Massachusetts Institute of Technology
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Tips and addenda for Problem Set 2

1. In this problem, you will need to compute quantities that look like $(x'_i x'_j x'_k)^{\rm STF}$ and $(\epsilon_{ikm} x'_j x'_k v_m)^{\rm STF}$. Since the STF technique is new to just about everyone, here is the octupole in STF form:

$$(x'_i x'_j x'_k)^{\rm STF} = x'_i x'_j x'_k - \frac{r^2}{5}\left(x'_i \delta_{jk} + x'_j \delta_{ki} + x'_k \delta_{ji}\right) \ .$$

This is clearly symmetric under exchange of any pair of indices; you should be able to verify that the trace on any pair of indices is zero.

Hopefully the STF mass quadrupole is easy! Also, you should find that, after symmetrizing on the free indices, the current quadrupole term is already trace free.

For the sake of uniformity in your solutions, please describe the binary as residing in the $x-y$ plane. Hence, for the mass $m_1$, we should have

$$x_i = \frac{m_2 R}{m_1 + m_2}\left(\cos\Omega t, \sin\Omega t, 0\right) \ .$$

For mass $m_2$, you should clearly have $-m_1/m_2$ times the position for $m_1$.

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It should become rapidly clear that the solution for the gravitational waveforms can be rather messy if you want it expressed as a function of general angle. When examining the relative contributions of the different multipoles to the total waveform, let's simplify things by considering only two particular positions:

- The equatorial plane: $\theta = \pi/2$, $\phi = 0$, so $n_i = (1, 0, 0)$;
- The "north" pole: $\theta = 0$, so $n_i = (0, 0, 1)$.

Don't forget that when you integrate for total energy carried by the radiation, you cannot just a pick a convenient point! However, if you leave your expressions general and integrate over the sphere using the tips provided, you shouldn't find this calculation to be too challenging.

Also, *please* use some kind of algebraic package like Mathematica as much as possible to evalute these tensors and work out the waveform! It took me about an hour to write a Mathematica notebook that solves problem 1. Doing it by hand would have driven me mad. Feel free to ask me for tips for how to automate the calculation.