Recap

- If two matrices commute they share the same eigenvectors, but with possibly different eigenvalues.

- Some symmetries (e.g. under exchange, shift, or permutation) can be described by matrices.

- It is typically easier to diagonalize matrices corresponding to symmetries; their eigenvectors can then be used to construct eigenvectors for problems sharing those symmetries.

- Translation (shift) symmetries are diagonalized by sine and cosine modes, presaging Fourier decomposition.