

# Consumer Credit Risk Models via Machine-Learning Algorithms\*

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## Abstract

We apply machine-learning techniques to construct nonlinear nonparametric forecasting models of consumer credit risk. By combining customer transactions and credit bureau data from January 2005 to April 2009 for a sample of a major commercial bank's customers, we are able to construct out-of-sample forecasts that significantly improve the classification rates of credit-card-holder delinquencies and defaults, with linear regression  $R^2$ 's of forecasted/realized delinquencies of 85%. Using conservative assumptions for the costs and benefits of cutting credit lines based on machine-learning forecasts, we estimate the cost savings to range from 6% to 25% of total losses. Moreover, the time-series patterns of estimated delinquency rates from this model over the course of the recent financial crisis suggests that aggregated consumer-credit risk analytics may have important applications in forecasting systemic risk.

**Keywords:** Household Behavior; Consumer Credit Risk; Credit Card Borrowing; Machine Learning; Nonparametric Estimation

**JEL Classification:** G21, G33, G32, G17, G01, D14

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# 1 Introduction

One of the most important drivers of macroeconomic conditions and systemic risk is consumer spending, which accounted for over two thirds of U.S. gross domestic product as of October 2008. With \$13.63 trillion of consumer credit outstanding as of the fourth quarter of 2008 (\$10.47 trillion in mortgages \$2.59 trillion in other consumer debt), the opportunities and risk exposures in consumer lending are equally outsized.<sup>1</sup> For example, as a result of the recent financial crisis, the overall charge-off rate in all revolving consumer credit across all U.S. lending institutions reached 10.1% in the third quarter of 2009, far exceeding the average charge-off rate of 4.72% during 2003 to 2007.<sup>2</sup> With a total of \$874 billion of revolving consumer credit outstanding in the U.S. economy as of November 2009,<sup>3</sup> and with 46.1% of all families carrying a positive credit-card balance in 2007,<sup>4</sup> the potential for further systemic dislocation in this sector has made the economic behavior of consumers a topic of vital national interest.

The large number of decisions involved in the consumer lending business makes it necessary to rely on models and algorithms rather than human judgment, and to base such algorithmic decisions on “hard” information, e.g., characteristics contained in consumer credit files collected by credit bureau agencies. Models are typically used to generate numerical “scores” that summarize the creditworthiness of consumers.<sup>5</sup> In addition, it is common for lending institutions and credit bureaus to create their own customized risk models based on private information about borrowers. The type of private information typically consist of both “within-account” as well as “across-account” data regarding customers’ past behavior.<sup>6</sup> These aspects of current consumer-credit risk analytics are ideally suited for computationally

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<sup>1</sup>U.S. Federal Reserve Flow of Funds data, June 11, 2009 release.

<sup>2</sup>Data available from the Federal Reserve Board at <http://www.federalreserve.gov/releases/chargeoff/>.

<sup>3</sup>See the latest release of Consumer Credit Report published by the Federal Reserve Board, available at <http://www.federalreserve.gov/releases/g19/Current/>

<sup>4</sup>See the *Survey of Consumer Finances, 2009* (SCF), released in February 2009 and available at <http://www.federalreserve.gov/pubs/bulletin/2009/pdf/scf09.pdf>. This report shows that the median balance for those carrying a non-zero balance was \$3,000, while the mean was \$7,300. These values have risen 25% and 30.4%, respectively, since the previous version of the SCF conducted three year earlier. The SCF also reports that the median balance has risen strongly for most demographic groups, particularly for higher-income groups.

<sup>5</sup>See Hand and Henley (1997) and Thomas (2009) for reviews of traditional and more recent statistical modeling approaches to credit scoring.

<sup>6</sup>The impact of such relationship information in facilitating banking engagement has been studied extensively in the area of corporate and small business lending (see, for example, Boot 2000) and, more recently, in consumer lending (see Agarwal, Chomsisengphet, Liu and Souleles, 2009).

intensive techniques for discerning patterns among extremely large datasets.

In this paper, we apply machine-learning algorithms such as radial basis functions, tree-based classifiers, and support-vector machines to the problem of predicting consumer default and delinquency behavior before and during the Financial Crisis of 2007–2009. By analyzing trends in consumer expenditures, savings, and debt payments using a proprietary dataset from a major commercial bank (which we shall refer to as the “Bank” throughout this paper to preserve confidentiality) from January 2005 to April 2009, we are able to identify subtle nonlinear relationships that are difficult to detect in these massive datasets using standard consumer credit-default models such as logit, discriminant analysis, or credit scores.

Machine-learning techniques are ideally suited for this particular challenge because of the large sample sizes and the complexity of the possible relationships among consumer transactions and characteristics.<sup>7</sup> The extraordinary speed-up in computing in recent years, coupled with significant theoretical advances in machine-learning algorithms, have created a renaissance in computational modeling, of which our consumer credit risk model is just one of many recent examples.

Our analysis shows strong links between *ex ante* observable characteristics and future delinquencies. To illustrate the potential forecast power of some simple predictors, consider two conditional probabilities estimated from our sample: customers with credit-card-balance-to-income ratios exceeding 10 have a 9.29% probability of falling 90-days-or-more delinquent on their payments over the subsequent 6 months, and customers experiencing a recent drop in income (as measured by sharp drops in direct deposits) have a 10.8% probability of 90-days-or-more delinquency over the subsequent 6 months. The same figure for the aggregate sample, i.e., without any conditioning, is 5.29%. Such predictors are statistically reliable throughout the entire sample period, and machine-learning models are able to generate many variants of such predictors and combine them in nonlinear ways to yield even more powerful forecasts.

One measure of this forecast power is to compare the machine-learning model’s forecasted scores of those customers who eventually default during the forecast period with the forecasted scores of those who do not. Significant differences between the forecasts of the two populations is an indication that the forecasts have genuine discriminating power. Over the

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<sup>7</sup>See, for example, Bellotti and Crook (2009) or Li, Shiue and Huang (2006) for applications of machine learning based model to consumer credit.

sample period from May 2008 to April 2009, the average forecasted score among individuals who do become 90-days-or-more delinquent during the 6-month forecast period is 61.9, while the average score across all customers is 2.1. The practical value-added of such forecasts can be estimated by summing the cost savings from credit reductions to high-risk borrowers and the lost revenues from “false positives”, and under a conservative set of assumptions, we estimate the potential net benefits of these forecasts to be 6% to 25% of total losses.

More importantly, by aggregating individual forecasts, it is possible to construct a measure of systemic risk in the consumer-lending sector, which accounts for one of the largest components of U.S. economic activity. We show that the time-series properties of our machine-learning forecasts are highly correlated with realized delinquency rates (linear regression  $R^2$ 's of 85%), implying that a considerable portion of the consumer credit cycle can be forecasted 6 to 12 months in advance. This has obvious implications for macroprudential risk management.

In Section 2, we describe our dataset, discuss the security issues surrounding it, and document some simple but profound empirical trends. Section 3 outlines our approach to constructing useful variables or feature vectors that will serve as inputs to the machine-learning algorithms we employ. In Section 4, we describe the machine-learning framework for combining multiple predictors to create more powerful forecast models, and present our empirical results. Using these results, we provide two applications in Section 5, one involving model-based credit-line reductions and the other focusing on systemic risk measures. We conclude in Section 6.

## 2 The Data

The data used in this study consists of transaction level, credit bureau and account balance data for a subset of the Bank’s customer base for the period from January 2005 to April 2009. This data was assembled from a number of different sources which we review in Section 2.1. Given the sensitive nature of the data and consumer privacy protection laws, all individual identification data elements such as names, addresses, and social security numbers were removed by the bank before sharing the data with us, and all computations were performed on machines physically located at the Bank’s office and within the bank’s electronic “firewalls”. We review these security protocols in Section 2.2. In Section 2.3, we document some trends in debt balances and payments over the sample period that clearly

point to substantial but gradual deterioration in the financial health of consumers over this period.

## 2.1 Data Sources

The raw data was provided to us in flat-file format. A set of programs were then used to pre-process the data, producing higher-level variables or “feature vectors” and time-aggregated to yield monthly observations that were stored in a database (see Figure 1). The following is a description of each of these components.

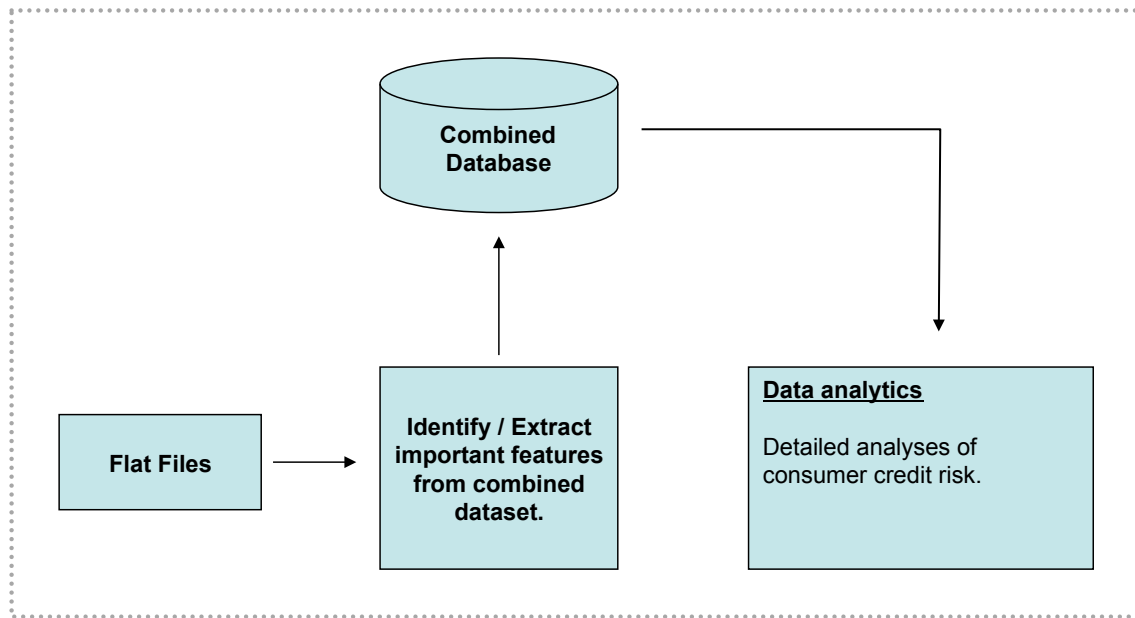


Figure 1: Construction of an integrated database of transactions and credit bureau statistics used in machine-learning model of consumer credit risk.

### Transactions Data

The Bank provided us with account-level transactions data for a subset of its customer base. In addition to identifying each transaction’s amount and direction (inflow or outflow), this dataset contains two other classification fields with each transaction: *channel* and *category*. The channel refers to the medium through which the transaction took place; a complete list of possible channels are: Automated Clearing House (ACH), Adjustments to Account (ADJ), Automated Teller Machine (ATM), Online Bill Pay (BPY), Credit Card (CC), Check

(CHK), Debit Card (DC), Account Fees (FEE), Interest on Account (INT), Wire Transfer (WIR), Internal Transfer (XFR) and Miscellaneous Deposit or Withdrawal (not in another channel) (MSC). There was also very rare occurrences of an Unidentifiable Channel (BAD).

The category classification is much broader and meant to capture the nature of the transaction. Examples of transaction categories include: restaurant expenditure, bar expenditure, grocery, food, etc. In the raw data, we observed 138 categories, which were produced by the Bank’s data team based on the information that was associated with each transaction. Naturally, the level of accuracy and coverage varied greatly across channels. In particular, transactions taking place via Credit Card (CC) or Debit Card (DC) channels had much more accurate categories associated with them than those associated with ATM or Check transaction channels.

The sheer size and scope of this dataset proved to be a significant challenge, even for computing basic summary statistics. To develop some basic intuition for the data, we began by computing the total number of transactions per month, total inflows, and total outflows across all transactions for each individual in the database, and then constructed the same metrics for each of the following channels separately: Automated Clearing House (ACH), Automated Teller Machine (ATM), Credit Card (CC), Debit Card (DC), Interest on Account (INT), Online Bill Pay (BPY) and Wire Transfer (WIR). These channels were chosen because they represent the vast majority of the entire sample.

We also attempted to distill the most important information contained in the 138 categories by selecting a smaller set of data elements that broadly represent the key aspects of consumers’ daily expenditures. In particular, Table 1 lists the subset of data items we selected from the transactions level data.<sup>8</sup> It should be emphasized that this data represents an incomplete picture of a typical consumer’s expenditures. For example, the ultimate purposes of transactions that take place via ATM and Check channels are not typically known. Also, while employment-related payments (e.g., salary, bonus, etc.) are captured if they are directed via ACH channel, these payments are missed and not classified if an individual deposits a check or cash into his or her account. Similarly, any expense paid in cash is not captured in this data. Finally, an individual may have banking relationships with multiple institutions, hence the data from the Bank may be an incomplete view of the individual’s

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<sup>8</sup>Note that not all of these items were used in creating our risk models because of legal restrictions (see Section 2.2 for further discussion). The list of data items used in our analysis is given in Section 4.2 and Table 4.

financial activities. However, to the extent that such levels of incompleteness are stable through time, the trends observed in the data may still be used to make inferences and predictions about the changing behavior of the population. While a certain degree of caution is warranted because of these issues, the apparent success of the statistical models created from this data (see Section 4.4 and 5) suggests that there is considerable incremental value for consumer credit risk management even in this limited dataset.

