Recap

- The two dimensional Laplacian in polar coordinates (r, ϕ) is

$$\nabla^2 h = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 h}{\partial \phi^2} = \frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h}{\partial \phi^2}\,. \tag{3.5.26}$$

- Its solutions can be formed from superposition of terms involving Bessel functions, as

$$h = \sum_n A_n J_n(kr)\cos(n\phi + \theta_n)\,. \tag{3.5.27}$$

- The Laplacian in spherical coordinates (r, θ, ϕ) is

$$\nabla^2 h = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial h}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial h}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 h}{\partial \phi^2}\,. \tag{3.5.28}$$

- It admits radially symmetric solutions proportional to $\sin(kr)/r$.