3.1.5 Initial conditions, Boundary conditions

In solving ODEs we had to supply initial values, one for the case of a first order equation and two for a second order equation. Solving for the full time evolution of PDEs also requires initial conditions in the form of functions, for example $u(x, t = 0)$ for a first order PDE, and additionally $\dot{u}(x, t = 0)$ for the second order one.

The **initial conditions** are related to, but distinct from **boundary conditions**. The latter arise because one typically needs to solve the problem over an interval of given extent in the space dimension, whose edges have to be treated carefully. Usually the complete description of a problem requires knowledge of what goes on at the boundaries. The most common boundary conditions are:

- **Dirichlet (closed):** The function is constrained to always be zero at an edge, such that
$$u(x = 0, t) = 0 \quad \text{for all times } t\,, \tag{3.1.22}$$
as in the case of the rubber band pinned to a wall at its edge.

- **Neumann (open):** The derivative is set to zero at an edge, i.e.
$$\frac{\partial u}{\partial x}(x = 0, t) = 0 \quad \text{for all times } t\,. \tag{3.1.23}$$
An open end that moves freely, and does not support a force is an example; no flux at edges of an interval in case of the diffusion equation is another.

- **Periodic:** When the two ends are joined together, forming a ring, like a snake biting its tail, we have
$$u(x = 0, t) = u(x = L, t) = 0.$$

There can also be mixed boundary conditions, e.g. a rope that is closed at one end, and open at the other, or **forced** at the edge to satisfy a particular motion, as in $u(x = 0, t) = a\cos(\omega t)$.