(\bar{t})]$ of the astronaut in the coordinates (t, x). Recall that at $t = \bar{t} = 0$, $X = \bar{x} = g^{-1}$. Sketch the astronaut's worldline on a spacetime diagram in the coordinates (t, x). You will return to and augment this sketch over the course of this problem, so you may want to do this on a separate piece of paper (and/or clean it up before handing in your assignment).

Solution: Integrating, we obtain

$$T(\bar{t}) = \int_0^{\bar{t}} \cosh(gs)ds = \frac{1}{g}\sinh(g\bar{t}) \tag{49}$$

and

$$X(\bar{t}) = \int_0^{\bar{t}} \sinh(gs)ds + \frac{1}{g} = \frac{1}{g}\cosh(g\bar{t}). \tag{50}$$

Figure 1 shows a plot of this trajectory.

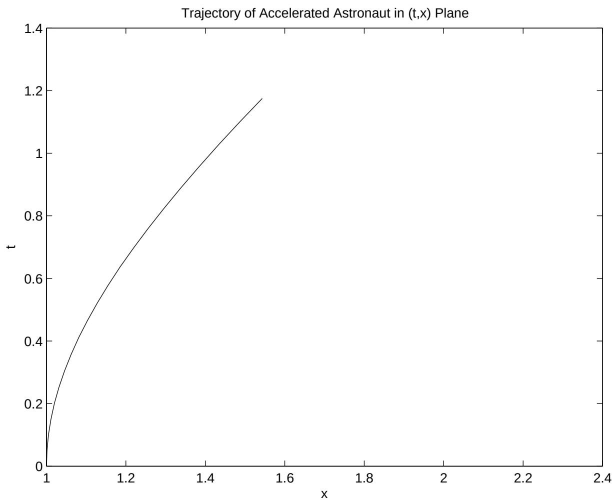

Figure 1: The trajectory of an accelerated observer who accelerates with $a^2 = 1$ and begins at the point $(0, 1)$ in the (t, x) plane.

(c) [5 pts] Find basis vectors $\vec{e}_{\bar{t}}$ and $\vec{e}_{\bar{x}}$ describing the momentarily inertial coordinate system at some time $\bar{t}$. **Hint**: In lecture, I noted that a body with a 4-velocity $\vec{u}$ has a very natural basis for the timelike direction. Now find an orthogonal vector in this problem; it will serve as $\vec{e}_{\bar{x}}$ (possibly modulo normalization). Add these vectors to the sketch of your worldline.

We now "promote" $\bar{t}$ to a coordinate (i.e., give it meaning not just on the astronaut's worldline, but everywhere in spacetime) by requiring that $\bar{t} =$ constant be a surface of constant time in the Lorentz frame in which the astronaut is instantaneously at rest.

Solution: A good vector for $\vec{e}_{\bar{t}}$ is $\vec{u}$. We want then to find $\vec{e}_{\bar{x}}$ such that

$$\vec{e}_{\bar{t}} \cdot \vec{e}_{\bar{x}} \quad \text{and} \quad \vec{e}_{\bar{x}} \cdot \vec{e}_{\bar{x}} = 1. \tag{51}$$

These equations tell us that we must have

$$e^t_{\bar{x}} = \sinh(g\bar{t}) \quad \text{and} e^x_{\bar{x}} = \cosh(g\bar{t}). \tag{52}$$

Note that $e_{\bar{x}}$ is parallel to the acceleration, $\vec{a}$ (since we're in two dimensions, all vectors perpindicular to $\vec{u}$ are parallel). Figure 2 shows these basis vectors attached to the trajectory from part (b).

(d) [5 pts] By noting that this "surface" must be parallel to $\vec{e}_{\bar{x}}$ and that it must pass through the point $[T(\bar{t}), X(\bar{t})]$, show that it is defined by the line