## Data from the Credit Bureaus

The Bank complements its account transactions data with credit file data provided by one of the credit bureaus for their customers. This data consists of variables listed under the “Individual Level” data items in Table 2. The generic credit score provided in this data, which we shall refer to as CScore, is similar in terms of statistical properties and predictive power to scores that are standard technique for measuring consumer credit quality.<sup>9</sup> We will use this credit score as a benchmark against which the performance of machine-learning models of credit risk can be compared.

Figure 2 depicts the relationship between CScore for the months of January of 2005, 2006, 2007 and 2008 and the realized default frequency in the following 6 months. In each case, we consider the initial CScore for individuals and calculate the default frequency in the subsequent 6 months for each level of the score. In addition, we plot second-order-polynomial-fitted curves in each graph, and Figure 2 shows that the CScore is remarkably successful in rank-ordering future default frequencies. In Figure 3, we plot the four polynomial approximations to compare the performance across different market conditions. As economic conditions began deteriorating in 2007 and 2008, the curves started to shift to the right, indicating a higher absolute default rate for each level of the CScore. This is to be expected since the distribution of the CScore is static overtime as shown in Figure 4.

In addition, this data contains information regarding all credit or loan facilities, also referred to as “Trade Line”, that each customer has across all financial institutions. For example, for a customer with a checking account with the Bank and a mortgage with another lender, this dataset would contain the mortgage balance, age, payment history and other relevant information without specifying the name of identity of the lender. Trade lines are divided into the following six types: Auto (AUT), Mortgage (MTG), Credit Card (CCA),

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<sup>9</sup>See Avery et al. (2003) for an excellent review of consumer data.



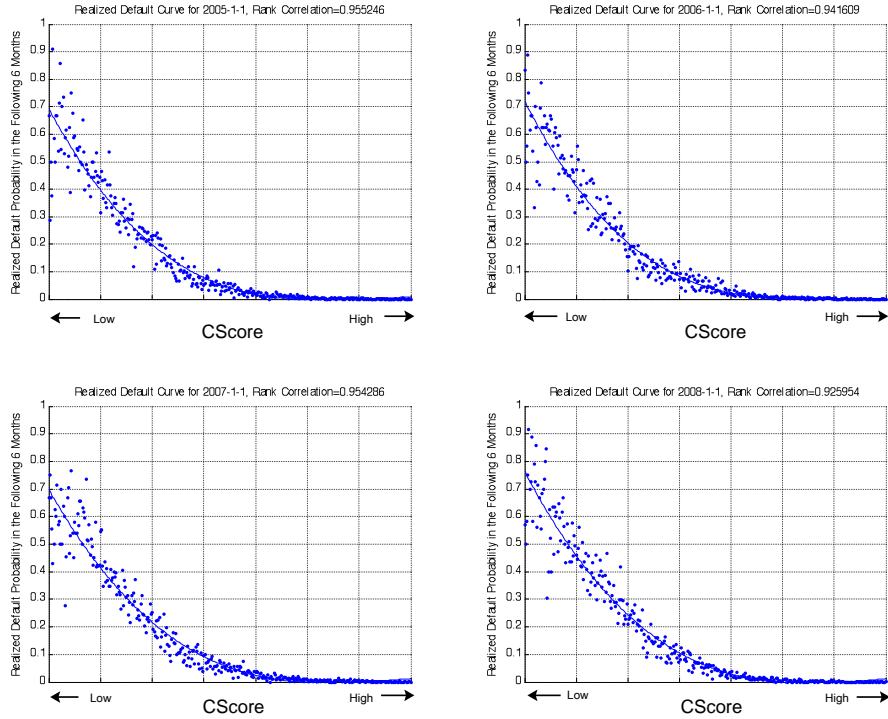


Figure 2: Relationship between CScore and subsequent default frequency (over the next 6 months) for January of 2005, 2006, 2007, 2008. Second-order polynomial approximations are also shown (solid lines).

Home Line of Credit (HLC), Home Loan (HLN), Other Loans (ILA), Other Lines of Credit (LOC), and Speciality type (SPC). The data collected for each Trade Line are listed in Table 2 under the “Trade Line Level” column.

### Account Balance Data

We also match the transaction-level and credit-bureau data with information about account balances from checking accounts and CDs that a customer has with the Bank. These items are listed in Table 3.

## 2.2 Data Security

Given the sensitivity of consumer data, a number of steps were taken to preserve the confidentiality of this data. First, the Bank de-identified all records—deleting all identification information such as names, addresses, social security numbers, and customer age—before

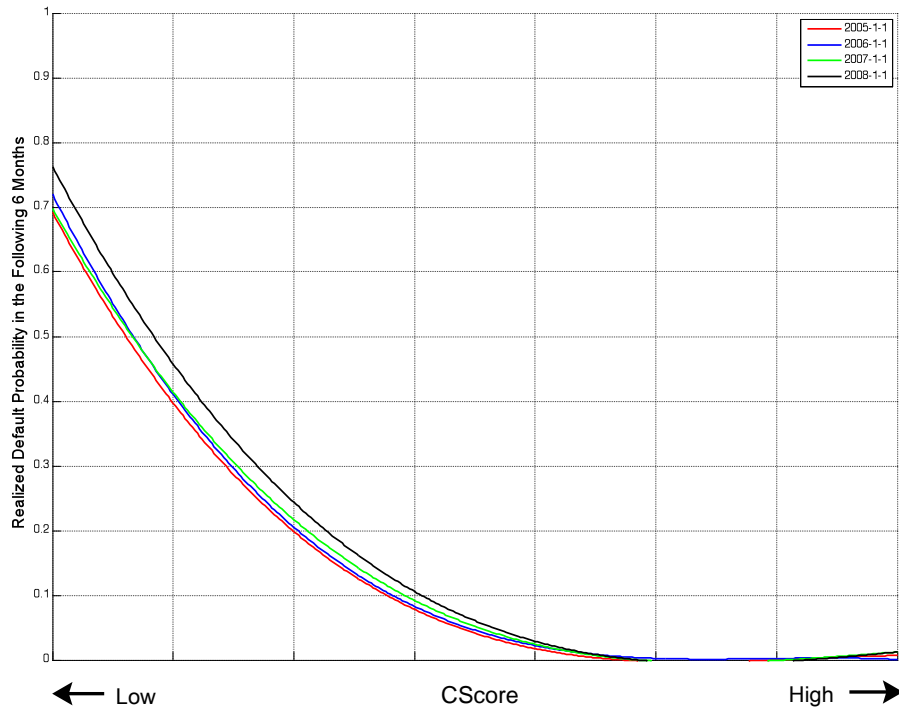


Figure 3: Second-order polynomial approximation of the relationship between CScore and subsequent default frequency (over the next 6 months) for January of 2005, 2006, 2007, 2008.

they were transferred into databases to which we were given access. All computations were performed on dedicated MIT computers installed on site at the Bank’s office, and all external network connections for these computers were disabled. At no point during this project was any raw consumer data transferred outside of the Bank’s electronic firewalls.

Moreover, several data items were excluded from our models due to legal restrictions such as the Fair Credit Reporting Act. In particular, all healthcare, insurance, unemployment, government, treasury, account and file age information were excluded from our analysis (see Section 4.2 and Table 4 for the list of data items used in our analysis).

### 2.3 Data Trends from 2005 to 2008

To provide some intuition for the empirical characteristics of the data, in this section we document several trends in the data over the period from January 2005 to December 2008. Since the main focus of our study is predictive models for credit-card defaults and delinquencies, we confine our attention in this section to observed trends in credit-card balances

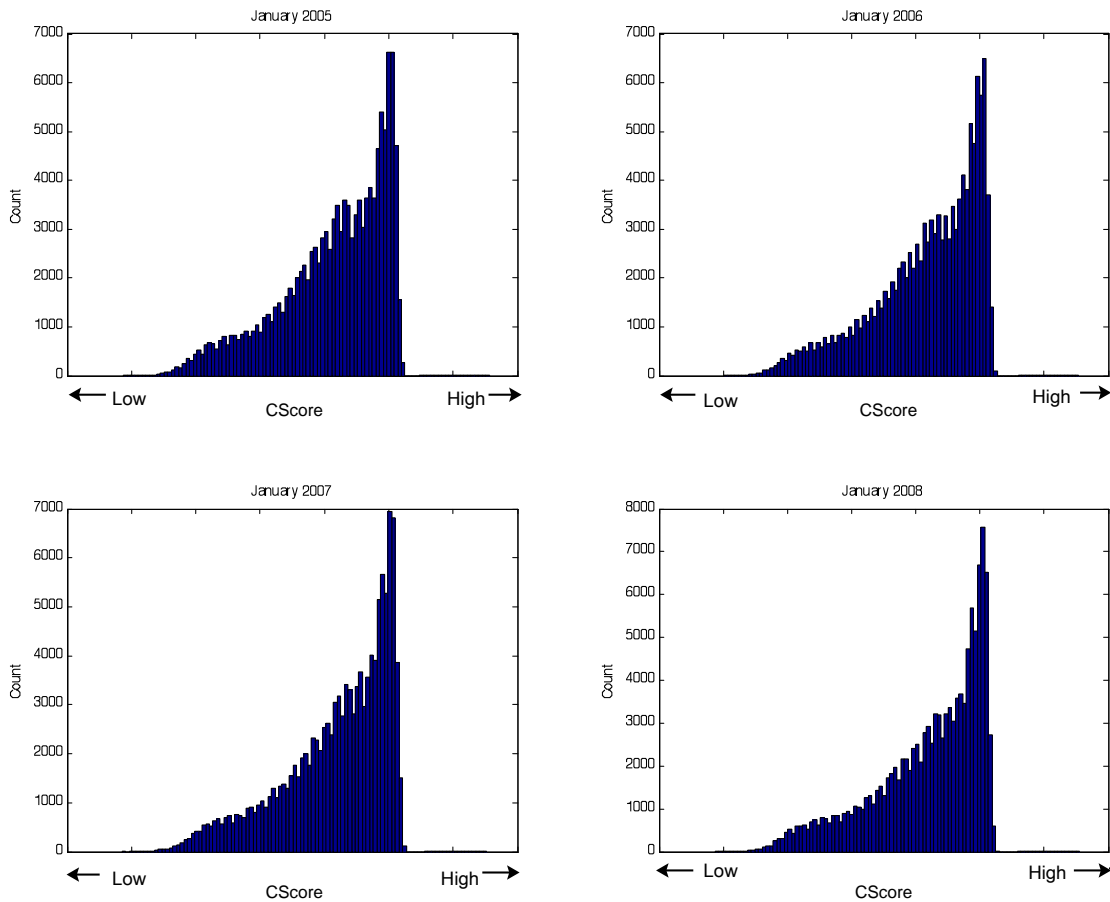


Figure 4: Histograms of CScore for January of 2005, 2006, 2007, 2008.

and payments.

Figure 5(a) shows the median, 75th, 90th, and 95th percentiles of credit-card balances, based on the total balance across all credit cards. Because the data used in this calculation are taken from the matched sample obtained from credit bureaus (see Table 2), the balance represents the total balance across credit cards provided by all financial institutions, and is an accurate representation of the total credit-card debt of individuals. This data shows that the top 5% of the sample had credit-card debt of approximately \$20,190 at the beginning of the sample, which increased to \$27,612 by the end of the sample. The pattern of increasing credit-card indebtedness is clear, and even more extreme for other percentiles of the distribution as shown in Figure 5(a). For example, the median credit-card balance almost doubled from

\$647 to \$1,259 over this period.

While increasing credit-card debt probably does not bode well for systemic risk, it need not represent a decline in the overall financial health of the consumer if the increased debt was accompanied by a comparable increase in income. For example, if increasing credit-card indebtedness is concurrent with a change in the usage among customers such that those consumers with high incomes—and who had not previously carried positive credit-card balances—started borrowing with their credit cards, this scenario may not represent a substantial change in systemic risk. If, on the other hand, customers increase their credit-card borrowing without any change in their income, the result would be an increase in systemic risk. To attempt to distinguish between these two scenarios, we plot the ratio of credit-card balances to monthly income in Figure 5(b).<sup>10</sup> This figure shows that in the first half of the sample, from January 2005 to January 2006, the increase in credit-card debt documented in Figure 5(a) was concurrent with increased monthly income or a shift in usage such that there was little or no change in the ratio of credit-card balances to income. However, starting in January 2007, a different pattern emerges—the ratio of the balances to monthly income starts to increase steadily. For example, for the top 5%, this ratio was approximately 11 for the first half of the sample but increased by about 2.5 to reach 13.5 over the period from January 2007 to December 2008.

While the trends shown in Figure 5(a) and 5(b) point to an increasing level of credit-card debt, both in absolute terms and relative to income, over this period, this by itself may be the result of a substitution effect where credit-card debt replaced another form of credit over this period. To explore this possibility, in Figure 5(c) we depict the same statistics for credit-card payments during this period.<sup>11</sup> We see that credit-card payments exhibited a positive trend from January 2005 to July 2007, but the increase stopped after July 2007. Finally, Figure 5(d) contains the ratio of credit-card payments to monthly income over this period, which shows that while credit-card payments were increasing as a percentage of monthly income for period of January 2005 to July 2007, this trend broke after July 2007.

However, this pattern must be interpreted with some care due to certain data issues. In particular, as discussed above, the monthly income data is only captured by the bank's

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<sup>10</sup>We have used the average of monthly income, as measures by the “Employment Inflow” data shown in Table 1, over the precessing 6 months, or as many month as available, as the denominator.

<sup>11</sup>In particular, we use average credit-card payments over the preceding 6 months, or as many month as are available.

transaction categorization engine if it comes through ACH (such as direct deposit), and there is no assurance that this system will capture an individual’s entire income, even if all of it is deposited with the Bank (for example, income in the form of deposited checks would not be categorized as income). Furthermore, there may be some outliers, as in the case where individuals use their credit cards to purchase big-ticket items such as vacations or major appliances, in which case their credit-card balances can exceed their monthly income. Nevertheless, we believe that the patterns in Figures 5(a)–5(d) are still revealing with respect to the deteriorating financial condition of consumers in months leading up to the financial crisis.

