4.3.2 Joint moments and cumulants

- *The expectation value* of a function $F(\mathbf{x})$, is obtained as before from

$$\langle F(\mathbf{x})\rangle = \int d^N\mathbf{x}\; p(\mathbf{x})F(\mathbf{x}) \; . \tag{4.3.6}$$

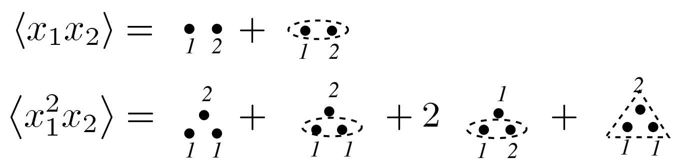
- *The joint characteristic function* is obtained from the N-dimensional Fourier transformation of the joint PDF,

$$\tilde{p}(\mathbf{k}) = \left\langle \exp\left(-i\sum_{j=1}^{N} k_j x_j\right)\right\rangle \quad . \tag{4.3.7}$$

- The *joint moments* and *joint cumulants* are generated by $\tilde{p}(\mathbf{k})$ and $\ln\tilde{p}(\mathbf{k})$ respectively, as

$$\begin{aligned}
\langle x_1^{n_1} x_2^{n_2}\cdots x_N^{n_N}\rangle &= \left[\frac{\partial}{\partial(-ik_1)}\right]^{n_1}\left[\frac{\partial}{\partial(-ik_2)}\right]^{n_2}\cdots\left[\frac{\partial}{\partial(-ik_N)}\right]^{n_N}\tilde{p}(\mathbf{k}=\mathbf{0}) \quad ,\\
\langle x_1^{n_1} * x_2^{n_2} * \cdots x_N^{n_N}\rangle_c &= \left[\frac{\partial}{\partial(-ik_1)}\right]^{n_1}\left[\frac{\partial}{\partial(-ik_2)}\right]^{n_2}\cdots\left[\frac{\partial}{\partial(-ik_N)}\right]^{n_N}\ln\tilde{p}(\mathbf{k}=\mathbf{0}) \quad .
\end{aligned} \tag{4.3.8}$$

- The previously described graphical relation between joint moments (all clusters of labeled points), and joint cumulant (connected clusters) is still applicable. For example, from



we obtain

$$\begin{aligned}
\langle x_1x_2\rangle &= \langle x_1\rangle_c\langle x_2\rangle_c + \langle x_1 * x_2\rangle_c \quad , \quad \text{and}\\
\langle x_1^2x_2\rangle &= \langle x_1\rangle_c^2\langle x_2\rangle_c + \langle x_1^2\rangle_c\langle x_2\rangle_c + 2\langle x_1 * x_2\rangle_c\langle x_1\rangle_c + \langle x_1^2 * x_2\rangle_c \quad .
\end{aligned} \tag{4.3.9}$$

The *connected correlation* $\langle x_\alpha * x_\beta\rangle_c$, is zero if x_α and x_β are independent random variables.