Using Collimators and Scrapers to Reduce Synchrotron Radiation in OLYMPUS

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1 Introduction

Synchrotron radiation can cause several problems in experiments at electron storage rings. Synchrotron photons can scatter near the target, showering particles that enter the detector becoming an annoying and even damaging form of background. Collimators or similar devices may be implemented to block or deflect this radiation, but they must be designed to dissipate this incident power.

In this report, I present a very rough calculation of the synchrotron radiation produced in the DORIS storage ring, and the consequences for the design of OLYMPUS. I also offer recommendations for the placement scrapers and the design of a collimator for the running of OLYMPUS at both energies of 2 GeV and 4.45 GeV.

2 A Rough Calculation

2.1 Approximations

In doing this calculation, I've had no information about the optics of DORIS, so I've made the approximation that electrons travel through the dipole section in a circular trajectory, of radius R, and that the beam height and width are negligible. Since I wanted to calculate average synchrotron power, I've assumed that the electrons flow continuously, instead of in bunches. I've also assumed that all of the synchrotron power will be radiated in the forward direction, with the total power being:

$$P_1 = \frac{ce^2\gamma^4}{6\pi\epsilon_0 R^2} \quad (1)$$

This equation comes, (with some modification) from Griffiths [1]. I assume that the acceleration of the electrion is perpendicular to its velocity.

2.2 Power Calculation

Equation 1 shows the power emanating from a single electron. Synchrotron radiation will be emitted from all of the electrons in the ring. Most of the radiation will hit the outside wall of the beam pipe, but some will make it through the straight section that leads to the target cell and may be problematic. This fraction of the synchrotron radiation will be emitted by electrons traversing the final portion of the last dipole magnet. If we call the length of this critical final portion S, we can assume that $S = R\theta$, where R is the radius of curvature for the electron trajectory and θ is the critical angle within which the radiation will pass into the straight section of the pipe.

Assuming that electrons come continuously, then their average position separation is $\Delta S_{avg} = \frac{cq}{I}$, where I is the beam current. Thus, the number of particles contributing to the synchrotron radiation of interest is described by:

$$\frac{S}{\Delta S_{avg}} = \frac{IR\theta}{ce} \tag{2}$$

The total power is thus:

$$P = \frac{S}{\Delta S_{avg}} \times P_1 \tag{3}$$

$$P = \frac{IR\theta}{ce} \times \frac{ce^2\gamma^4}{6\pi\epsilon_0 R^2} \tag{4}$$

$$P = \frac{Ie\gamma^4\theta}{6\pi\epsilon_0 R} \tag{5}$$

2.3 Angle Calculation

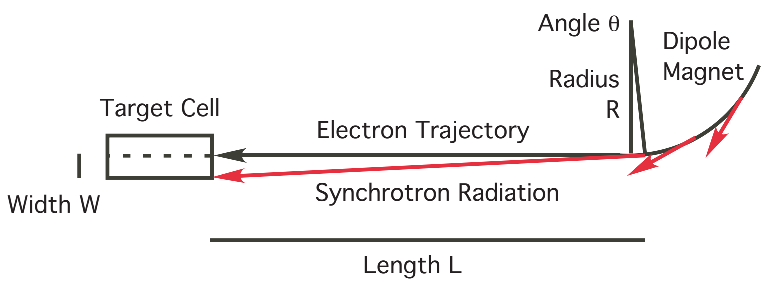

Figure 1: Electrons moving from right to left produce synchrotron radiation (red). Inside angle θ the radiation will hit the cell at the far right.

The question remains: what is the angle θ? Figure 1 shows how I have defined θ as the maximum angle for a line tangent to the electron trajectory to intersect some downstream object with half-width w, at distance of length L. According to this definition, θ can be solved for (see Appendix A:

$$\theta \approx \sin\theta = \frac{-LR + (R+W)\sqrt{L^2 + W^2 + 2RW}}{L^2 + (R+W)^2} \tag{6}$$

$$P \approx \frac{Ie\gamma^4}{6\pi\epsilon_0 R} \times \frac{-LR + (R+W)\sqrt{L^2 + W^2 + 2RW}}{L^2 + (R+W)^2} \tag{7}$$

2.4 Critical Energy

The frequency spectrum of synchrotron radiation is peaked near a critical frequency, given by [2]:

$$\omega_c = \frac{3c\gamma^3}{2R} \tag{8}$$

$$E_c = \frac{3c\gamma^3\hbar}{2R} \tag{9}$$

In the recent calculation by B. Nagorny [3] was run at 4.45 GeV, indicating a critical energy of 16 keV. At this energy, $\gamma \approx 8700$, so by rearranging, we can solve:

$$R = \frac{3c\gamma^3\hbar}{2E_c} \approx 12.3\ m \tag{10}$$

If the experiment were run at 2 GeV, then $\gamma \approx 4000$ and the critical energy would be $E_c = 1.56$ keV, in the soft X-ray region.