Figures 6(a) and 6(b) contain other trends that confirm the developing problems in consumer credit during this period. Figure 6(a) shows the ratio of monthly payments for credit cards, auto loans, student loans, and mortgages to monthly income, and the pattern is consistent with Figure 5(d); monthly payments across all debt instruments were increasing as a percentage of monthly income (but again, we may not have identified all sources of an individual’s income). To develop a better sense of these trends, Figure 6(b) plots the ratio of debt payment to total monthly outflow over this period.<sup>12</sup> The total outflow provides a useful robustness check since the direction of each transaction (inflow or outflow) is known with certainty, so the total monthly outflow is a complete picture of all expenditures from all Bank accounts over the course of the month. The pattern that emerges from this analysis, see 6(b), supports the trends in the previous figures.

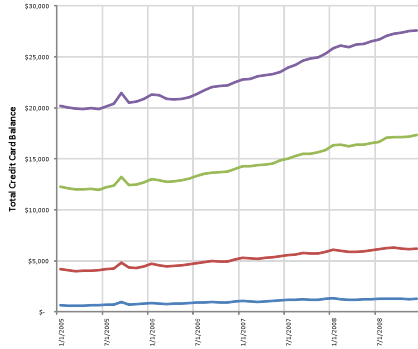
The patterns reported in this section are a very small subset of the many trends that are readily observed in this extremely rich data set. Even based on this very limited set of metrics, it is clear that credit-card balances were increasing, both in absolute and relative terms, long before the credit crisis hit, and similar trends were apparent among a broader set of debt instruments. Such patterns suggest that predictive systemic risk indicators in the consumer lending business are feasible.

### 3 Constructing Feature Vectors

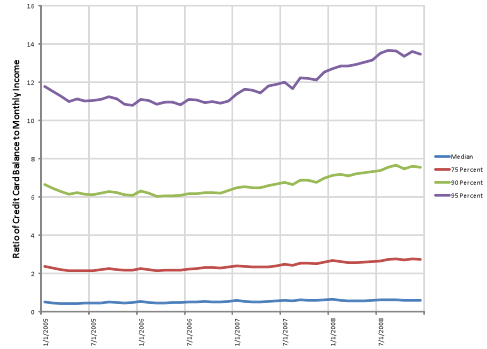
The objective of any machine-learning model is the identification of statistically reliable relationships between certain features of the input data and the target variable or outcome.

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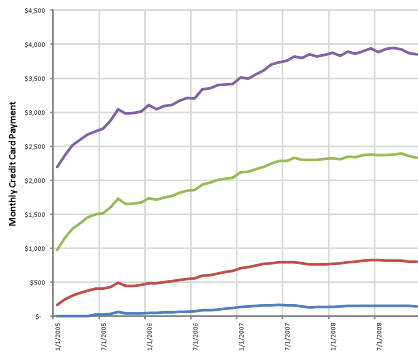
<sup>12</sup>Total monthly outflow is measured as the average monthly outflow over the preceding 6 months, or as many month as available.



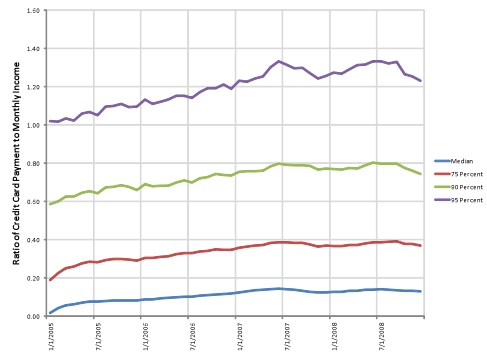
(a) Historical Monthly Credit-Card Balance



(b) Historical Monthly Credit-Card Balance to Income Ratio

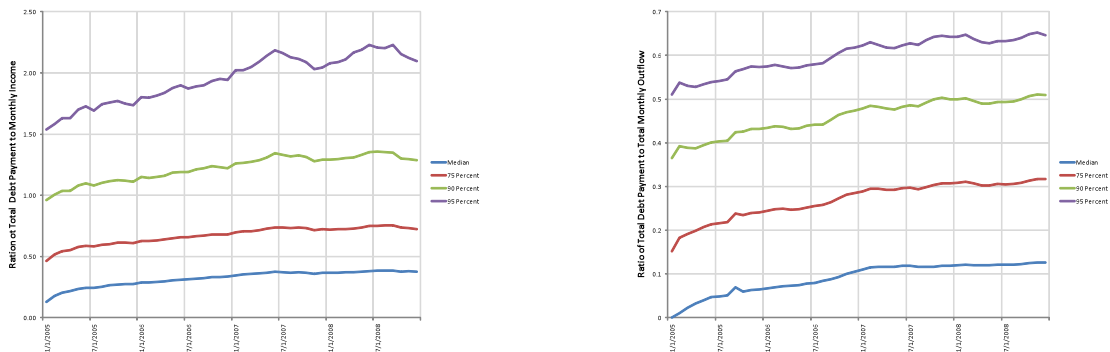


(c) Historical Monthly Credit-Card Payment



(d) Historical Monthly Credit-Card Payment to Income Ratio

Figure 5: Characteristics of credit-card balances, payments and their relationship to monthly income, based on data from January 2005 to December 2008. In Panel (a), the credit-card balance, which is the total balance across all credit cards of the customer in the month ending shown on the horizontal axis, is shown. In Panel (b), the ratio of credit-card balance to monthly income is shown. Panel (c) shows credit-card payments over this period while Panel (d) shows the ratio of credit-card payments to monthly income. For this calculation, the average income and credit-card payment over 6 months preceding the date on the horizontal axis or as many months as available, is used for the calculation. See Section 2 for further details.



(a) Historical Monthly Ratio of All Debt Payments to Income      (b) Historical Monthly Ratio of All Debt Payments to Total Outflow

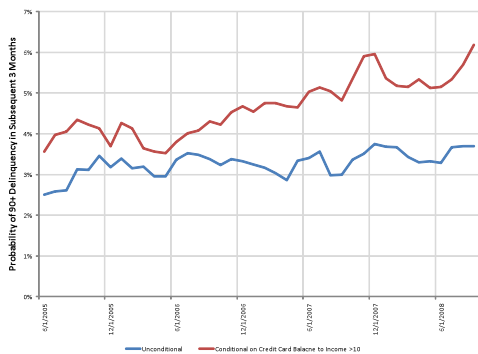
Figure 6: Characteristics of payments of all forms of debt to monthly income and total monthly outflow based on data from January 2005 to December 2008. Payments for the following debt instruments are included in this calculation: auto loans, student loans, banks loans, mortgages, and credit cards. The average of monthly figures over 6 months preceding the date on the horizontal axis or as many months as available, is used for this calculation. See Section 2 for further details.

In the models that we construct in later sections, the features we use include data items such as total inflow, total income, credit-card balance, etc. and the target variable is a binary outcome that indicates whether an account is delinquent by 90 days or more within the subsequent 3-, 6-, or 12-month window. In this section, we provide two illustrative examples of the relationship between certain features and subsequent delinquencies to develop intuition for both the data and our machine-learning algorithms. In Section 3.1, we consider high levels of the ratio of credit-card balances to income, and in Section 3.2, we explore the impact of sharp income declines.

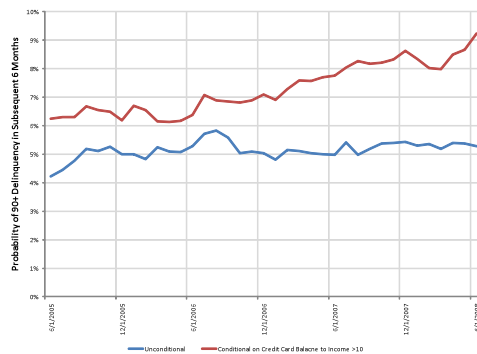
### 3.1 High Balance-to-Income Ratio

For each month in our sample (January 2005 to December 2008), we estimate the probability of 90-days-or-more delinquencies on any of the credit-card accounts in the following 3 and 6 months for all customers. We then compute the same estimates for customers with credit-card-balance-to-income ratios greater than 10.<sup>13</sup> This threshold is chosen arbitrarily, and is

<sup>13</sup>We define “credit-card balance” as the total credit-card balances across all financial institutions obtained from credit bureau data. Also, we measure the rate of 90-days-or-more delinquencies on any credit-card account across all financial institutions.



(a) Impact of High Balance-to-Income Ratios on Future 3-Month Delinquency Rates



(b) Impact of High Balance-to-Income Ratios on Future 6-Month Delinquency Rates

Figure 7: Delinquency rates of customers with high credit-card-payment-to-income ratios. For each month, the rates of 90-days-or-more delinquencies during the following 3- or 6-month window for all customers as well as customers with total-credit-card-debt-to-income ratios greater than 10 are computed. The data covers the period from January 2005 to December 2008. A 6-month moving-average of income is the denominator used in calculating the ratio of credit-card balance to income, hence the horizontal axis starts in June 2005 and ends in June or Sept 2008 depending on whether 6- or 3-month forecast windows apply.

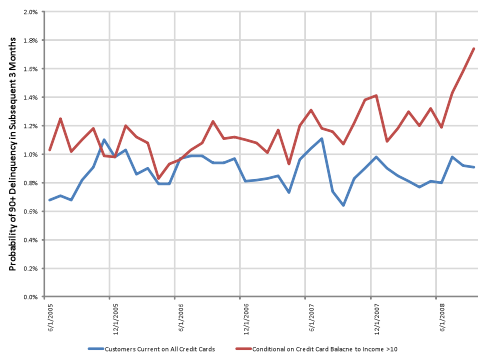
meant to identify consumers experiencing great financial distress. As shown in Figure 5(b), the top 5% of the sample have credit-card-balance-to-income ratios exceeding 10.

Figures 7(a) and 7(b) contain the outcome of this stratification for both 3-month and 6-month evaluation periods. Observe that the last month shown on the horizontal axis is June or Sept 2008 since we need 6 or 3 months, respectively, to calculate subsequent realized delinquency rates, and our sample ends in December 2008. Each axis also begins in June 2005 since we use the average income level over the prior 6 months for the denominator, hence we lose the first 6 months of the sample. This is consistent with the analysis presented in Figure 5(a)–5(d) and Figure 6(a)–6(b).

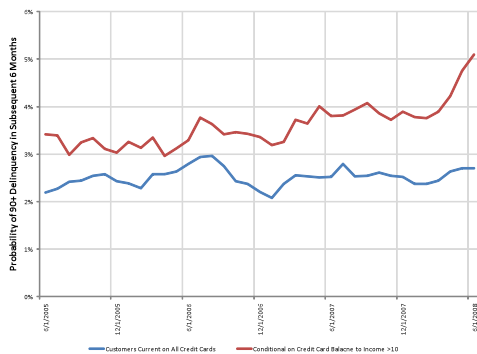
As shown in Figures 7(a) and 7(b), customers with very high credit-card-balance-to-income ratios are much more likely to experience delinquencies in subsequent 3- and 6-month periods. Furthermore, the separation has become wider in the latter part of the sample.

To evaluate the reliability of the relationship identified in Figures 7(a) and 7(b), in Figures 8(a) and 8(b) we report the same statistics but on the limited sample of customers who were current on *all* their credit-card accounts as of the conditioning date. The rate of future delinquency is lower (compare the blue lines in Figures 7(a) and 7(b) with those in Figures





(a) Impact of High Balance-to-Income Ratios on Future 3-Month Delinquency Rates



(b) Impact of High Balance-to-Income Ratios on Future 6-Month Delinquency Rates

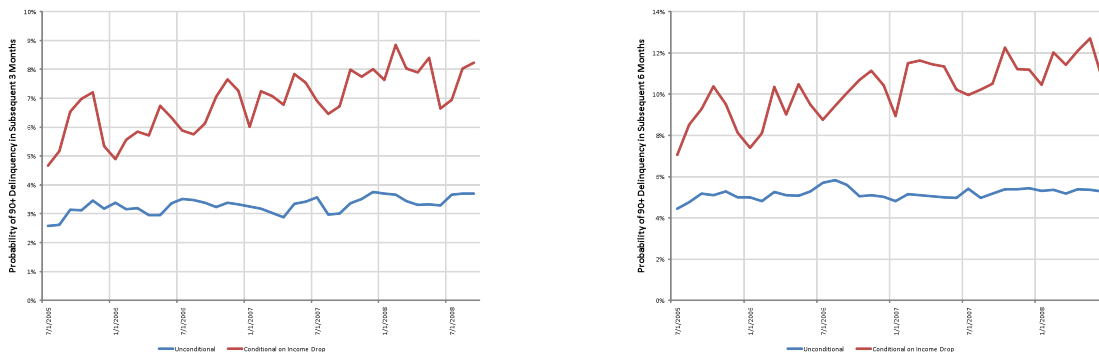
Figure 8: Delinquency rates for customers with high credit-card-payment-to-income ratios. For each month, the rates of 90-days-or-more delinquencies during the following 3- or 6-month window for customers who are current on all their credit-card accounts are computed, as well as customers who are current and have total-credit-card-debt-to-income ratios greater than 10. The data covers the period from January 2005 to December 2008. A 6-month moving-average of income is the denominator used in calculating the ratio of credit-card balance to income, hence the horizontal axis starts in June 2005, and ends in June or Sept 2008 depending on whether 6- or 3-month forecast windows apply.

8(a) and 8(b)), but the gap between delinquency rates for typical customers and for highly indebted customers is still clearly apparent.

