
Recap

- (i) Any second order ODE can be solved by using energy as a first integral, as

$$\ddot{x} = F(x), \quad \implies \quad \frac{dx}{dt} = \pm \sqrt{2(E - V(x))}, \quad \implies \quad \pm t = \int_{x_0}^{x(t)} \frac{dx'}{2(E - V(x'))}, \quad (1.3.19)$$

- (ii) Complex exponentials, solutions to the linear ODR $\ddot{x} = -\omega_0^2 x$, follow the *Euler relation*

$$e^{i\omega_0 t} = \cos \omega_0 t + i \sin \omega_0 t, \quad (1.3.20)$$

which can be used to switch between polar and cartesian representation, and to derive various trigonometric identities.