When appropriate, the relevant equations for Problem 1 of Chapter 2, Problem 1.2, and Problem 2 of Chapter 3 are provided.

PROBLEM 1: COSMIC STRINGS

(a) [12 pts] Solve the vacuum equations and find the metric around the massive source assuming that the Ricci curvature tensor vanishes.

(b) [4 pts] Use a change of coordinates to put the metric in the form

(c) [6 pts] Consider the metric on a spatial slice (let $r$ be the spatial coordinates) that depends on the source field $A(r)$. The metric is

(d) [12 pts] The metric is

ANSWER:

PROBLEM 2: INTEGRAL STARBURST

The metric is

PROBLEM 3: CLASSICAL STRINGS

The metric is

PROBLEM 4: COSMIC STRINGS

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PROBLEM 5: INTEGRAL STARBURST

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PROBLEM 6: CLASSICAL STRINGS

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PROBLEM 7: COSMIC STRINGS

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PROBLEM 8: INTEGRAL STARBURST

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PROBLEM 10: COSMIC STRINGS

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PROBLEM 11: INTEGRAL STARBURST

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PROBLEM 13: COSMIC STRINGS

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PROBLEM 69: CLASSICAL STRINGS

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PROBLEM 70: COSMIC STRINGS

The metric is

DUE DATE: Thursday, May 3, 2018, at 5:00 pm.
would be unavoidable. However, as long as the particles are spread out, then a local
area approximation is quite reasonable, thus suggesting the inequality

\[ \frac{1}{2} \left( \sum_{j=1}^{N} A_{j} \right) + \sum_{1 \leq i < j \leq N} A_{ij} = \sum_{j=1}^{N} \mathcal{A}_{j} \]

1. (a) For this deficit angle, the metric can be written

\[ ds^2 = \left( 1 - \frac{2}{\sqrt{r} \rho} \right) \left( dr^2 + r^2 d\theta^2 \right) + \left( 1 - \frac{2}{\sqrt{r} \rho} \right)^{-1} \left( dx^2 + dy^2 + dz^2 \right) \]

2. (b) In two dimensions these rotation angles simplify considerably. The components of the nonzero components of the Riemann tensor are

\[ R_{\theta \theta} = -\frac{\kappa^2}{\rho^4} \]

3. (c) Given that parallel transport counterclockwise around the source produces

\[ \frac{1}{2} \left( \sum_{j=1}^{N} A_{j} \right) + \sum_{1 \leq i < j \leq N} A_{ij} = \sum_{j=1}^{N} \mathcal{A}_{j} \]

4. (d) For this deficit angle, the metric can be written

\[ ds^2 = \left( 1 - \frac{2}{\sqrt{r} \rho} \right) \left( dr^2 + r^2 d\theta^2 \right) + \left( 1 - \frac{2}{\sqrt{r} \rho} \right)^{-1} \left( dx^2 + dy^2 + dz^2 \right) \]

5. (e) Finally, the nonzero components of the Riemann tensor are

\[ R_{\theta \theta} = -\frac{\kappa^2}{\rho^4} \]

\[ \frac{1}{2} \left( \sum_{j=1}^{N} A_{j} \right) + \sum_{1 \leq i < j \leq N} A_{ij} = \sum_{j=1}^{N} \mathcal{A}_{j} \]

\[ 1. (a) \quad \frac{1}{2} \left( \sum_{j=1}^{N} A_{j} \right) + \sum_{1 \leq i < j \leq N} A_{ij} = \sum_{j=1}^{N} \mathcal{A}_{j} \]

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(13.1) \[ x_\rho Y (\rho) \Lambda = \varepsilon \] reproduces the previous result.

(13.1) \[ x_\rho Y (\rho) \Lambda = \varepsilon \]

which leads to

(13.1) \[ x_\rho Y (\rho) \Lambda = \varepsilon \]

where \( \delta \) is antisymmetric with \( \xi = \varepsilon \). To check the normalization of \( x_\rho Y (\rho) \Lambda \), we find

(13.1) \[ x_\rho Y (\rho) \Lambda = \varepsilon \]

\( \delta \Lambda = 0 \) \( \varepsilon \) is the total curvature.