This example illustrates the presence of statistically stable relationships that can be exploited in creating a statistical model for predicting future delinquencies and defaults. However, since the gap between the future delinquency rates of average customers versus highly indebted customers has become wider in more recent periods, it is clear that the data contain important non-stationarities. Therefore, any machine-learning model of delinquencies and defaults must be re-calibrated frequently enough to capture such changes in a timely manner. We will discuss this issue in more detail in Section 4.

### 3.2 Negative Income Shocks

A sharp and sudden drop in income—most likely due to unemployment—is a plausible potential predictor of future delinquency and default, and we proceed in a fashion similar to Section 3.1 to evaluate this possibility. The challenge is to identify income shocks that are “significant”, i.e., large relative to his/her historical levels and variability of income. To that



(a) Impact of Income Ratio Drop on Future 3-Month Default Rates

(b) Impact of Income Ratio Drop on Future 6-Month Default Rates

Figure 9: Delinquency rates for customers experiencing negative income shocks. For each month and customer, the income-shock variable is defined as the difference between the current month’s income and the 6-month moving-average of income over the prior 6 months, divided by the standard deviation of income over that period. For each month, the rates of 90-days-or-more delinquencies during the following 3- or 6-month window for all customers as well as customers with income shocks less than  $-2$  are calculated. The data covers the period from January 2005 to December 2008. Because of the 6-month moving-average for income, the horizontal axis starts in June 2005, and ends in June or Sept 2008 depending on whether 6- or 3-month forecast windows apply.

end, in each month, we construct an income-shock variable by taking the difference between the current month’s income and the 6-month moving-average of the income, and then divide this difference by the standard deviation of income over the same trailing 6-month window. By dividing the income drop by the estimated volatility, we are able to account for those individuals with jobs that produce more volatile income streams. We can stratify the sample according to this income-shock variable, and compare the 3- and 6-month subsequent 90-days-or-more delinquency rates for individuals with and without significant income shocks during each month in our sample (January 2005 to December 2008).

Figure 9(a) and 9(b) plots the 90-days-or-more delinquency rates for all customers (blue) and for those customers with income-shock variables less than  $-2$  (red). The gap between the delinquency rates of these two groups is clear. Furthermore, like the balances-to-income ratio of Section 3.1, this gap has also become wider over time, consistent with the patterns in Figures 7(a)–7(b) and Figures 8(a)–8(b).

## 4 Modeling Methodology

In this section, we describe the machine-learning algorithms we use to construct a delinquency forecast model for the Bank’s consumer credit and transactions data from January 2005 to April 2009.<sup>14</sup> This challenge is well suited to be formulated as a supervised learning problem, which is one of the most widely used techniques in the machine-learning literature. In the supervised learning framework, a learner is presented with input/output pairs from past data, in which the input data represent pre-identified attributes to be used to determine the output value. Such input data are commonly represented as a vector and, depending on learning algorithm, can consist of continuous and/or discrete values with or without missing data. The supervised learning problem is referred as a “regression problem” when the output is continuous, and as a “classification problem” when the output is discrete.

Once such input/output data are presented, the learner’s task is to find a function that correctly maps such input vectors to the output values. One brute force way of accomplishing this task is to “memorize” all previous values of input/output pairs. Although this correctly maps the input/output pairs in the training data set, it is unlikely to be successful in forecasting the output values if the input values are different from the ones in the training data set, or when the training data set contains noise. Therefore, the challenge of supervised learning is to find a function that generalizes beyond the training set, so that the resulting function also accurately maps out-of-sample inputs to out-of-sample outcomes.

Using this machine-learning approach, we build a credit-risk forecast model that predicts delinquencies and defaults for individual consumers. For example, an output of our model is a continuous variable between 0 and 1 that can be interpreted (under certain conditions) as an estimate of the probability of 90-days-or-more delinquency sometime within the next 3 months of a particular credit-card account, given specific input variables from that account. Using this model, we address two questions:

1. How can this model be used to improve decisions about credit-line cuts?
2. What is the rate of credit-card delinquencies in the aggregate economy?

In addition, we need to take model accuracy into account in addressing these two questions, and more detailed discussion of this and other modeling issues are provided in Section 5.

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<sup>14</sup>There is an extensive literature on machine learning, and we refer readers to Bishop (2006), Duda et al. (2000), and Vapnik (1998) for excellent overviews of this large and still-growing literature.

For both questions, selecting a parsimonious set of input variables that are relevant to credit risk is critical, and requires intuition for the underlying drivers of consumer credit, which cannot be resolved by the learning algorithm alone. The approach we described in Section 3 can be used to identify such variables, and Figure 10 summarizes our supervised learning framework.

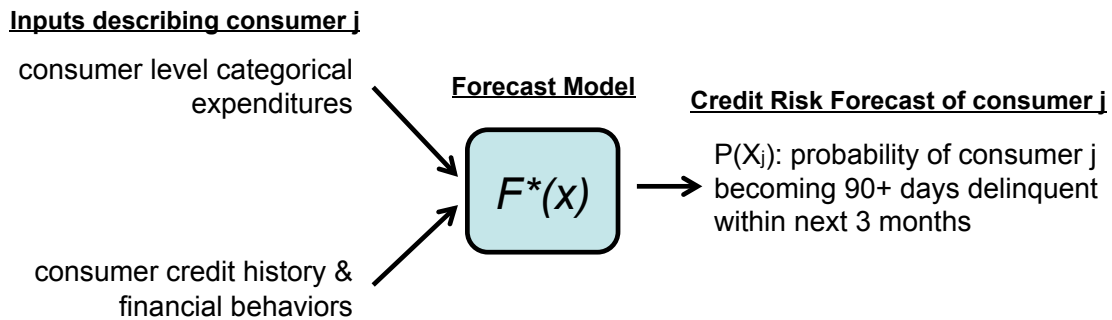


Figure 10: A summary of the machine-learning algorithm used to construct the consumer credit risk model.

## 4.1 Machine-Learning Framework

We use a Bayesian tree-based approach to construct our forecast model. Related approaches have been used by Atiya (2001), Shin et al. (2005), and Min et al. (2005), in which artificial neural networks (ANN) and support vector machines (SVM) were applied to the problem of predicting corporate bankruptcies. Ong et al. (2005) conclude that genetic-algorithms-based consumer credit-scoring models outperform models based on ANNs, decision trees, rough sets, and logistic regression. Martens et al. (2007) introduce rule-extraction methods for SVMs that can generate consumer-credit models with human-interpretable rules and little loss in accuracy. Galindo and Tamayo (2000) test CART decision-tree algorithms on mortgage-loan data to identify defaults, and also compare their results to K-Nearest Neighbor (KNN), ANNs, and probit models. Huang et al. (2004) provides an excellent survey of these and other studies.

In the remainder of this section, we describe the attributes we selected for our inputs and discuss the model’s performance. To identify the most powerful set of attributes, we combine bank transaction, consumer credit, and account balance databases and process them

in parallel to construct our feature vectors (see Figure 1).

## 4.2 Model Inputs

Table 4 lists the data elements from the three sources discussed in Section 2.1 that were used as inputs to our delinquency prediction model. As discussed in Section 2.2 not all data elements can be used because of legal restrictions. We conducted extensive exploratory data analysis, similar to illustrative examples in Section 3, and finalized the set of input variables to be used in constructing our feature vectors listed in Table 4. For all transaction-related items, their average values over the prior 6 months, or as many months as available, are used.

## 4.3 Comparison to CScore

The empirical results of Section 2.1 show that the CScore is indeed useful in rank-ordering customers by their delinquency and default rates, hence it should serve as a reasonable benchmark for the machine-learning forecasts.

Figure 11 compares the machine-learning forecasts of 90-days-or-more delinquencies of the Bank’s credit-card accounts to CScore for December 2008. The machine-learning forecasts are constructed by training the model on delinquency data from October 2008 to December 2008 and using input data from September 2008 to construct the feature vectors, and then computing “fitted values” using features in December 2008, which yields December-2008 forecasts of delinquencies in the 3-month period from January 2009 to to March 2009. CScore scores from December 2008 are plotted on the x-axis and the corresponding December-2008 machine-learning forecasts are plotted on the y-axis.

This figure shows that machine-learning forecasts differ substantially from CScore scores. In particular, there are a handful of accounts with relatively high CScore scores (note that higher CScore scores indicate higher credit quality or lower credit risk) that have high forecasted delinquency and default risk according to the machine-learning model. To provide further intuition for these conflicts, in Figure 12 we reproduce the same plot but have included color-coding for status of each account status as of the prediction date (December 2008)—green for accounts that are current (no late payments), blue for accounts that are 30-days delinquent, yellow for accounts 60-days delinquent, and red for accounts that are 90-days-or-more delinquent. Individuals with high CScore and high credit-risk forecasts are

those already delinquent in their payments as of the prediction date (in some cases 90-days-or-more). However, those with low CScore but low forecasted credit risk are individuals that are current in their payments. While these patterns are not meant to be a conclusive comparison of the predictive ability of the two measures, Figure 12 does suggest that machine-learning forecasts contain different information than traditional credit scores like CScore. We provide a more quantitative assessment in the next section.

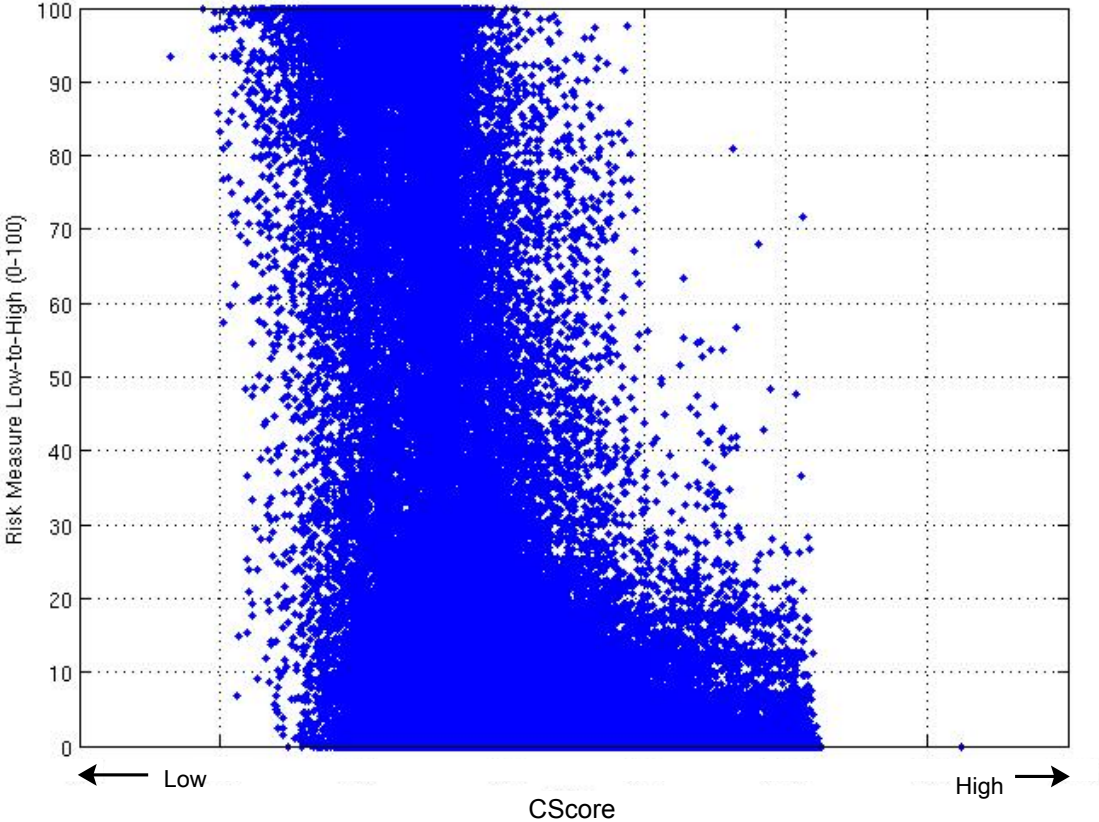


Figure 11: Comparison of December 2008 CScore (x-axis, and where higher values are associated with lower credit risk) and machine-learning forecasts of 90-days-or-more delinquency rates over subsequent 3-month windows using realized delinquency events from October to December 2008 and September 2008 feature vectors. Once calibrated, the machine-learning model is applied to the December 2008 feature vector, and the resulting “fitted values” are plotted (y-axis) against the December 2008 CScores.

<b>Transaction Data</b>	
Transaction Count	<b>By Category (Cont.)</b>
Total Inflow	
Total Outflow	Hotel expenses
	Travel expenses
<b>By Channel:</b>	Recreation
	Department Stores Expenses
ACH (Count, Inflow and Outflow)	Retail Stores Expenses
ATM (Count, Inflow and Outflow)	Clothing expenses
BPY (Count, Inflow and Outflow)	Discount Store Expenses
CC (Count, Inflow and Outflow)	Big Box Store Expenses
DC (Count, Inflow and Outflow)	Education Expenses
INT (Count, Inflow and Outflow)	Total Food Expenses
WIR (Count, Inflow and Outflow)	Grocery Expenses
	Restaurant Expenses
<b>By Category</b>	Bar Expenses
	Fast Food Expenses
Employment Inflow	Total Rest/Bars/Fast-Food
Mortgage payment	Healthcare related expenses
Credit car payment	Fitness expenses
Auto loan payment	Health insurance
Student loan payment	Gas stations expenses
All other types of loan payment	Vehicle expenses
Other line of credit payments	Car and other insurance
Brokerage net flow	Drug stores expenses
Dividends net flow	Government
Utilities Payments	Treasury
TV	Pension Inflow
Phone	Collection Agencies
Internet	Unemployment Inflow

Table 1: Inflows and outflows into accounts are collected by the Bank. Direction, amount, channel, and category information is associated with each transaction. The elements outlined in this table are collected and aggregated from a much larger set of channel and category data collected by the the Bank. Not all fields are used because of legal restrictions, e.g., insurance, healthcare related, social security, collection agencies, unemployment and several more were excluded.

