9.65: November 26, 2001 Judgment Handout

Outline:

Judgment under uncertainty: Inductive reasoning

1. Introduction: deductive reasoning versus inductive reasoning
2. Probability judgments
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b. Bayes' theorem
3. Heuristics and fallacies
a. Representativeness heuristic
b. Base rate neglect
c. Availability heuristic
4. Conclusion: rational or irrational?

I. Introduction: deductive reasoning versus inductive reasoning

2. Probability judgments

a. Kolmogorov's axioms:

The field of statistics is based on a theory about probability that attempts to turn induction into a deductive system.

These axioms are the basis of probability theory:

P = probability, $P(S_1)$ = probability of statement 1.

1. For every statement, P(S) is equal to or greater than 0.

2. If S_1 is logically true, then $P(S_1) = 1$ (highest P).

3. If S_1 and S_2 are mutually exclusive, then

$P(S_1 \text{ or } S_2) = P(S_1) + P(S_2)$.

4. If $P(S_1)$ is not = 0, then

$P(S_1 \text{ and } S_2) = P(S_1 \mid S_2) \times P(S_2)$.

b. Bayes' theorem

Bayes' theorem follows from the above axioms: here I use X for S_1, and Y for S_2:

If P(X) is not equal to 0, then (Bayes' theorem):

$$P(X \mid Y) = P(X) \times \frac{P(Y \mid X)}{P(Y)}$$

Note that P(X) is the base rate or prior odds of X. Y is a piece of relevant information--e.g., getting a positive result on a medical test such as a mammogram. P(Y) is the overall probability of Y (independent of X).

Example: Probability that Julia's positive result on a mammogram means that she has breast cancer (see text pp. 396-397 for the numbers in this example).

3. Heuristics and fallacies

Are people Bayesian?

Consider the following sketch:

Mary X was active in the environmental and nuclear-freeze movements in college, and organized demonstrations at her university. She graduated a few years ago.

On the basis of this knowledge about Mary, please give an estimate of the probability that each of the following statements is true of her.

1. Mary X is a strong supporter of George W. Bush. ___

2. Mary X is "pro-choice." ___

3. Mary X works in a bank. ___

4. Mary X likes pizza. ___

5. Mary X is demonstrating against the current war. ___

6. Mary X works in a bank and is pro-choice. ___

a. Conjunction fallacy and representativeness:

Representativeness heuristic:

b. Base rate neglect

Sample size:

Misconceptions of chance:

-coin toss:

-the Gambler's Fallacy:

Regression to the mean (e.g., Rookie of the Year: Figure 12.6)

c. Availability heuristic

-Anchoring

4. Conclusion: rational or irrational?