3.6 Dynamics of vector fields

Problem 3.19: *Maxwell's Equations:* Maxwell's equations relate the electric and magnetic fields $\vec{E}$ and $\vec{B}$, to the charge and current densities ρ and $\vec{j}$, by the differential equations

$$\begin{cases} \nabla \cdot \vec{E} = \dfrac{\rho}{\epsilon_0} \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j} \\ \nabla \cdot \vec{B} = 0 \end{cases} .$$

1. Show that these equations imply the following relation between ρ and $\vec{j}$,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0,$$

which represents the conservation of charge.

2. Write down explicitly the 8 equations for $\vec{E} = (E_r, E_\theta, E_z)$ and $\vec{B} = (B_r, B_\theta, B_z)$ in cylindrical coordinates (r, θ, z).

3. Write down explicitly the 8 equations for $\vec{E} = (E_r, E_\theta, E_\phi)$ and $\vec{B} = (B_r, B_\theta, B_\phi)$ in spherical coordinates (r, θ, ϕ).