**Recap**

- On a finite interval of size $L$, $\sin(2n\pi x/L)$ and $\cos(2n\pi x/L)$ for $n = 0, 1, 2, \cdots$ form a complete basis, in terms of which any function $f(x)$ on the interval can be decomposed in a Fourier series.

- Along the infinite line $\sin(kx)$ and $\cos(kx)$ (for $k \geq 0$), or $e^{ikx}$ (for real $k$) acts as a basis, for Fourier decomposition:

$$f(x) = \int_{-\infty}^{\infty} dx e^{ikx} \tilde{f}(k)\,, \quad \text{and} \quad \tilde{f}(k) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{-ikx} f(x)\,. \tag{3.3.37}$$

- The orthogonality condition of Fourier modes is expressed in terms of the Dirac delta-function, as

$$\int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(x-x')} = \delta(x - x')\,, \tag{3.3.38}$$