1.2.4 Universality and critical slowing down

There are many examples in nature when the behavior of a system undergoes a drastic change. Transitions between different phases of matter are a prime example from physics. The bifurcations described above provide a mathematical model for such phenomena. The concept of *universality* captures the applicability of the same mathematical formalism to diverse phenomena. As indication of the power of this concept, note that in both Eq. (1.2.10) and Eq. (1.2.14) the time dependence is controlled by an exponential, with a characteristic time scale $\tau = 1/|r|$, or $\tau = 1/|\epsilon|$ that diverges at the transition point. The slow-down of relaxation near points of extinction or symmetry breaking is an important characteristic of these phenomena. We have thus found an underlying mathematical reason for this observation which is completely independent of various details of the process!

At the point $\epsilon = 0$, the exponential dependence gives way to a power-law decay, as

$$\frac{dy}{dt} = -y^p, \quad \Longrightarrow \quad t = \frac{p-1}{y_0^{p-1}} - \frac{p-1}{y^{p-1}}, \quad \Longrightarrow \quad \lim_{t\to\infty} y(t) \propto t^{\frac{1}{p-1}}, \tag{1.2.16}$$

with $p = 2$ or $p = 3$ for transcritical or pitchfork bifurcations. You should convince yourself that Eq. (1.2.10) and Eq. (1.2.14) indeed exhibit such power-law decays at their transition points.