Normal modes for N blocks

It appears a daunting task to find all the normal modes for the general case of n blocks. However, there is a simple formula that generates all normal modes and frequencies. For the αth normal mode, we will try eigenvectors of the form

$$\vec{A}_\alpha = \begin{pmatrix} \sin\left(\frac{\pi\alpha}{N+1}1\right) \\ \sin\left(\frac{\pi\alpha}{N+1}2\right) \\ \vdots \\ \sin\left(\frac{\pi\alpha}{N+1}(N-1)\right) \\ \sin\left(\frac{\pi\alpha}{N+1}N\right) \end{pmatrix}, \quad \text{for} \quad \alpha = 1, 2, \cdots, N\,. \tag{2.3.32}$$

When multiplied by $\mathbf{T}_n$, a typical element is

$$\begin{aligned} &2\sin\left(\frac{\pi\alpha}{N+1}k\right) - \sin\left(\frac{\pi\alpha}{N+1}(k+1)\right) - \sin\left(\frac{\pi\alpha}{N+1}(k-1)\right) = \\ &2\sin\left(\frac{\pi\alpha}{N+1}k\right)\left[1 - \cos\left(\frac{\pi\alpha}{N+1}\right)\right], \end{aligned} \tag{2.3.33}$$

where the trigonometric identity in Eq. (1.3.11) was used to convert the sum of two sines to the product of sine and cosine appearing in the second line. From this we can identify the normal mode frequencies

$$\omega_\alpha = \omega_0\sqrt{2\left[1 - \cos\left(\frac{\pi\alpha}{N+1}\right)\right]} = 2\omega_0 \sin\left(\frac{\pi\alpha}{2(N+1)}\right), \tag{2.3.34}$$

by taking advantage of another trigonometric identity, $1 - \cos(2\theta) = 2\sin^2\theta$. The reasoning that allows us to identify the eigenvector in Eq. (2.3.32) is explained in the next section.