## 5.2 Continuum limit

1. *The Moran process,* named after Patrick Moran, is a simple method for modeling a population of constant size. At each step one individual from the population is randomly selected for duplication/reproduction, and another for elimination/death, thus maintaining a fixed size.

   (a) For a haploid population of size $N$, with one locus and two alleles $A_1$ and $A_2$, compute the changes $\langle \Delta N_1 \rangle$ and $\langle \Delta N_1^2 \rangle$ in number of individuals with allele $N_1$ after one step.

   (b) Construct the drift-diffusion equation for this model, assuming that $N/2$ steps of the Moran process correspond to one generation time.

   (c) How would you modify the process to implement differing fitness values for the two alleles?

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2. *Treadmilling Actin:* Actin filaments are long, asymmetric, polymers involved in a variety of cellular functions. In some cases the filaments are in a dynamic state in which monomers are removed from one end and added to the other. (The two ends are called minus and plus respectively, and this process is known as treadmilling.)

   (a) Assume that monomers are added to the plus-end at rate $a$, and removed from the minus end at rate $b$. Write down the equations governing the rate of change of the probabilities $\{p(\ell, t)\}$, for finding a filament of length $\ell$ at time $t$. Note that $\ell = 1,\ 2,\ ,3, \cdots$, and that the equation of $p(1, t)$ is different from the rest.

   (b) It is possible to have a dynamic steady state with probabilities $p^*(\ell)$ that do not change with time. Find the (properly normalized) distribution $p^*(\ell)$ in such a case.

   (c) What is the condition for the existence of a time independent steady state, and the mean length of the filament in such a case?

   (d) For $a > b$, what is the average length of a filament at time $t$, starting from individual monomers at time $t = 0$? Calculate the fluctuations (variance) in length, and write down an approximate probability distribution $p(\ell, t)$ with the correct first and second moment.

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3. *Activation/deactivation reaction:* Many molecules in biology can be made active or inactive through the addition of a phosphate group. The enzyme that adds the phosphate group is usually termed a kinase, while a phosphatase removes this group. Let

us consider a case where a finite number $N$ of such molecules within a cell can be exchanged between the two forms at rates $a$ and $b$, i.e.

$$\mathrm{A} \rightleftharpoons_b^a \mathrm{B} \quad ,$$

where we have folded the probabilities to encounter the enzymes in the reaction rates.

(a) Write down the Master equation that governs the evolution of the probabilities $p(N_A = n, N_B = N - n, t)$.

(b) Assuming that initially all molecules are in state A, i.e. $p(n, t = 0) = \delta_{n,N}$, find $p(n, t)$ at all times. You may find it easier to guess the solution, but should then check that it satisfies the equations obtained before.

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