$$x = t \coth g\bar{t} \ .$$

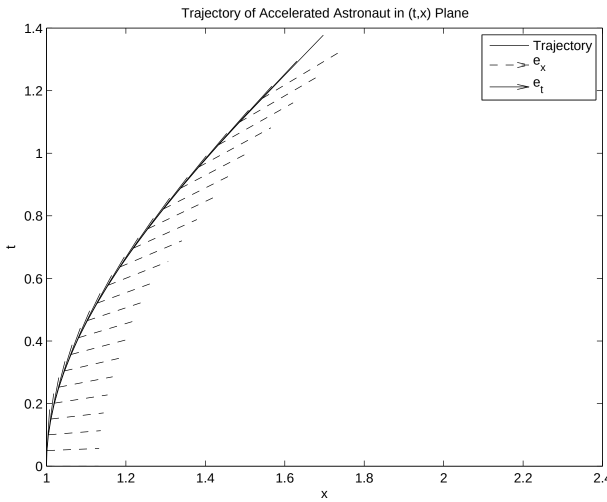

Figure 2: Trajectory and basis vectors for an accelerated observer who begins at the point $(0, 1)$ in the (t, x) plane and accelerates with $a^2 = 1$.

In other words, it's just a straight line going through the origin with slope $\coth g\bar{t}$.

We've now defined the time coordinate $\bar{t}$ that the astronaut uses to label spacetime. Next, we need to come up with a way to set her spatial coordinates $\bar{x}$.

Solution: To be parallel to $\vec{e}_{\bar{x}}$, we require that the surface be a line with slope $m \equiv e^t_{\bar{x}}/e^x_{\bar{x}} = \tanh(gt)$. The unique line with this slope which passes through the point $(T(\bar{t}), X(\bar{t}))$ is given by the equation

$$t = \tanh(g\bar{t})x \quad \text{or} \quad x = \coth(g\bar{t})t. \tag{53}$$

These are the surfaces of constant $\bar{t}$. Up to this point, the vectors $\vec{e}_{\bar{t}}$ and $\vec{e}_{\bar{x}}$ lived only on the trajectory. We have now extended $\vec{e}_{\bar{t}}$ to the entire spacetime (it is everywhere perpindicular to surfaces of constant $\bar{t}$, which we just defined). It has now become a *vector field.*

(e) [5 pts] Recalling that the CSOs must themselves be accelerated observers, argue that their worldlines are hyperbolae[1], and thus that a CSO's

[1]If your sketch of the astronaut's worldline isn't a hyperbola, this would be a good moment to revise your solution!

position in (t, x) must take the form

$$t = \frac{A}{g} \sinh g\bar{t} \,,$$
$$x = \frac{A}{g} \cosh g\bar{t} \,.$$

From the initial conditions, find A.

Solution: There was nothing special about our derivation of the trajectory in part (b) except the initial condition that $x = 1/g$ when $t = \bar{t} = 0$. For an accelerated observer with $x = \bar{x}$ when $t = \bar{t} = 0$ (these are the CSOs in question), we have

$$t = \bar{x} \sinh(g\bar{t}) \quad \text{and} \quad x = \bar{x} \cosh(g\bar{t}). \tag{54}$$

(You can verify this by noting that it solves the differential equations in (b) and, when specialized to the initial condition in (b) reproduces the result we derived there.) We see that $A = g\bar{x}$. Now that we have the complete coordinate transformation between the barred and un-barred coordinates, we have extended both $\vec{e}_{\bar{t}}$ and $\vec{e}_{\bar{x}}$ to be vector fields on spacetime.

(f) [10 pts] Show that the line element $ds^2 = d\vec{x} \cdot d\vec{x}$ in the new coordinates takes the form

$$ds^2 = -dt^2 + dx^2 = -(g\bar{x})^2 d\bar{t}^2 + d\bar{x}^2.$$

This is known as the *Rindler metric*[2]. As this exercise illustrates, it is just the flat spacetime of special relativity; but, it is expressed in coordinates that introduce some features that are very important in general relativity. We will return to this spacetime in later exercises.

Solution: We have the differentials of the coordinate transformation from $(\bar{t}, \bar{x})$ to (t, x) given in (e):

$$dt = \sinh(g\bar{t}) d\bar{x} + g\bar{x} \cosh(g\bar{t}) d\bar{t} \tag{55}$$
$$dx = \cosh(g\bar{t}) d\bar{x} + g\bar{x} \sinh(g\bar{t}) d\bar{t}, \tag{56}$$

from which

$$ds^2 = -dt^2 + dx^2 = -\left(g\bar{x}\right)^2 d\bar{t}^2 + d\bar{x}^2 \tag{57}$$

[2]Rindler often appears in textbooks with $(1 + g\bar{x})^2$ rather than $(g\bar{x})^2$. The difference amounts to an uninteresting shift of the origin.