<b>Individual Level</b>	<b>Account Level</b>
Credit Score File Age Bankruptcy (Date & Code) MSA & Zip Code	Age of Account Open/Closed Flag & Date of Closure Type (CC, MTG, AUT, etc) Balance Limit if applicable Payment Status 48-Month Payment Status History

Table 2: Data items extracted from credit bureaus. Not all fields extracted were used because of legal restrictions, e.g., MSA, zip code, file age, and account age data were not used.

<b>Deposit Data</b>	
Checking Account Balance Brokerage Account Balance Saving Account Balance	CD Account Balance IRA Account Balance

Table 3: Balance data for accounts held with the Bank. Not all fields extracted were used due to legal restrictions.



<b>Model Inputs</b>	
<p><b>Credit Bureau Data</b></p> <p>Total Number of Trade Lines  Number of Open Trade Lines  Number of closed trade lines  Number and balance of auto loans  Number and balance of credit cards  Number and balance of home line of credits  Number and balance of home loans  Number and balance of all other loans  Number and balance of all other lines of credit  Number and balance of all mortgages</p> <p>Balance of all auto loans to total debt  Balance of all credit cards to total debt  Balance of all home line of credit to total debt  Balance of all home loans to total debt  Balance of all other loans to total debt  Balance of all other lines of credit to total debt  Ratio of total mortgage balance to total debt</p> <p>Total credit-card balance to limits  Total home line of credit balances to limits  Total balance on all other lines of credit to limits</p> <p><b>Transaction Data</b></p> <p>Number of Transactions  Total inflow  Total outflow  Total pay inflow</p> <p>Total all food related expenses  Total grocery expenses  Total restaurant expenses  Total fast food expenses  Total bar expenses</p>	<p><b>Transaction Data (Cont.)</b></p> <p>Total expenses at discount stores  Total expenses at big-box stores  Total recreation expenses  Total clothing stores expenses  Total department store expenses  Total other retail stores expenses</p> <p>Total utilities expenses  Total cable TV &amp; Internet expenses  Total telephone expenses</p> <p>Total net flow from brokerage account  Total net flow from dividends and annuities</p> <p>Total gas station expenses  Total vehicle related expenses</p> <p>Total logging expenses  Total travel expenses</p> <p>Total credit-card payments  Total mortgage payments  Total outflow to car and student loan payments</p> <p>Total education related expenses</p> <p><b>Deposit Data</b></p> <p>Savings account balance  Checking account balance  CD account balance  Brokerage account balance</p>

Table 4: The final inputs used in the machine-learning model of consumer credit risk.

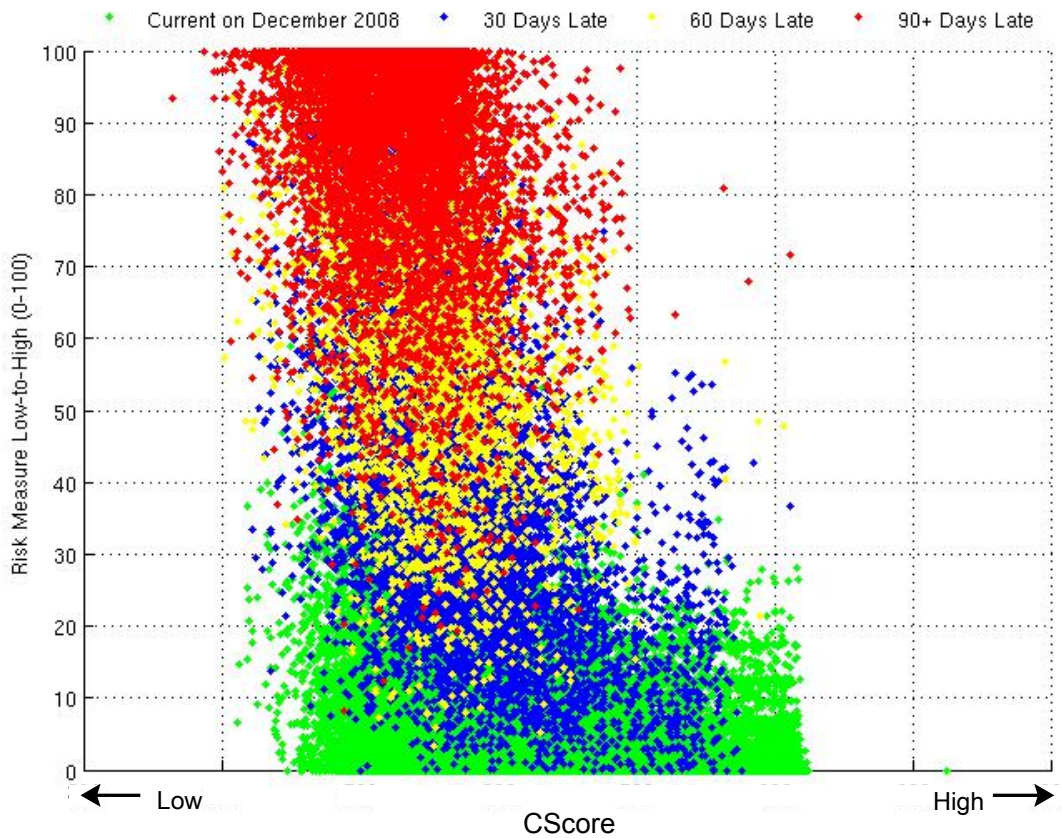


Figure 12: Color-coded comparison of December 2008 CScore (x-axis, and where higher values are associated with lower credit risk) and machine-learning forecasts of 90-days-or-more delinquency rates over subsequent 3-month windows using realized delinquency events from October to December 2008 and September 2008 feature vectors. Once calibrated, the machine-learning model is applied to the December 2008 feature vector, and the resulting “fitted values” are plotted (y-axis) against the December 2008 CScores. The color coding indicates whether an account is current (green), 30-days delinquent (blue), 60-days delinquent (yellow), or 90-days-or-more delinquent as of December 2008.

Input	Date	Training Period		Prediction Date	Evaluation Period	
		Start Date	End Date		Start Date	End Date
	Jan-08	Feb-08	Apr-08	Apr-08	May-08	Jul-08
	Feb-08	Mar-08	May-08	May-08	Jun-08	Aug-08
	Mar-08	Apr-08	Jun-08	Jun-08	Jul-08	Sep-08
	Apr-08	May-08	Jul-08	Jul-08	Aug-08	Oct-08
	May-08	Jun-08	Aug-08	Aug-08	Sep-08	Nov-08
	Jun-08	Jul-08	Sep-08	Sep-08	Oct-08	Dec-08
	Jul-08	Aug-08	Oct-08	Oct-08	Nov-08	Jan-09
	Aug-08	Sep-08	Nov-08	Nov-08	Dec-08	Feb-09
	Sep-08	Oct-08	Dec-08	Dec-08	Jan-09	Mar-09
	Oct-08	Nov-08	Jan-09	Jan-09	Feb-09	Apr-09

Table 5: The 10 calibration and testing periods used in evaluating the performance of machine-learning forecasts of consumer credit risk.

#### 4.4 Model Evaluation

We now turn to a more rigorous evaluation of our models. We will focus on the model used to produce forecasts of 90-days-or-more delinquencies over subsequent 3-month windows for the period from January 2008 to April 2009. We choose this period because it includes the most severe deterioration in consumer credit quality in recent history, providing a natural laboratory for gauging the performance of machine-learning models during periods of financial dislocation. To minimize the effects of look-ahead bias, we only train the model based on delinquencies over 3-month windows that were observable at the time of the forecast. For example, the first model was calibrated using input data from January 2008 and realized delinquencies over the 3-month window from February to April 2008, and then the parameter estimates were applied to the input data in April 2008 to produce forecasts of delinquencies over the 3-month window from May to July 2008. This rolling-window approach yields 10 calibration and testing periods listed in Table 5, and Table 6 reports sample sizes for each of the 10 prediction periods. The percentage of 90-days-or-more delinquent accounts varies from 2.0% to 2.5%.

The output of the machine-learning model is a continuous variable that, under certain conditions, can be interpreted as an estimate of the probability of an account becoming 90-days-or-more delinquent during the subsequent 3-month forecasting window. One measure of the model’s success is the difference between the forecasts of those accounts that did

3 Months Starting Date	3 Months Ending Date	Customers going 90+ days delinquent (%)	Customers NOT going 90+ days delinquent (%)
May-08	Jul-08	2.4	97.6
Jun-08	Aug-08	2.2	97.8
Jul-08	Sep-08	2.1	97.9
Aug-08	Oct-08	2.0	98.0
Sep-08	Nov-08	2.1	97.9
Oct-08	Dec-08	2.1	97.9
Nov-08	Jan-09	2.1	97.9
Dec-08	Feb-09	2.3	97.7
Jan-09	Mar-09	2.6	97.4
Feb-09	Apr-09	2.5	97.5

Table 6: Sample sizes of accounts and 90-days-or-more delinquencies for each of the 10 3-month evaluation windows from May 2008 to April 2009. This data is based on Bank-issued credit cards only.

become delinquent and those that did not; if accounts that subsequently became delinquent had the same forecasts as those that did not, the forecasts have no value. Table 7 reports the difference between the average forecast associated with accounts that did and did not fall into the 90-days-or-more delinquency category over the 10 evaluation periods. For example, during the period from May to July 2008, the model’s average forecast among the 2.4% of accounts (see Table 6) that did become 90-days-or-more delinquent was 61.2, whereas the average forecast among the 97.6% of accounts that did not fall into this category was only 1.0. We should emphasize that these forecasts are truly out-of-sample, being based on input data available at the end of April 2008, and calibrated using data from February to April 2008 (see Table 5). This implies significant forecast power in distinguishing between accounts that will and will not become delinquent over subsequent 3-month periods. Moreover, this forecast power does not diminish over the 10 calibration and evaluation periods, likely due to the frequent re-calibration of the model that captures some of the changing dynamics of consumer behavior documented in Section 3.

Another interesting challenge for the machine-learning model is the identification of “straight-rollers”, accounts that are current as of the forecast date but become 90-days-or-more delinquent within the subsequent 3-month window. Such accounts typically do not

become current after short delinquencies, but usually “roll straight” into the most extreme delinquency level that is commonly used as an indication of likely default and a charge-off loss for that bank. Recall from the examples of Section 3 (see, in particular, Figures 8(a) and 8(b)) that certain input variables do seem to be related to future delinquencies, even among customers with current accounts as of the forecast date.

To evaluate the machine-learning model’s ability to identify straight-rollers, we compare the model’s average forecast among customers who were current on their accounts but became 90-days-or-more delinquent with the average forecast among customers who were current and did not become delinquent. Since this is a harder learning problem, we expect the model’s performance to be less impressive than the values reported in Table 7. Nevertheless the values reported in Table 8 still indicate that the model is remarkably powerful, and is clearly able to separating the two populations. For example, using input date from April 2008, the average model forecast among customers who were current on their account and did not become 90-days-or-more delinquent from May to July 2008 was 0.7, while the average forecast for straight-rollers was 10.3. And as before, the degree of separation is remarkably consistent across the 10 evaluation periods listed in Table 8.

Training Window		Prediction Date	Evaluation Window		Average Predicted Probability of Going 90+ Delinquent		
Start	End		Start	End	Among All	Among Customers Going 90+ Days Delinquent	Among Customers <b>NOT</b> Going 90+ Days Delinquent
Feb-08	Apr-08	<b>Apr-08</b>	May-08	Jul-08	2.5	<b>61.2</b>	1.0
Mar-08	May-08	<b>May-08</b>	Jun-08	Aug-08	2.0	<b>62.1</b>	0.6
Apr-08	Jun-08	<b>Jun-08</b>	Jul-08	Sep-08	1.9	<b>60.4</b>	0.7
May-08	Jul-08	<b>Jul-08</b>	Aug-08	Oct-08	1.9	<b>62.5</b>	0.6
Jun-08	Aug-08	<b>Aug-08</b>	Sep-08	Nov-08	2.0	<b>62.4</b>	0.7
Jul-08	Sep-08	<b>Sep-08</b>	Oct-08	Dec-08	2.1	<b>63.6</b>	0.8
Aug-08	Oct-08	<b>Oct-08</b>	Nov-08	Jan-09	2.1	<b>62.5</b>	0.8
Sep-08	Nov-08	<b>Nov-08</b>	Dec-08	Feb-09	2.2	<b>59.8</b>	0.8
Oct-08	Dec-08	<b>Dec-08</b>	Jan-09	Mar-09	2.4	<b>60.8</b>	0.8
Nov-08	Jan-09	<b>Jan-09</b>	Feb-09	Apr-09	2.4	<b>62.8</b>	0.9

Table 7: Performance metrics for machine-learning forecasts over 10 3-month evaluation windows from May 2008 to April 2009. For each evaluation period, the model is calibrated on credit-card delinquency data over the 3-month period specified in the *Training Window* columns, and predictions are based on the data available as of the date in the *Prediction Date* column. For example, the first row reports the performance of the model calibrated using input data available in January 2008 and credit-card delinquency data from February to April 2008, and applied to the April 2008 feature vector to generate forecasts of delinquencies from May to July 2008. Average model forecasts over all customers, and customers that (ex-post) did and did not become 90-days-or-more delinquent over the evaluation period are also reported.

