

### 3.3 Fourier analysis

**Problem 3.6:** Show the following relations

$$\begin{aligned}\int_{-\pi}^{\pi} \sin nt \cos mtdt &= 0 \\ \int_{-\pi}^{\pi} \cos nt \cos mtdt &= \begin{cases} 0 & n \neq m \\ \pi & n = m \neq 0 \\ 2\pi & n = m = 0 \end{cases} \\ \int_{-\pi}^{\pi} \sin nt \sin mtdt &= \begin{cases} 0 & n \neq m, n = m = 0 \\ \pi & n = m \end{cases}\end{aligned}$$

**Problem 3.7:** Find the Fourier series for the following examples

1.  $f(x) = |x|, -\pi < x < \pi$
2.  $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$
3.  $f(x) = 1 + x$  on  $[-\pi, \pi]$
4.  $f(x) = 1 + \sin^2 t$
5.  $f(x) = \begin{cases} 1 & -1 \leq x < 0 \\ \frac{1}{2} & x = 0 \\ x & 0 < x \leq 1 \end{cases}$  on  $[-1, 1]$
6.  $f(x) = \begin{cases} -1 & -3 \leq x < 0 \\ 1 & 0 < x \leq 3 \end{cases}$  on  $[-3, 3]$
7.  $f(x) = x^2$  on  $[-\pi, \pi]$
8.  $f(x) = f(x + 2), f(x) = (x - 1)(x - 3)$  on  $[1, 3]$ .
9.  $f(x) = x$  on  $[0, 1]$ .
10.  $f(t) = \begin{cases} \frac{4}{\pi}t & 0 \leq t < \frac{\pi}{2} \\ \frac{-4}{\pi}t & \frac{-\pi}{2} \leq t \leq 0 \end{cases}$