### 3.4.6 Planar and Circular Travelling Waves

One can also generate travelling waves on a soap film, or (more easily) on the surface of water, although the dispersion relation in the latter case is more complicated. The simplest type of solution varies only in one direction, e.g.

$$h(x,y,t) = A\cos(kx-\omega t+\theta)\,, \tag{3.4.35}$$

and is effectively one dimensional. These solutions also exist in three dimension and are called *plane waves*.

More interesting are traveling solutions with circular symmetry. It is indeed easy to verify that

$$h(r,t) = AJ_0\left(kr-\omega t\right)\,, \tag{3.4.36}$$

is a solution to

$$\frac{1}{v^2}\frac{\partial^2 h}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) = \frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\,, \tag{3.4.37}$$

provided that $\omega = kv$. This solution corresponds to a wave propagating out from the center. At small patch of this wave-from at large distances $r$ should effectively look like a plane wave, hence $J_0(x)$ should be proportional to $\cos(x+\theta)$ at large $x$. The decay of the amplitude with $r$ can be explained by noting that the input power at the origin, after travelling a distance $r$ is uniformly distributed over a parameter of size $2\pi r$. Hence the local energy density must decay as $1/r$. Since the energy density is proportional to the square of the amplitude, the amplitude itself must decay as $1/\sqrt{r}$, as indicated in the asymptotic formula. This reasoning does not explain the phase factor of $\pi/4$, and the precise proportionality factor, which require matching to the solution at small $r$.