2.4.3 Pinned chain of blocks

While the eigenvalues of the periodic chain in Eq. (2.4.15) are non-negative, there is one mode for $\alpha = N$ with $\lambda_N = 0$. This mode describes the free motion of the center of mass of the chain which does not experience any restoring force. Such a mode is absent for the chain with the 2 ends pinned to walls presented in Eq. (2.3.8). However, the eigenvectors in Eq. (2.3.32) and the eigenvalues in Eq. (2.3.34) are almost identical to those of the periodic chain. How is this possible?

Consider the sine eigenvector in Eq. (2.4.16) for a *longer* chain of $M = 2(N+1)$ blocks. Ignore the first N components of this vector that may or may not be non-zero. The $(N+1)$th component is $\sin\left(\frac{2\pi\alpha(N+1)}{2(N+1)}\right) = 0$ is zero for all integer α. There are then again N components, coinciding exactly with those in Eq. (2.3.32), since $\sin\left(\frac{2\pi\alpha(N+1+k)}{2(N+1)}\right) = \sin\left(\frac{2\pi\alpha k}{2(N+1)}\right)$. that may or may not be non-zero, followed by the last element of the sine eigenvector which is always zero. The mid-point and final zero components of this eigenvector act precisely as the stationary walls to which the end blocks are connected. Thus normal modes of the pinned chain are found embedded in normal modes of a longer periodic chain!