Recap

- Normal modes in a potential $V = K_{ij}x_ix_j/2$, are eigenvectors of the symmetric matrix with elements $M_{ij} = K_{ij}/\sqrt{m_im_j}$.
- The normal mode frequencies are related to the eigenvalues of the matrix, and obtained from solutions to $\det(\mathbf{M} - \omega^2\mathbf{1}) = 0$.
- For a frictionless chain of n blocks connected in series by springs, the equations of motion

$$\ddot{x}_i = \omega_0^2 \left(x_{i+1} + x_{i-1} - 2x_i\right), \qquad \text{for } i = 1, 2, \cdots, n\,, \tag{2.3.35}$$

  allow for normal modes

$$x_i^{(\alpha)}(t) = a_\alpha \sin\left(\frac{\pi\alpha i}{N+1}\right)\cos\left(\omega_\alpha t + \phi_\alpha\right), \qquad \text{for } \alpha = 1, 2, \cdots, n\,, \tag{2.3.36}$$

  with frequenceis

$$\omega_\alpha^2 = 2\omega_0^2\left[1 - \cos\left(\frac{\pi\alpha}{N+1}\right)\right], \quad \Rightarrow \quad \omega_\alpha = 2\omega_0 \sin\left(\frac{\pi\alpha}{2(N+1)}\right). \tag{2.3.37}$$