5.3.3 From dissipation to fluctuation

A stone dropped in water, sinks to the bottom. In the process, it dissipates (loses) energy to the surrounding fluid. We described this form of motion by the simple ODE for the particle position h:

$$\frac{dh}{dt} = \mu F(h) = -\mu \frac{dV}{dh}\,. \tag{5.3.12}$$

In case of a mass m falling under gravity $V(h) = mgh$; more generally the particle dissipates energy until it rests at the position of *mechanical equilibrium* at the minimum of the potential $V(h)$ (where the force $F(h)$ is zero). On the other hand, a microscopic grain placed at rest at the minimum of the potential $V(h)$ undergoes Brownian motion that takes it away from this location, described as in Eq.(5.3.2) by a Langevin equation

$$\frac{dh}{dt} = -\mu \frac{dV}{dh} + \eta(t)\,. \tag{5.3.13}$$

Ultimately, its position can only be described probabilistically as in Eq. (5.3.11).

Both phenomena are manifestations of the exchange of energy with the fluid bath: The larger stone dissipates energy to the bath. while the small grain gains energy from fluctuating forces exerted by the fluid. Of course while energy is exchanged in the these processes, the total energy (including the bath) is conserved and constant. The microscopic mechanisms through which energy is exchanged between the fluid molecules and immersed bodies can be quite complex. Fortunately, we can avoid describing these mechanisms by taking a probabilistic approach:

- Consider several bodies, labelled by $i = 1, 2 \cdots .\ell$, immersed in the fluid bath. Each body could be different, incorporating distinct internal components, with its total energy indicated by E_i composed of various vibrational and rotational components. For example, one (additive) component to E_i could be the potential energy $V_i(x_i)$ of its center of mass in an external trap.

- The bodies are sufficiently far apart not interact directly with each other, but they do exchange energy with the bath.

- In lieu of describing the dynamics by which the bodies exchange energy with the bath, we assume that their energies are independent random variables, described by probability densities $p_i(E_i)$.

- We assume that in steady state, the bath does not *on average* remove or deposit energy to the bodies, merely facilitating exchange of energy between them, such that

$$\langle E_1 + E_2 + \cdots E_\ell \rangle = \sum_{i=1}^{\ell} \langle E_i \rangle = \text{constant}\,. \tag{5.3.14}$$

What is the best choice of probabilities $\{p_i(E_i)\}$ given the above assumptions?