1.3 Second order ordinary differential equations

Problem 1.22: Find values of k so that $y = e^{kx}$ is a solution of:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$$

Hence state the general solution.

Problem 1.23: Find the general solution of: $\frac{d^2y}{dx^2} + 4y = 0$

Problem 1.24: Given $ay'' + by' + cy = 0$, write down the auxiliary equation. If the roots of the auxiliary equation are complex (one root will always be the complex conjugate of the other) and are denoted by $k_1 = \alpha + \beta\mathrm{i}$ and $k_2 = \alpha - \beta\mathrm{i}$ show that the general solution is:

$$y(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

Problem 1.25: Find the auxiliary equation for the differential equation $L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$ Hence write down the general solution.

Problem 1.26: Use a computer algebra program such as mathematica to solve the ODE subject to the given conditions

$$y''(x) = y'(x)\cos(xy(x)), \qquad y(0) = 1, y'(1) = 0$$

and plot the result for y(x) over the range $-5 \leq x \leq 5$.

Problem 1.27:

1. Write down the general solution for the differential equation

$$\frac{d^2y}{dt^2} + y = \cos rt$$

and by substituting a suitable trial function, determine a particular integral for $r \neq 1$.

2. Express the complete solution to this equation in terms of the initial values $y(0)$ and $\dot{y}(0)$

3. By taking the limit of $r \to 1$ of the complete solution, determine the solution of this equation for $r = 1$.

4. Show that the same result follows if a trial solution $y_p = t(A\cos t + B\sin t)$ is used to determine the particular integral.

Problem 1.28: Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = (1+x)e^{-x}; \quad y(0) = 0, \quad y'(0) = 1$$

by solving first for the general solution of the homogenous equation, and looking for an appropriate trial function to obtain a particular integral.

Problem 1.29: Show Eq. (1.3.6). What are the initial position and velocity of the solution in (1.3.6)

Problem 1.30: Generalise the discussion of beats to the case where one tuning fork has twice the amplitude of the other.

Problem 1.31: Plot and discuss the position, velocity and acceleration of the solution $x = \tanh(t)$ in ()1.3.18).