Recap

- The *divergence theorem* relates the flux of a vector field $\vec{v}$ through any closed surface to the integral of the divergence of $\vec{v}$ though the enclosed volume:

$$\int_{\text{volume } V} dV \ \text{div}\vec{v} = -\int_{\text{surface } S} \vec{v} \cdot d\vec{S}\,. \tag{3.6.45}$$

- For a general coordinate system, the expressions for divergence of a vector field and the Laplacian of a scalar field are given by

$$\begin{aligned} \text{div}\vec{v} = \vec{\nabla} \cdot \vec{v} &= \frac{1}{|\det \mathbf{J}|} \sum_\alpha \frac{\partial}{\partial y_\alpha} \left(\frac{|\det \mathbf{J}|}{h_\alpha} v_\alpha \right) , \\ \nabla^2 f &= \frac{1}{|\det \mathbf{J}|} \sum_\alpha \frac{\partial}{\partial y_\alpha} \left(\frac{|\det \mathbf{J}|}{h_\alpha^2} \frac{\partial f}{\partial y_\alpha} \right) . \end{aligned} \tag{3.6.46}$$

- In spherical coordinates, the corresponding expressions are

$$\begin{aligned} \nabla^2 \Phi &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial \Phi}{\partial r}) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta \frac{\partial \Phi}{\partial \theta}) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 \Phi}{\partial \phi^2} , \\ \vec{\nabla} \cdot \vec{v} &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 v_r \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta v_\theta) + \frac{1}{r \sin\theta} \frac{\partial v_\phi}{\partial \phi} . \end{aligned} \tag{3.6.47}$$

- The isotropic vector field in d-dimensions admits one longitudinal mode parallel to the wave-vector, and $(d-1)$ transverse modes perpendicular to it.