(13.1) \[ x_\rho Y (\rho) \Lambda = \varepsilon \]

\( x_\rho Y (\rho) \Lambda = \varepsilon \) written in terms of the Riemann tensor. This can be computed in a 2D space only for independent components of the Riemann tensor, namely \( R_{ij} \), where \( g \) is the genus of the number of handles. Note that the Euler characteristic \( \chi \) of the compactification matrix, one sees that the rotation angle can be written as

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\[ (2.2) \quad 0 = \nabla_{\sigma} Y^\sigma = \frac{\Delta}{2} = \nabla^{\sigma} Y_{\sigma} \]

For any vector field \( \sigma \), which vanishes since the first factor is antisymmetric in two terms. The third term vanishes, for example, because \( \nabla_{\rho} \nabla_{\rho} Y^{\sigma} \) would have to be symmetric in its two indices (Carroll Eq. (3.133), p. 127):

\[ (2.3) \quad \nabla_{\rho} \nabla_{\rho} Y^{\sigma} = \nabla^{\rho} Y_{\rho} \]

The second proof is to observe that if \( \nabla_{\rho} \nabla_{\rho} Y^{\sigma} \) vanishes, then Killing's equation reduces to the statement that the metric is independent of \( \sigma \).

The right-hand side can be further simplified by using the symmetries of the Riemann tensor, and then we use the Bianchi identity (Carroll Eq. (3.134), p. 128) to rewrite the expression so that two indices are interchanged and we have:

\[ (2.4) \quad \nabla_{\rho} \nabla_{\rho} Y^{\sigma} = \nabla^{\rho} Y_{\rho} \]

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\[ (2.5) \quad \nabla_{\rho} \nabla_{\rho} Y^{\sigma} = \nabla^{\rho} Y_{\rho} \]

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\[ (2.6) \quad \nabla_{\rho} \nabla_{\rho} Y^{\sigma} = \nabla^{\rho} Y_{\rho} \]
since there are only 10 independent Killing equations, but it is useful to tabulate
where primes denote derivatives with respect to $\mu$. These equations are redundant,

$$0 = \partial_\rho u^\rho$$

Thus there are 10 Killing vectors in all, corresponding to what is called the Poincaré

$$\mu, \nu, \rho, \sigma$$

$$\partial_\rho u^\rho = \partial_\mu u^\mu = \partial_\nu u^\nu = \partial_\sigma u^\sigma = 0$$

We can combine the Killing vector fields as follows, since $\partial$ is not in the Killing equation

$$(\partial \cdot 0 \cdot 0 \cdot x \cdot x) = \delta^{(10)}_{20} M \iff 1 = \partial \cdot 0 \cdot \delta$$

are the vector indices, so the left hand sides are the labels for the 10 Killing vectors, while the indices without parentheses are the labels for the 10 Killing vectors in the shorthand space. We use a notation where indices in

$$\alpha = \mu, \nu, \rho, \sigma$$

...
The Killing vector can be found by exactly the same method as the fourth, but
without the direction used on the x-direction. The equation in
square brackets is interpreted as the standard coordinate system
as in the preceding problem. Thus, a Killing vector is a vector
parallel to the Killing vector field, and can be determined by
the Killing equation. This equation tells us that the Killing
vector is a vector field that is invariant under the Lie derivative
of the metric tensor by the Killing vector field. Thus, we can
write the Killing equation in the form
\[ \nabla_a \nabla_b \xi^a = 0. \]

The Killing vector is written as
\[ \xi^a = (\xi_x, \xi_y, \xi_z). \]

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The solutions correspond to the terms proportional to $\phi$, $G_{a}^{b}$, and $F_{a}$.

\[ \phi + (n)\delta \phi = \phi \]
\[ \phi + (n)\delta \phi = \phi \]
\[ \delta \phi + x\delta \phi + q\phi = \phi \]
\[ 0 = \phi \]

Thus, the general solution is constrained to have the form

\[ 0 = \phi = \phi \]

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The Killing vector $\mathbf{\kappa}$ is a vector field with the property that an infinitesimal coordinate transformation is induced by the flow of $\mathbf{\kappa}$, as defined in lecture.

This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric. This is the form that we obtained in lecture by starting with the definition that a Killing vector is a vector field whose flow preserves the metric.

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Further Additional Discussion: Why should we believe this? We would like to show there is a solution in which $f$ is constant and $g$ is an arbitrary function. Let's start by considering the Killing equation:

\[ (\partial x, \partial y) \partial v = (\partial x, \partial y) C \partial v = 0 \]

This implies that $f$ is constant and $g$ is an arbitrary function.

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The volume of the closed universe is a function of $t$, but the solution is a closed expression in terms of $t$ as well as in terms of the radius $a$ and the scale factor $\rho$. The final integral can be done by using:

$$\int_0^\infty \frac{\rho^2}{\sin \phi} \, d\rho = \frac{\pi}{2}.$$

The result for a FRW cosmology, where the closed universe, which we will learn

\[ PROBLEM 4: THE VOLUME OF A CLOSED UNIVERSE \]

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