Training Window		Prediction Date	Evaluation Window		Average Predicted Probability of Going 90+ Delinquent		
Start	End		Start	End	Among All	Among Customers Going 90+ Days Delinquent	Among Customers NOT Going 90+ Days Delinquent
Feb-08	Apr-08	<b>Apr-08</b>	May-08	Jul-08	<b>0.7</b>	<b>10.3</b>	0.7
Mar-08	May-08	<b>May-08</b>	Jun-08	Aug-08	<b>0.4</b>	<b>8.0</b>	0.4
Apr-08	Jun-08	<b>Jun-08</b>	Jul-08	Sep-08	<b>0.5</b>	<b>9.7</b>	0.4
May-08	Jul-08	<b>Jul-08</b>	Aug-08	Oct-08	<b>0.4</b>	<b>8.6</b>	0.3
Jun-08	Aug-08	<b>Aug-08</b>	Sep-08	Nov-08	<b>0.4</b>	<b>9.6</b>	0.4
Jul-08	Sep-08	<b>Sep-08</b>	Oct-08	Dec-08	<b>0.5</b>	<b>9.5</b>	0.4
Aug-08	Oct-08	<b>Oct-08</b>	Nov-08	Jan-09	<b>0.5</b>	<b>10.1</b>	0.4
Sep-08	Nov-08	<b>Nov-08</b>	Dec-08	Feb-09	<b>0.5</b>	<b>10.6</b>	0.5
Oct-08	Dec-08	<b>Dec-08</b>	Jan-09	Mar-09	<b>0.5</b>	<b>8.6</b>	0.5
Nov-08	Jan-09	<b>Jan-09</b>	Feb-09	Apr-09	<b>0.5</b>	<b>10.2</b>	0.5

Table 8: Performance metrics for machine-learning forecasts over 10 3-month evaluation windows from May 2008 to April 2009 for “straight-rollers”, customers who are current as of the forecast date but who become 90-days-or-more delinquent in following 3-month window. For each evaluation period, the model is calibrated on credit-card delinquency data over the 3-month period specified in the *Training Window* columns and predictions are based on the data available as of the date in the *Prediction Date* column, using only customers who are current as of the forecast date. For example, the first row shows the performance of the model calibrated using input data available in January 2008 and credit-card delinquency data from February to April 2008, and applied to the April 2008 feature vector to generate forecasts about delinquencies from May to July 2008.

## 5 Applications

In this section we apply the models and methods of Sections 3 and 4 to two specific challenges in consumer-credit risk management: deciding when and how much to cut individual-account credit lines, and forecasting aggregate consumer credit delinquencies for the purpose of enterprise-wide and macroprudential risk management.

The former application is of particular interest from the start of 2008 through early 2009 as banks and other financial institutions responded to the developing crisis with massive risk reductions. In Section 5.1, we use input data from December 2008 to make forecasts about individual-account delinquencies from January to March 2009, which are then compared with realized delinquencies during that period to evaluate the out-of-sample predictive ability of our machine-learning model. By making a few simplifying but plausible assumptions, in Section 5.2 we are able to estimate the cost savings or value-added that would have been generated by this approach.

In Section 5.3, we investigate the robustness of machine-learning forecasts by comparing their performance across subsets of the data stratified by the number of features available for each account. We find that delinquencies among accounts with more features are significantly easier to predict than those with few features. This difference underscores the potential value of the conditioning information we have constructed from the combined data items in Table 4.

In Section 5.4, we combine the individual-account forecasts to produce an aggregate forecast of consumer-credit delinquencies, which we propose as a measure of systemic risk. Despite the fact that our data represents a small subset of the Bank’s customer base, the machine-learning model is surprisingly effective in capturing the changing risk characteristics of the population. In fact, our results indicate near-monotonic increases in aggregate consumer-credit risk from January 2005 to April 2009, suggesting that consumer-based measures of financial distress may play an important role in macroprudential risk management.

### 5.1 Credit-Line Risk Management

Once a line of credit is extended to a customer by a bank, the bank actively monitors the creditworthiness of the customer. The credit line may be increased or the interest rate reduced in the case of improving creditworthiness, while the opposite may occur if there is



a decline in the customer’s credit profile. Therefore, the particular application considered in this section is the primary focus of any consumer-credit business. For the business to be successful, credit-line decisions must be based on a model that captures changes in a customer’s risk profile in a timely and accurate manner. In fact, during the sample period of our dataset, the major decision facing banks and other consumer lenders was identifying high-risk customers and taking the appropriate actions with respect to their credit lines, i.e., lowering credit limits, increasing interest rates, or some combination of both. However, to simplify our analysis, we assume that once a customer is identified as high-risk, his/her remaining (unused) credit line will be reduced to zero.

As noted in Section 4, the type of problem in which the target variable or output is binary is known as a *classification* problem in the machine-learning literature. Although the forecasts produced by the model described in Section 4 can be interpreted, under certain assumptions, as estimated delinquency probabilities, these numerical scores can be easily converted to binary decisions by comparing the forecast to a specified threshold and classifying customers with scores exceeding that threshold as high-risk. Setting the appropriate level for this threshold involves a tradeoff. A very low threshold results in many customers being classified as high-risk, and while it may correctly identify customer who are genuinely high-risk and about to fall behind on their payments, this choice may also result in many low-risk customers incorrectly classified as high-risk. On the other hand, a high threshold may allow too many high-risk customers to be classified as low-risk, yielding greater subsequent delinquencies and losses.

This type of trade-off is present in any classification problem, and involves the well-known statistical balancing act between Type-I (false positives) and Type-II (false negatives) error in classical hypothesis testing contexts. In the more practical context of credit-risk management, the trade-off can be made explicitly by a cost/benefit analysis of false positives vs. false negatives, and selecting the threshold that will optimize some criterion function in which such costs and benefits are inputs. Before turning to this cost/benefit analysis, we wish to develop further intuition for the statistical properties of this tradeoff.

To understand the statistical behavior of any classification algorithm, one commonly used performance metric in the machine-learning and statistics literatures is a  $2 \times 2$  contingency table often called the “confusion matrix” for reasons that will become obvious once we define

its entries (see the left side of Figure 13).<sup>15</sup> The two rows correspond to the two types of customers in our sample, low-risk or “Good” and high-risk or “Bad”, or *ex post* realizations of delinquency. In our application, we define Good customers as those who did not become 90-days-or-more delinquent during the forecast period, and Bad customers of those who did. The two columns correspond *ex ante* classifications of the customers into these same two categories, Good and Bad. When any forecast model is applied to a given set of customers, each customer falls into one of these four cells, hence the performance of the model can be gauged by the relative frequencies of the entries (see Figure 13). In the traditional Neyman-Pearson hypothesis-testing framework, the lower-left entry is defined as Type-I error and the upper right as Type-II error, and the objective of the classical statistician is to find testing procedures to reduce Type-II error (or maximize “power”) subject to a fixed level of Type-I error (or “size”).

The right side of Figure 13 contains the confusion matrix for our machine-learning model for December 2008 using a 3-month forecasting horizon from January to March 2009 and a threshold of 20%, i.e., accounts with estimated delinquency probabilities greater than 20% are classified as “Bad” and 20% or below as “Good”. For this month, the machine-learning model classified  $96.37\% + 0.44\% = 96.81\%$  of the accounts as good, of which 96.37% were, in fact, good and 0.44% of which were bad, i.e., were 90-days-or-more delinquent sometime during January to March 2009. Similarly, of the  $0.44\% + 2.13\% = 2.57\%$  that became 90-days-or-more delinquent during the three-month forecast period, the model correctly identified 2.13% of the accounts, a correct-classification rate of 82.9%. Of course this success comes at the cost of labeling 1.06% of customers who ended up not falling into the 90-days-or-more delinquency state as bad.

Four specific performance metrics are typically computed from the entries of the confusion matrix, and their formulas are given in Figure 13. “Precision” refers to the model’s accuracy in instances that the model classified an account as bad, while “Recall” refers to the number of bad accounts identified by the model divided by the actual number of bad accounts. “True positive rate” and “false positive rate” are self-explanatory. Ideally, we would like our model to have very high precision and recall, or, equivalently, to have a high “true positive rate” and a low “false positive rate”. The role of the classification threshold is now clear: if a lender

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<sup>15</sup>A confusion matrix is typically used in supervised learning to describe actual and predicted classifications by the learning model. See Kohavi and Provost (1998) for a formal definition of this and other machine-learning terminology.

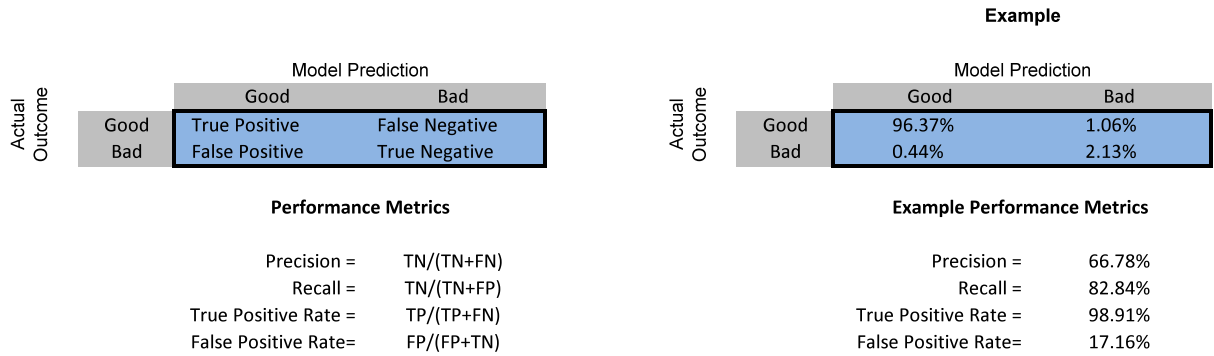


Figure 13: Confusion matrix framework, model performance metrics, and numerical example based on December 2008 machine-learning forecasts of 90-days-or-more delinquencies over the 3-month forecast horizon from January to March 2009. Rows correspond to actual customer types (“Bad” is defined as 90-days-or more delinquent) and columns correspond to forecasted types. TN: True Negative, FN: False Negative, FP: False Positive; TN: True Negative.

wishes to aggressively identify bad accounts by using a lower threshold, such aggressiveness will result in more incorrect classifications of good accounts as bad, leading to the opportunity cost of lost interest income from cutting the credit lines of low-risk customers. Figure 14 illustrates such tradeoffs for four different threshold levels.

By applying our forecast model with varying levels of the classification threshold, we can trace out explicitly the trade-off between true and false positives, and this trade-off is plotted in Figure 15. The blue line, known as the Receiver Operating Characteristic (ROC) curve in the machine-learning literature, is the pairwise plot of true and false positive rates for various classification thresholds (green line), and as the threshold increases, Figure 15 shows that the true positive rate increases, but the false positive rate increases as well.

The ROC curve shows that the trade-offs are not linear, meaning that increase in true positive rates is not always commensurate with the increase false positive rates. If the cost of false positives is equal to the gain of true positives, the optimal threshold will correspond to the tangent point of the ROC curve with the 45° line. If the cost/benefit trade-off is not one-for-one, the optimal threshold is different, but can easily be determined from the ROC curve. We provide a simple framework for such optimization in next section.

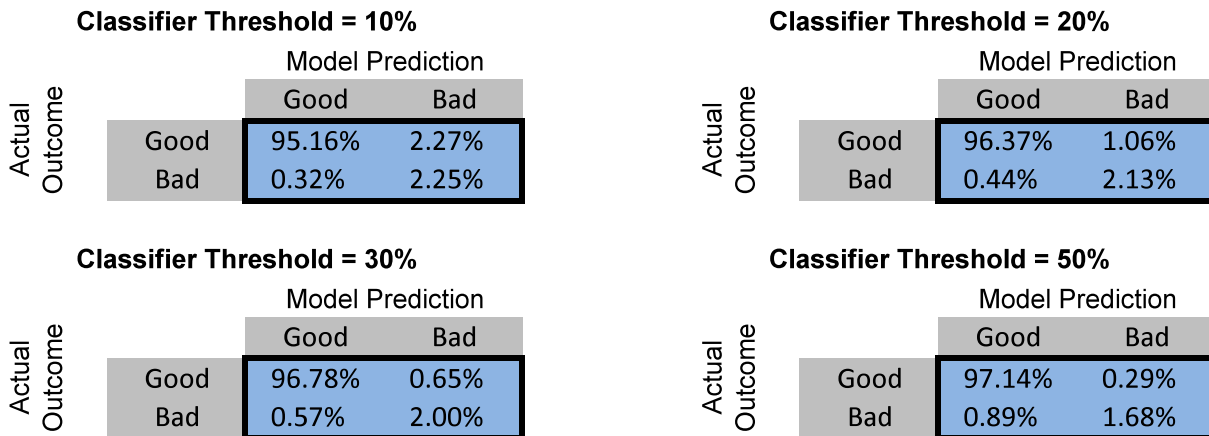


Figure 14: Confusion matrices of machine-learning forecasts of 90-days-or-more delinquencies for four classification thresholds. Rows correspond to actual customer types (“Bad” is defined as 90-days-or more delinquent). The numerical example is based on the December 2008 model forecast for the 3-month forecast horizon from January to March 2009.