2.5 Photon Rate

The total emission rate of photons is related to the power and critical energy by [2]:

$$N = \frac{15\sqrt{3}P}{8E_c} \tag{11}$$

3 Numbers

3.1 Power through the target

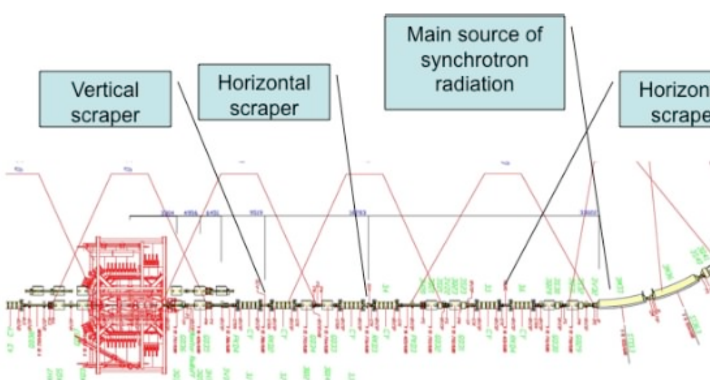

Figure 2: the beamline leading up to OLYMPUS, with electrons coming from the right; taken from [4]

To plug numbers into equation 7, I've estimated 12.3 m for the value of R. For the value of I, I've matched Nagorny's calculation using 140 mA [3]. I have also used $\gamma \approx 4000$ at 2 GeV, and ≈ 8700 at 4.45 GeV.

Using these numbers we can estimate the amount of power that will go through the target cell. Referring to Brinker's talk (the relevant diagram is shown as Figure 2). I choose $L = 32.7\,m$ and $W = 12.5mm$, and get:

$$P = 1.07\,W\ (E = 2\,GeV)$$
$$P = 23.9\,W\ (E = 4.45\,GeV)$$

Applying equation 11, the rate of photons through the target will be $1.39 \cdot 10^{16}$ Hz at 2 GeV, and $4.43 \cdot 10^{16}$ Hz at 4.45 GeV.

3.2 Comparison with Nagorny's Results

I modeled my calculation to match Nagorny's in curvature, current and energy. Due to the way he positioned the scrapers for his calculation, he essentially measured the amount of power that will pass through the target. His calculation measures 23 Watts passing through the target, while my calculation predicts a value closer to 24 Watts. This agreement justifies to a great extent my approximations.

3.3 A Conservative Estimate of Power on the Collimator

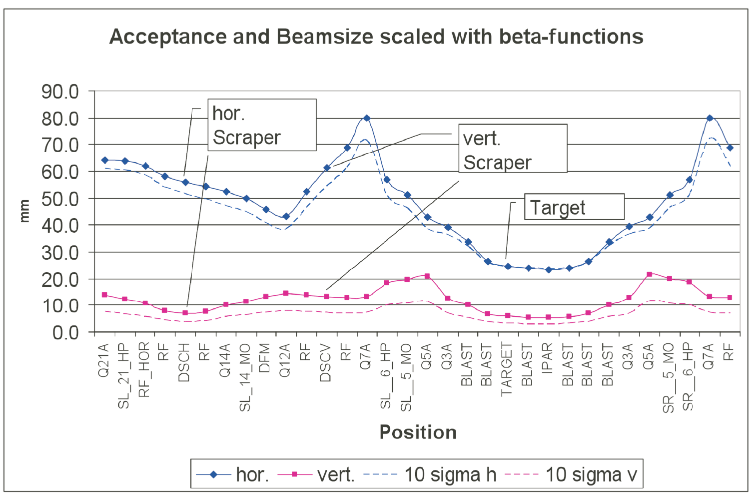

Figure 3: the width of the DORIS beam at 4.45 GeV; taken from [4]; It should be noted that the width of the beam scales linearly with energy, so at 2 GeV, the beam width will simply be reduced by a factor of 4.45 / 2.

