5.3 Brownian motion

1. *Foraging:* Typical foraging behavior consists of a random search for food, followed by a quick return to the nest. For this problem, assume that the nest is at the origin, and the search consists of a random walk *in two dimensions* around the nest.

   (a) Modeling the search as a random walk with diffusion constant D, what is the probability density for the searcher to be a distance r from the nest, at a time t after leaving the nest?

   (b) Assume that durations of search segments are exponentially distributed, i.e. with probability $p(t) \propto e^{-t/\tau}$. Further assume that the times spent in returning to the nest, and stay at nest between searches, are negligible compared to search times. After times much longer than τ, what is the probability to find the searcher at a distance r from the nest. Use saddle-point integration to find the asymptotic probability for large r.

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2. *Chemotaxis:* The motion of *E. Coli* in a solution of nutrients consists of an *alternating* sequence of *runs* and *tumbles*. During a run the bacterium proceeds along a straight line for a time t_r with a velocity v. It then tumbles for a time t_t, after which it randomly chooses a new direction $\hat{n}$ to run along. Let us assume that the times t_r and t_t are independently selected from probability distributions

$$p_r\left(t_r\right) = \frac{4t_r}{\tau_r^2}\exp\left(-\frac{2t_r}{\tau_r}\right) \quad , \quad \text{and} \quad p_t\left(t_t\right) = \frac{4t_t}{\tau_t^2}\exp\left(-\frac{2t_t}{\tau_t}\right) \quad .$$

   (a) Assuming values of $\tau_r \approx 2$s, $\tau_t \approx 0.2$s, and $v \approx 30\mu\text{ms}^{-1}$, calculate the diffusion coefficient D for the bacterium at long times.

   (b) In the presence of a chemical gradient the run times become orientation dependent, and are longer when moving in a favorable direction. For preferred motion up the z axis, let us assume that the average run time depends on its orientation $\hat{n}$ according to $\tau_r\left(\hat{n}\right) = \tau_0 + g\hat{n}\cdot\hat{z}$. Calculate the average drift velocity at long times.

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