4.2 Continuous random variable

4.2.1 Probability distribution function

Let us next discuss a random variable whose allowed values are not discrete, but continuous. In particular, consider a random variable x, whose outcomes are real numbers, i.e. $\mathcal{S} \in \{-\infty < x < \infty\}$.

- *The cumulative probability function* (CPF) $P(x)$, is the probability of an outcome with *any value* less than x, i.e. $P(x) = \text{prob.}(E \subset [-\infty, x])$. $P(x)$ must be a monotonically increasing function of x, with $P(-\infty) = 0$ and $P(+\infty) = 1$.

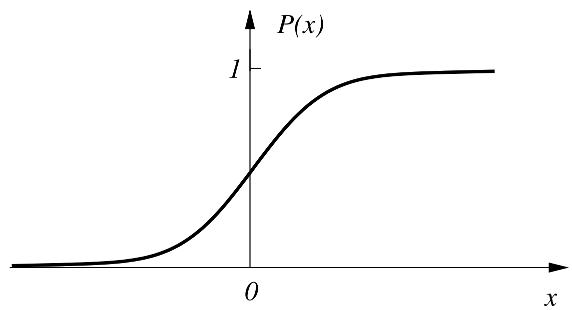

Figure 4.1: A typical cumulative probability function.

- *The probability density function* (PDF) is defined by $p(x) \equiv dP(x)/dx$. Hence, $p(x)dx = \text{prob.}(E \in [x, x+dx])$. As a probability density, it is *positive*, and normalized such that

$$\text{prob.}(\mathcal{S}) = \int_{-\infty}^{\infty} dx\ p(x) = 1\ . \tag{4.2.1}$$

Note that since $p(x)$ is a *probability density*, it has dimensions of $[x]^{-1}$, and changes its value if the units measuring x are modified. Unlike $P(x)$, the PDF has no upper bound, i.e. $0 < p(x) < \infty$, and may contain divergences as long as they are integrable.

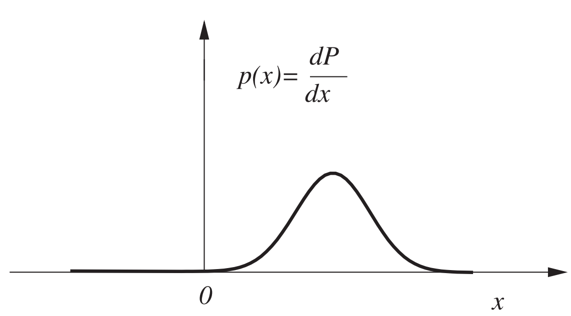

Figure 4.2: A typical probability density function.