The important number is how much power will fall on the collimator. The collimator will likely be no wider than 15 cm in diameter. Thus we can plug in 33.7 m for length, and 7.5 cm for width to find how much power at most will fall on the collimator.

$$P = 6.41\,W\ (E = 2\,GeV)$$

$$P = 143.4\,W\ (E = 4.45\,GeV)$$

Subtracting the power that passes through the cell, the power incident on the collimator will be:

$$\begin{aligned} P &= 5.34\,W\ (E = 2\,GeV) \\ P &= 119.5\,W\ (E = 4.45\,GeV) \end{aligned}$$

At most, the collimator will have to dissipate 3-4 W of synchrotron power in the 2 GeV case, and 120 W of power in the 4.45 GeV case.

It is important to remember that in this calculation, the incident power will scale proportionally to I and to γ^4, should these values change.

It is also important to remember that this incident power is only the result of synchrotron radiation from the final dipole. The 12 quadrupole magnets in the region preceding the target will also generate synchrotron power, though most of this will pass directly through the target. The collimator will also see power from scattered electrons, though this will be insignificant relative to the synchrotron power (See Appendix B for a quick justification).

3.4 Comparison with the Hermes Synchrotron Calculation

There is a synchrotron radiation calculation in the design proposal for the Hermes experiment [5]. A calculation from a different experiment can provide a valuable check of my approximations. I obtained numbers for some of my parameters from Nagorny's calculation, so while consistency between our calculations is reassuring, it would be even more reassuring to show that my method agrees reasonably well with an experiment that has a different beam energy, curvature, and collimator spacing.

From the Hermes Experiment proposal [5], there are three collimators, spaced 50 mm, 1.5 m and 101.5 m from the target opening, with half-width apertures of 7.1 mm, 6.3 mm, and 16.3 mm respectively. The beam current for the experiment is 60 mA. The beam energy is 35 GeV, meaning that $\gamma \approx 68500$. In the report, the radius of curvature is slightly ambiguous, but the table on p. 14 suggests a value of 608 m. The distance between the last dipole and the target is never explicitly given, but I have decided to take a value of 180 m, half the length of the HERA straight section. Using these values, I find that the power incident on the middle collimator is 359 W compared to 325 W listed in the report. The relatively good agreement is reassuring. The report also lists 325 W passing through the target, compared to my value of 75 W. I think the report is including in this value the radiation produced by downstream quadrupoles, so the comparison isn't relevant.

3.5 Synchrotron Beam Width

Up to this point I have assumed that synchrotron radiation is emitted only in the forward direction, when in fact, there is some angular spread in emission. A good approximation [1] is that the radiation will be emitted within $\frac{1}{\gamma}$ of the forward direction. It is important to know how much the radiation will spread by the time it reaches the collimator. For $E = 2$ GeV, $\frac{1}{\gamma} = 2.50 \times 10^{-4}$ and for $E = 4.45$ GeV, $\frac{1}{\gamma} = 1.15 \times 10^{-4}$. At a distance of 32.7 m, this corresponds to full widths of 16.4 mm and 7.5 mm respectively. I would like to point out that at the lower energy, this spread exceeds the vertical width of the colimator aperture. This means that if the downstream scraper were used vertically, it could be used to intercept some synchrotron power. This scraper is 23.5 m downstream of the dipole, where the synchrotron full width will be approximately 11.8 mm. The 10 σ beam width at that point (and at that energy) is only 3.6 mm.

The majority of the power incident on the collimator will be on the beam-left side. The absorbing region will for the most part be a horizontal stripe of constant flux, the height of which depends on the beam energy.

4 Positioning of the Scrapers

In addition to having an aperture that can be adjusted, the upstream scrapers can also be rotated to be either horizontal or vertical. In this section, I'll discuss what I believe to be the best scraper configurations and their advantages.

The scrapers consist of water-cooled copper rods, each with a 30 mm diameter. There are currently two, positioned 16.2 m and 23.5 m from the final dipole, with the down stream scraper being currently designated as vertical.