## 5.2 The Value-Added of Credit Risk Management

To calculate the practical value-added of machine-learning forecasts, we require some simplifying assumptions regarding the revenues and costs of the consumer lending business. As a starting point, suppose that in the absence of any forecasts, a lender will take no action with respect to credit risk, implying that customers who default or are delinquent will generate losses for the lender, and customers who are current in their payments will generate financing fees on their running balances. To make the analysis simpler, assume that all non-defaulting customers have a running balance of  $B_r$ , and defaulting customers also start with a balance of  $B_r$  but will increase their balance to  $B_d$  between the point at which the lender’s credit-risk management decision is made and the point when they default. For example, an empirically plausible set of parameters are  $B_r = \$1,000$  and  $B_d = \$1,200$ . As it will be clear shortly, only the ratio of run-up balance,  $B_d - B_r$ , and the running balance,  $B_r$ , enters into the potential savings calculation. In this particular example, the critical factor is the 20% run-up in the balance before default.

While the lender’s loss from default is simply  $B_d$ , calculating the profit due to a customer in good-standing requires further discussion. For now, let the present value of the profit from such a customer be given by a fraction of the running balance, denoted by  $P_m$  for “Profit

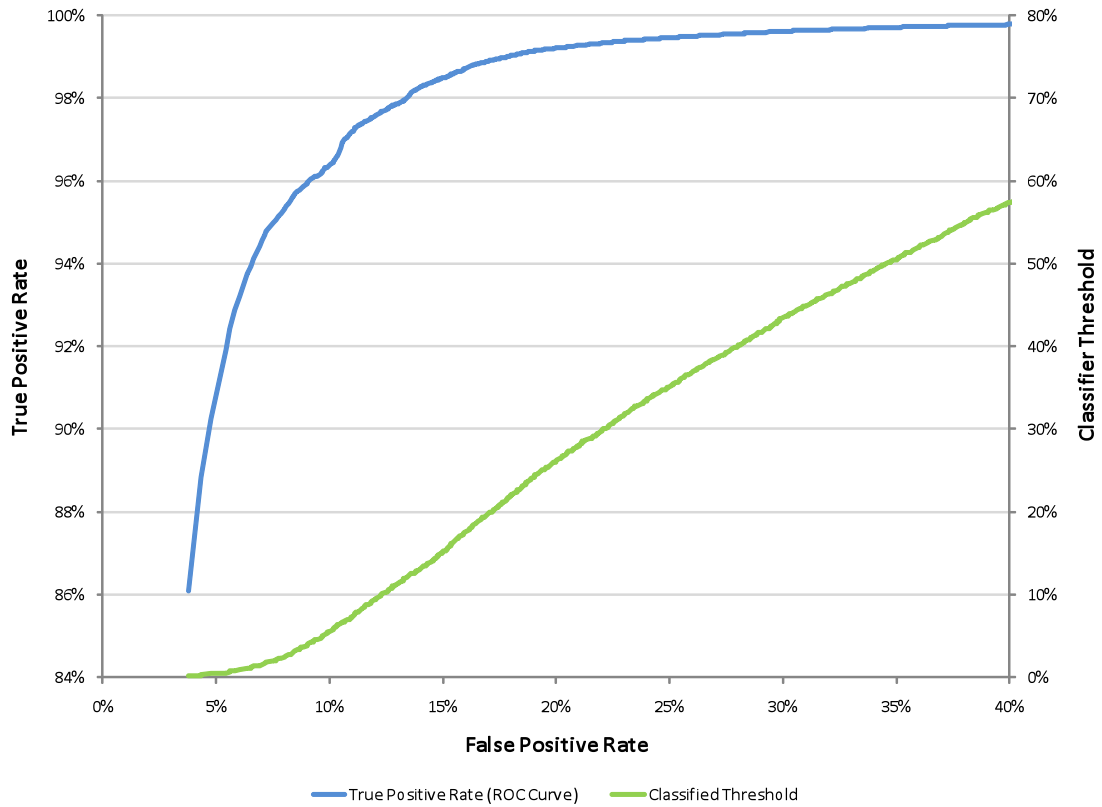


Figure 15: A Receiver Operating Characteristic (ROC) curve and corresponding classification threshold value of December 2008 machine-learning forecasts of 90-days-or-more delinquencies over the 3-month forecast horizon from January to March 2009.

Margin Rate”, which can be a function of the interest rate and the expected length of time the account holder will use this credit line.<sup>16</sup> Given these assumptions, we can calculate the profit with and without a forecast model as follows:

$$\begin{aligned}\text{Profit without Forecast} &= (\text{TP} + \text{FN}) \cdot B_r \cdot P_m - (\text{FP} + \text{TN}) \cdot B_d \\ \text{Profit with Forecast} &= \text{TP} \cdot B_r \cdot P_m - \text{FP} \cdot B_d - \text{TN} \cdot B_r\end{aligned}$$

where (see Figure 13):

TN  $\equiv$  True Negative

TP  $\equiv$  True Positive

FN  $\equiv$  False Negative

FP  $\equiv$  False Positive .

Then the dollar savings is simply given by the difference:

$$\text{Savings} = \text{TN}(B_d - B_r) - \text{FN} \cdot B_r \cdot P_m \quad (1)$$

where

$\text{TN}(B_d - B_r)$  = Savings due to correct decision

$\text{FN} \cdot B_r \cdot P_m$  = Opportunity cost due to incorrect decision .

We can simplify this even further by dividing the savings by the savings that would have been possible under the perfect-foresight case, i.e., the case in which all bad customers, and only bad customers, are identified and their credit was reduced. We will call this ratio the “value added” (VA) of the forecast, which is given by:

$$\text{Value Added} \equiv \frac{\text{TN} - \text{FN} \cdot P_m \cdot \frac{B_r}{B_d - B_r}}{\text{TN} + \text{FP}} \quad (2)$$

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<sup>16</sup>For simplicity, we make the assumption that all “good” customers who are subjected to a line reduction immediately pay off their balances and close their accounts. To the extent that most customers do not behave in this manner, our estimates of the potential cost-savings are conservative.

Note that the final expression depends on the running and default balance only through the ratio of the running balance to the difference between the default and running balance. For example, it is clear from (1) that if this difference is zero, no value added can be achieved, which is expected because it is only the difference between  $B_d$  and  $B_r$  that can potentially be captured by a correct and timely decision by the lender.

Before attempting to evaluate VA, we must first specify how the profit margin rate,  $P_m$ , is calibrated. This factor is meant to capture the total financing charge that customers pay to the lender on their outstanding balances. This value depends on the credit-card interest rate as well as the speed with which customers pay off their balances. For simplicity, we assume that the difference between the credit-card interest rate and the lender's funding cost is a fixed rate given by  $r$ , and customers pay back their entire credit card debt over an  $N$ -year period. In this simple setting, the financing charge is an annuity hence  $P_m$  is given by: <sup>17</sup>

$$P_m = r \cdot \frac{1 - (1 + r)^{-N}}{r} = 1 - (1 + r)^{-N}. \quad (3)$$

Combining (2) and (3) yields the following expression for VA:

$$VA(r, N, TN, FN, FP) = \frac{TN - FN \cdot [1 - (1 + r)^{-N}] \cdot [\frac{B_d}{B_r} - 1]^{-1}}{TN + FP} \quad (4)$$

Using this expression and the trade-off shown in Figure 15, we can evaluate the potential value-added of our forecasting model. To develop intuition for the properties of VA and its robustness to the various assumptions, we evaluate VA for four levels of run-up before default (10%, 20%, 30% and 50%) and three horizons over which customers amortize their credit-card balances (3 year, 5 years and 10 years). In all cases, we assume  $r = 5\%$ . Figure 16 contains the results.

In all cases, there are optimal classification thresholds for maximizing value-added. An aggressive line reduction strategy, i.e. a very low threshold, can actually result in net losses for the lender. VA is always higher for shorter amortization periods, which is to be expected

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<sup>17</sup>In the interest of parsimony and simplicity, we have implicitly assumed that the lender's funding cost is also  $r$ . For example, if  $r = 5\%$  implies that the lender pays 5% for funds and charges an interest rate of 10% on credit-card balances.

since the lender collects financing fees over that period. Longer amortization periods imply more fees collected from non-defaulting customers and increases the cost of incorrect decisions. Furthermore, larger run-ups before default also implies higher potential VA from correct decisions. Finally, each set of assumptions corresponds to a different optimal threshold. For lower run-ups before default and longer amortization periods, forecasts should be used less aggressively (higher thresholds), and for higher run-ups before default and shorter amortization periods, a more aggressive policy is appropriate. Depending on the parameters, the value-added ranges from 30% (assuming a 10% run-up before default and a 10-year amortization period) to 70% (assuming a 50% run-up before default and a 3-year amortization period).

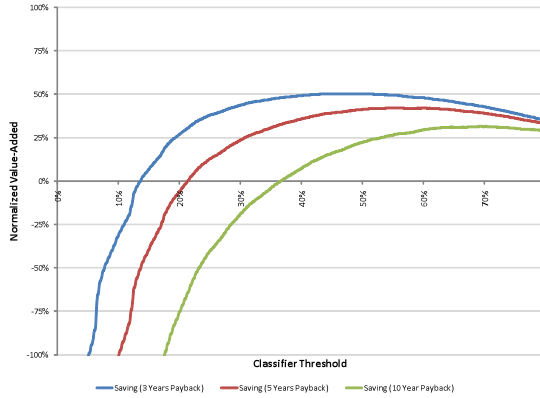
### 5.3 Robustness

Although the dataset used in our analysis is from a large consumer bank in the U.S., it does not cover the entirety of a consumer’s spending and saving activities. Consumers usually have business relationships with multiple banks and credit-card companies, hence it is rare that all elements of the feature vectors we described in Section 4.2 are available for a given account. To check the robustness of our model to missing features, we divide our data set into equally sized samples separated by the availability of features. More sophisticated analysis can be performed—using feature-selection techniques widely used in machine learning—to assign weights on features by their importance, however in this study, we focus on the simpler approach of rank-ordering the data by simply counting number of missing values, without differentiating among the features that are missing.

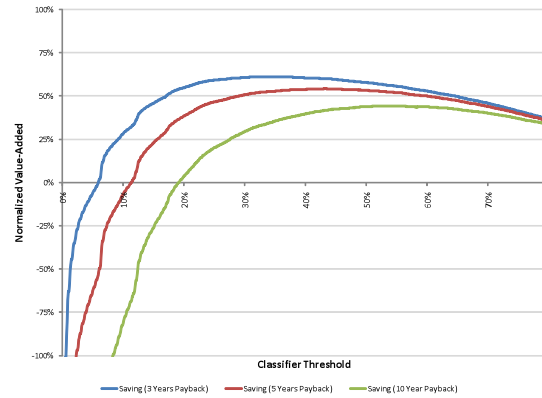
After dividing the data set in this manner, for each group we perform 10-fold cross validation (CV), i.e., stratifying the dataset into 10 bins, using 9 of them for training of the model and the remaining one for testing, and repeating this step 9 times using different set of bins for testing and training. Figure 17 contains the average confusion matrix (see Section 5.1) across the 10-fold CV results for the 90-days-or-more delinquency forecasts over a 6-month forecast horizon. The panels are labeled Group I through Group IV where Group I contains accounts with the largest number of missing features, and Group IV contains accounts with the fewest.

This analysis shows a significant improvement in both precision and recall values from Group I to Group II. Group I customers are typically single-account holders, i.e., individuals

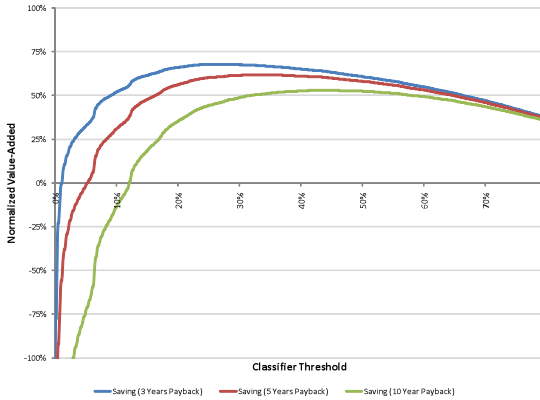




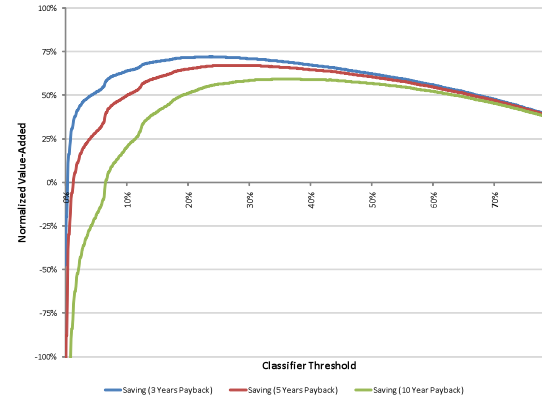
(a) Value-added for 10% run up before default ( $B_d = 1.1B_r$ )



(b) Value-added for 20% run-up before default ( $B_d = 1.2B_r$ )



(c) Value-added for 30% run up before default ( $B_d = 1.3B_r$ )



(d) Value-added for 50% run-up before default ( $B_d = 1.5B_r$ )

Figure 16: Value-added (VA) of December 2008 machine-learning forecasts of 90-days-or-more delinquencies over the 3-month forecast horizon from January to March 2009, with parameters  $r = 5\%$ , run-up before default of 10%, 20%, 30% or 50%, and amortization period  $N$  of 3, 5 or 10 years.

with only a checking account and a low number of transactions, also known as “thin-file” customers because their files contain few records. However, as a customer’s file becomes “thicker” with more transactions and accounts, the ability of machine-learning models to forecast future delinquencies improves, presumably because of the additional features available for such customers.

Figure 18 plots the VA measure (4) of Section 5.2,<sup>18</sup> estimated via the the 10-fold CV performed on four different forecasts of 90-days-or-more delinquencies over forecast horizons of 3, 6, 9, and 12 months. As expected, the model’s forecast accuracy becomes better as the prediction horizon lengthens because the forecast problem is easier, e.g., predicting 90-days-or-more delinquency over a 12-month horizon is considerably easier than over a 3-month horizon. Also, across all four models, there is a significant improvement in VA from Group I to Group II, whereas the performance of the model in Groups III and IV are comparable to that of Group II, with Group IV yielding slightly lower performance.

