4.2 Continuous random variable

1. *Characteristic functions:* Calculate the characteristic function, the mean, and the variance of the following probability density functions:

   (a) *Uniform* $p(x) = \frac{1}{2a}$ for $-a < x < a$, and $p(x) = 0$ otherwise;

   (b) *Laplace* $p(x) = \frac{1}{2a} \exp\left(-\frac{|x|}{a}\right)$;

   (c) *Cauchy* $p(x) = \frac{a}{\pi(x^2+a^2)}$.

   The following two probability density functions are defined for $x \geq 0$. Compute only the mean and variance for each.

   (d) *Rayleigh* $p(x) = \frac{x}{a^2} \exp(-\frac{x^2}{2a^2})$,

   (e) *Maxwell* $p(x) = \sqrt{\frac{2}{\pi}} \frac{x^2}{a^3} \exp(-\frac{x^2}{2a^2})$.

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2. *Diode:* The current I across a diode is related to the applied voltage V via $I = I_0 \left[\exp(eV/kT) - 1\right]$. The diode is subject to a random potential V of zero mean and variance σ^2 which is Gaussian distributed. Find the probability density $p(I)$ for the current I flowing through the diode. Find the most probable value for I, the mean value of I, and indicate them on a sketch of $p(I)$.

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3. *Tchebycheff inequality:* Consider any probability density $p(x)$ for $(-\infty < x < \infty)$, with mean λ, and variance σ^2. Show that the total probability of outcomes that are more than $n\sigma$ away from λ is less than $1/n^2$, i.e.

$$\int_{|x-\lambda|\geq n\sigma} dx p(x) \leq \frac{1}{n^2}.$$

   Hint: Start with the integral defining σ^2, and break it up into parts corresponding to $|x - \lambda| > n\sigma$, and $|x - \lambda| < n\sigma$.

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