The width of the beam can be found by consulting [4] (See figure 3). This plot shows the beam size at 4.5 energy. The width of the beam scales linearly with energy, so at 2 GeV the beam width will be reduced by a factor of 4.45/2. Though the final positions of the scrapers must be determined empirically, the scrapers will generally start to degrade the beam when they are pulled within 6σ.

The more upstream the synchrotron radiation can be blocked, the less background from showering will enter the detector.

4.1 2 GeV

As was discussed in section 3.5, the downstream scraper could be used vertically to interecept some synchrotron power before it hits the collimator. This has the advantage of moving upstream the interception of some of the synchrotron power, as well as allowing the vertical scraper to remove the vertical tails of the beam. The beam's 6σ vertical half-width at this point is 1.6 mm, so the scraper can be pulled in this far without significant attenuation.

The upstream scraper should be used horizontally, pulled in to the beam's 6σ half-width of 7.2 mm. In this position, 1.24 W will pass through the aperture.

My model is unable to determine how much power is interecepted by the downstream vertical scraper. Since 1.07 W will pass through the target, we can at least know that less than 0.17 W will be incident on the collimator.

4.2 4.45 Gev

At higher energy, the vertical scraper will offer no advantage in reducing synchrotron power, so it should be turned to a horizontal position. If the upstream scraper is pulled into the 6σ half-width of 16 mm, then 61.7 W will pass through the aperture. With the downstream scraper in the horizonal position pulled to the 6σ half-width of 17 mm, then

45.2 W will pass through the aperture while 16.5 W are intercepted by the scraper. That leaves 21.3 W incident on the collimator and 23.9 W through the target.

5 Design of the Collimator

5.1 Dimensions

The proposed target cell will be an eliptic cylinder, 600 mm long, 9 mm high, 27 mm wide. The collimator should therefore have an aperture 7 mm high, and 25 mm wide. It should be made of a heavy metal with desirable properties under vaccuum, like Tungsten. The radiation length of Tungsten is 3.5 mm, so 10 cm ($\approx$ 29 radiation lenghts) will be a more than sufficient thickness. Assuming a 15 cm diameter (which is excessive since it is larger than the beam-pipe), the block of Tungsten will have a volume of 1712 cm^3. Tungsten has a density of 19/3 g/cm^3, meaning, the collimator will weigh in at a hefty 33 kg.

5.2 Power Dissipation

With the judicious use of scrapers, the power on the collimator will be certainly no greater than 50 W. In the 4.45 GeV case, the dipole synchrotron radiation, as we saw in section 4.2 could be reduced to 21.3 W. Downstream quadrupole magnets might produce radiation on the same scale. Scattered electrons will contribute a negligible amount of power (shown in Appendix B).

The power incident on the collimator will cause a change in temperature according to:

$$\Delta Q = CM\Delta T \tag{12}$$

$$P = CM\frac{dT}{dt} \tag{13}$$

$$\tag{14}$$

Taking the specific heat of tungsten to be 0.134 J$/g\,{}^\circ$C, we can solve to get:

$$\frac{dT}{dt} = 0.011\frac{{}^\circ C}{s} \tag{15}$$

Over the course of a 2500 s run, the collimator will heat up 27.5 °C.

If power is dissipated from the collimator strictly by thermal radiation, then we can apply the Stefan-Boltzman law to get a rough approximation of the equilibrium temperature of the collimator.

$$j = \frac{P}{A} = \sigma T^4 \tag{16}$$

This equation describes the power radiating from an object. However, the object will also be absorbing thermal radiation from the surroundings. We can modify the equation to reflect this:

$$\frac{P}{A} = \sigma(T_c^4 - T_0^4) \tag{17}$$

In this equation, T_c is the temperature of the collimator while T_0 is the temperature of the surroundings. We can solve for T_c to get:

$$T_c = (\frac{P}{A\sigma} + T_0^4)^{\frac{1}{4}} \tag{18}$$

Taking power to be 50 W, to match the incoming synchrotron power, T_0 to be 300 K, and A to be 825 cm^2, then the equilibrium temberature will be:

$$T_c = 370K$$

While this is hot, it certainly isn't hot enough to melt tungsten (3700 K melting point).

There is a possibility that the collimator could be cooled, using the same cooling system as the target.