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<sup>18</sup>We assume an amortization period  $N = 3$  years, a run-up before default of 30% ( $B_d = 1.3B_r$ ), and an interest rate differential ( $r = 5\%$ ).

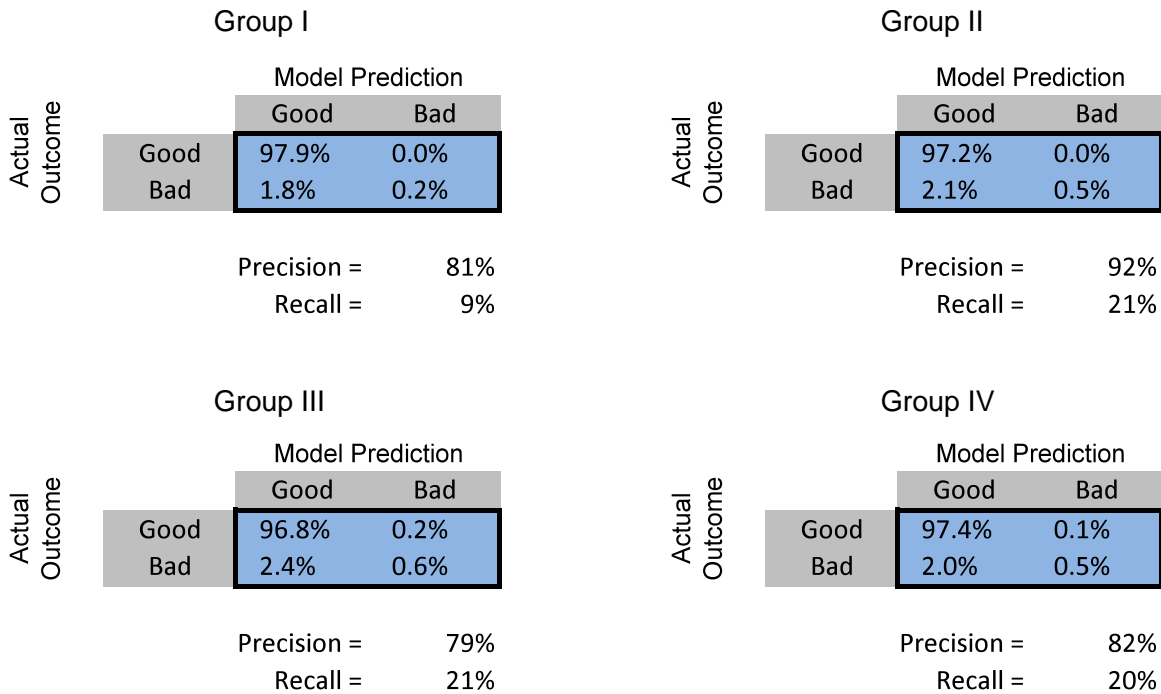


Figure 17: Average confusion matrices, averaged over 10 trials of machine-learning forecasts of 90-days-or-more delinquency over a 6-month forecast horizon on data from January 2005 through April 2008, with each trial based on a randomly chosen 10% subset of the entire dataset. For each trial, four models are estimated, one for each of four stratified sub-samples of equal size, stratified by on the number of non-missing features. Group I is the sample of accounts with the fewest available features (“thinnest”), and Group IV is the sample with the most (“thickest”).

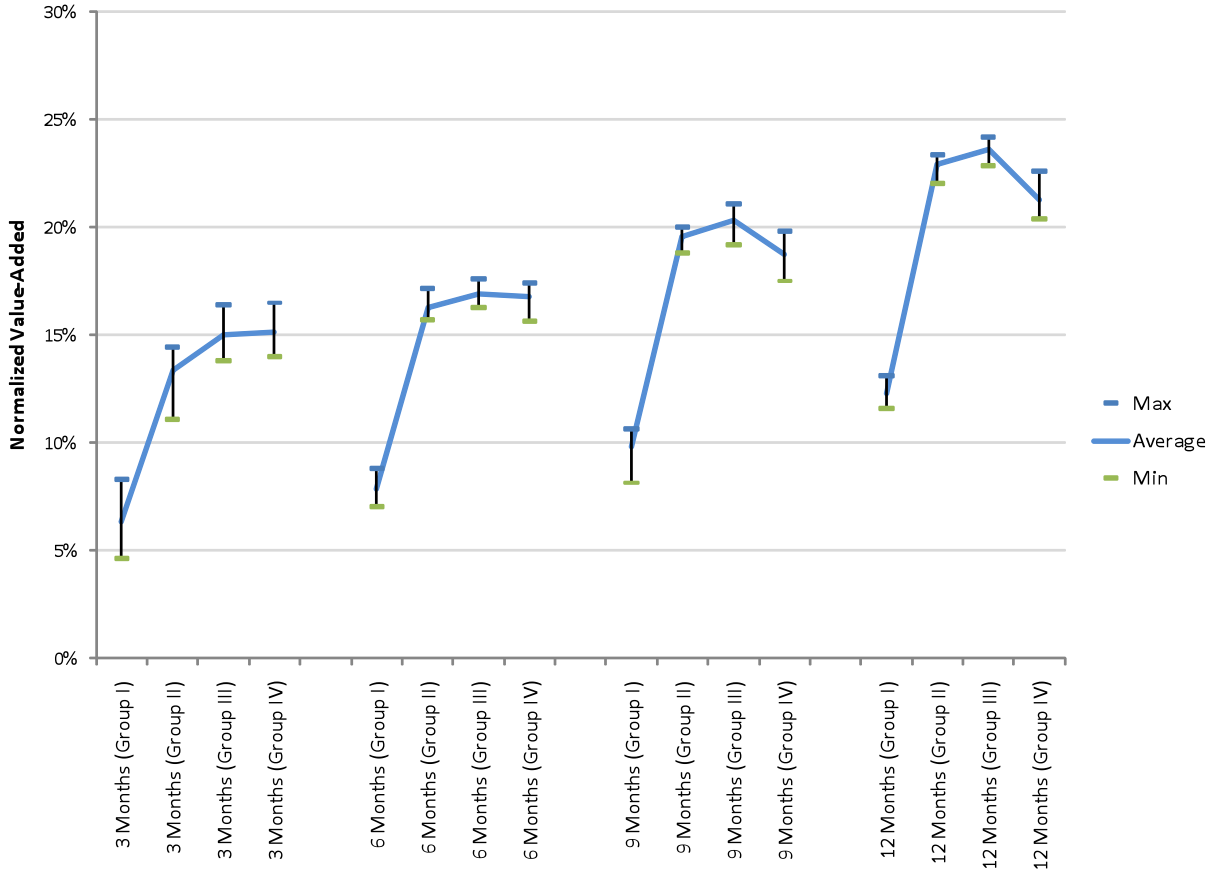


Figure 18: Value-added of machine-learning forecasts of 90-days-or-more delinquency over 3-, 6-, 9-, and 12-month forecast horizons on data from January 2005 through April 2008, for four equal stratified sub-samples of a randomly chosen 10%-subset of the data, stratified by on the number of non-missing features. Group I is the sample of accounts with the fewest available features (“thinnest”), and Group IV is the sample with the most (“thickest”). Minimum and maximum VA values are calculated using 10-fold cross validation from the entire data set using equation (4) with amortization period  $N=3$  years, run-up before default of 30% ( $B_d=1.3B_r$ ),  $r=5\%$ , and the ROC-tangency classification threshold.

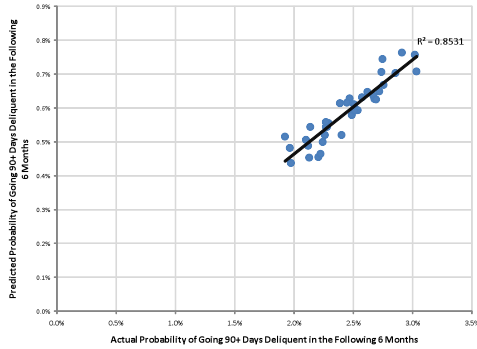
## 5.4 Macroprudential Risk Management

Machine-learning forecasts for individual accounts can easily be aggregated to generate macroeconomic forecasts of credit risk in the consumer lending business by simply tabulating the number of accounts forecasted to be delinquent over a given forecast period and for a specific classification threshold. The proportion of forecasted delinquencies in the population may then be used as an indicator of systemic risk for consumer lending, and

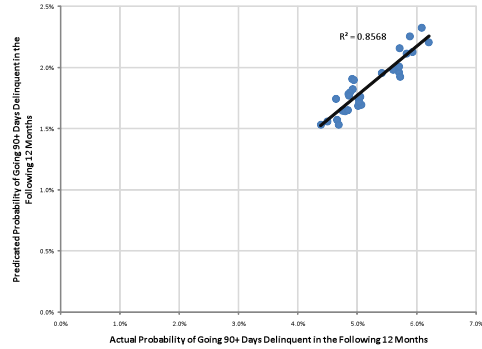
its properties can also be summarized with confusion matrices with different classification thresholds. For purposes of aggregate risk measurement, a longer forecast horizon may be more appropriate, so we apply the same machine-learning algorithm to data from January 2005 to December 2008 and define the target variable as 90-days-or-more delinquency over a 6- or 12-month forecast horizon.

Figure 19(a) and 19(b) show the aggregate results the machine-learning forecasts as compared to realized delinquencies for 6- and 12-month forecast horizons using the ROC-tangency classification threshold (in which the trade-off between true and false positive rates is one-to-one; see Section 5.1). The forecasted delinquencies are highly correlated with realized delinquencies—with linear regression  $R^2$ 's of 85% for both horizons—hence our forecasts are capturing the dynamics of the consumer credit cycle over the sample.

However, note that the forecasts consistently under-estimate the absolute level of delinquencies by a scalar multiple, which may be an artifact of the particular classification threshold selected. To correct for this underestimation, and to reduce the effects of overfitting, we use 10-fold cross validation to estimate the model's recall value (the number of accounts classified as bad as a percentage of the total number of bad accounts). Then we simply multiply the forecasted default rate by the reciprocal of this estimated recall rate. Figures 20(a) and 20(b), which contain time series plots of the adjusted forecasts as well as the realized aggregate delinquency rates, show that the adjustment factor is reasonably successful in correcting the underestimation bias, yielding forecasts that are accurate both in terms of level and dynamics.



(a) Predicted and actual 90-days-or-more delinquency rates (6-month)

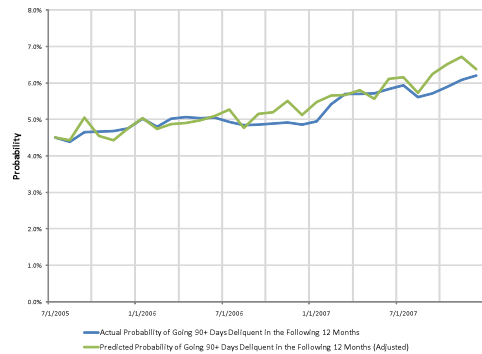


(b) Predicted and actual 90-days-or-more delinquency rates (12-month)

Figure 19: Forecasted and realized 90-days-or-more delinquency rates for 6- and 12-month forecast horizons, with ROC-tangency classifier threshold, based on data from January 2005 to December 2008. The horizontal axis starts in July 2005 since some features require 6 months of training data, and ends June 2008 or December 2007 since either 6 or 12 months of data are needed to calculate the realized delinquencies events to be compared again the predicted values.



(a) Time series of actual and predicted 90-days-or-more delinquency rates (6-month)



(b) Time series of actual and predicted 90-days-or-more delinquency rates (12-month)

Figure 20: Time series of predicted and realized 90-days-or-more delinquency rates for 6- and 12-month forecast horizons, with ROC-tangency classifier threshold, based on data from January 2005 to December 2008. The horizontal axis starts in July 2005 since some features require 6 months of training data, and ends June 2008 or December 2007 since either 6 or 12 months of data are needed to calculate the realized delinquencies events to be compared again the predicted values.

## 6 Conclusion

In the aftermath of one of the worst financial crises in modern history, it has become clear that consumer behavior has played a central role at every stage—in sowing the seeds of crisis, causing cascades of firesale liquidations, and bearing the brunt of the economic consequences. Therefore, any prospective insights regarding consumer credit that can be gleaned from historical data has become a national priority.

In this study, we develop a machine-learning model for consumer credit default and delinquency that is surprisingly accurate in forecasting credit events 3 to 12 months in advance. Although our sample is only a small percentage of the Bank’s total customer base, the results are promising. Our out-of-sample forecasts are highly correlated with realized delinquencies, with linear regression  $R^2$ ’s of 85% for monthly forecasts over 6- and 12-month horizons. Moreover, crude estimates of the value-added of these forecasts yield cost savings ranging from 6% to 23% of total losses. Given the scale of industry wide charge-offs during 2008, even a 6% cost-savings would amount to hundreds of millions of dollars.

From a macroprudential risk management perspective, the aggregation of machine-learning forecasts of individuals may have much to contribute to the management of enterprise and systemic risk. Current credit-bureau analytics such as credit scores are based on slowly varying consumer characteristics, and therefore not as relevant for tactical risk management decisions by chief risk officers and policymakers. We find that machine-learning forecasts are considerably more adaptive, and are able to pick up the dynamics of changing credit cycles as well as the absolute levels of default rates.

We believe that our results are indicative of considerably more powerful models of consumer behavior that can be developed via machine-learning techniques, and are exploring further refinements and broader datasets in ongoing research.

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