4.2.2 Change of variables

- Consider a function $F(X)$ of the random variable X. as before the *expectation value* of the function is given by

$$\langle F(x) \rangle = \int_{-\infty}^{\infty} dx\ p(x) F(x)\ . \tag{4.2.2}$$

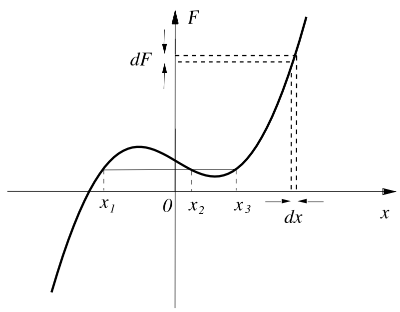

Figure 4.3: Obtaining the PDF for the function $F(x)$.

However, the function F is itself a random variable, with an associated PDF of $p_F(f)df = \text{prob.}(F \in [f, f + df])$. There may be multiple solutions x_i, to the equation $F(x) = f$, and

$$p_F(f)df = \sum_i p(x_i)dx_i\,, \quad \Longrightarrow \quad p_F(f) = \sum_i p(x_i) \left| \frac{dx}{dF} \right|_{x=x_i} \quad . \tag{4.2.3}$$

The factors of $|dx/dF|$ are the *Jacobians* associated with the change of variables from x to F.

- As an example, consider $p(x) = \lambda \exp(-\lambda|x|)/2$, and the function $F(x) = x^2$. There are two solutions to $F(x) = f$, located at $x_\pm = \pm\sqrt{f}$, with corresponding Jacobians $|\pm f^{-1/2}/2|$. Hence,

$$P_F(f) = \frac{\lambda}{2} \exp\left(-\lambda\sqrt{f}\right) \left(\left| \frac{1}{2\sqrt{f}} \right| + \left| \frac{-1}{2\sqrt{f}} \right| \right) = \frac{\lambda \exp\left(-\lambda\sqrt{f}\right)}{2\sqrt{f}}\,,$$

for $f > 0$, and $p_F(f) = 0$ for $f < 0$. Note that $p_F(f)$ has an (integrable) divergence at $f = 0$.

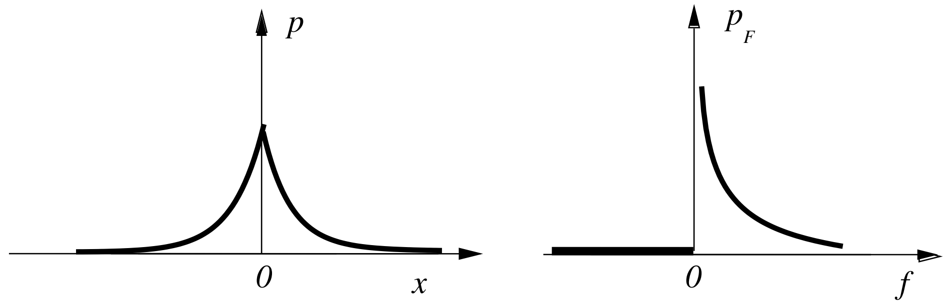

Figure 4.4: Probability density functions for x, and $F = x^2$.