6 Conclusion

The synchrotron radiation generated by DORIS will really only be of consequence during higher energy runs. At 2 GeV, a few watts of power will be generated, but it should be easily intercepted and dissipated. In order to run OLYMPUS at a higher beam energy, there are measures that need to be taken: the scrapers must be pulled in tightly, the collimator must be properly designed and possibly even cooled.

While synchrotron radiation will not be much of a hazard for this experiment, the use of collimators and scrapers provide another: electrons at the edges beam can scatter and shower off the collimator and scrapers. It will be important to study the levels of background one can expect from using the scrapers and collimators, in order to refine their use and positioning.

A Derivation of the Critical Angle θ

Referring back to Figure 1, I will use a coordinate system with an origin at the center of the circular trajectory, with positive x pointing right, and positive y pointing right. A line tangent to the trajectory at the critical angle will have slope $m = \tan\theta$. The tangent point will be at $(R\sin\theta,-R\cos\theta)$. The equation for this tangent line will thus be:

$$y = \tan\theta(x - R\sin\theta) - R\cos\theta \tag{19}$$

This line will interesect the corner of the target cell at point $(-L,-R-W)$. Plugging this in to equation 19, we can solve for $\sin\theta$:

$$\begin{aligned} -R - W &= \tan\theta(-L - R\sin\theta) - R\cos\theta & (20) \\ R + W &= L\tan\theta + R\tan\theta\sin\theta + R\cos\theta & (21) \end{aligned}$$

$$R + W = \frac{L\sin\theta + R\sin^2\theta + R\cos^2\theta}{\cos\theta} \quad (22)$$

$$R + W = \frac{L\sin\theta + R}{\cos\theta} \quad (23)$$

$$(R + W)\cos\theta = L\sin\theta + R \quad (24)$$

$$(R + W)\sqrt{1 - \sin^2\theta} = R + L\sin\theta \quad (25)$$

$$(R + W)^2(1 - \sin^2\theta) = R^2 + 2LR\sin\theta + L^2\sin^2\theta \quad (26)$$

$$0 = [L^2 + (R + W)^2]\sin^2\theta + 2LR\sin\theta - (W^2 + 2RW) \quad (27)$$

From here we can use the quadratic formula. The descriminant will always be positive because two points on the circle will have tangent lines pointing to the corner of the target cell. Only the larger solution is physical. Thus:

$$\sin\theta = \frac{-2LR + \sqrt{4L^2R^2 + 4[L^2 + (R + W)^2](W^2 + 2RW)}}{2[L^2 + (R + W)^2]} \quad (28)$$

$$= \frac{-LR + \sqrt{L^2R^2 + [L^2 + (R + W)^2](W^2 + 2RW)}}{L^2 + (R + W)^2} \quad (29)$$

$$= \frac{-LR + \sqrt{L^2R^2 + L^2W^2 + 2RWL^2 + W^2(R + W)^2 + 2RW(R + W)^2}}{L^2 + (R + W)^2} \quad (30)$$

$$= \frac{-LR + \sqrt{L^2(R + W)^2 + W^2(R + W)^2 + 2RW(R + W)^2}}{L^2 + (R + W)^2} \quad (31)$$

$$\sin\theta = \frac{-2LR + (R + W)\sqrt{L^2 + W^2 + 2RW}}{L^2 + (R + W)^2} \quad (32)$$

B Estimating Power Dissipation by Electrons

The DORIS ring has a 289.2 m circumference, meaning electron will traverse the ring in a little less than 1 μs [6]. Since $Q = It$, the ring will store $9 \cdot 10^{11}$ electrons. These electrons will slowly dissipate over a life time $\tau \approx 0.7$ hours ≈ 2500 s [6]. Thus the electron dissipation rate is about $3.6 \cdot 10^8$ Hz. Assuming all of these electrons are lost in the collimator (which they are most definitely not), the power deposited would be ≈ 0.1 W in the case of 2 GeV electrons, and ≈ 0.25 W in the case of 4.5 GeV electons. We can see that the power deposited by electrons is miniscule compared to that of synchrotron photons.

References

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[3] B. Nagorny, "Synchrotron radiation olympus," in *OLYMPUS Collaboration April Meeting, 2009* [7].

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[6] "A proposal to definitely determine the contribution of multiple photon exchange in elastic lepton-nucleou scattering," tech. rep., The OLYMPUS Collaboration, 2008.

[7] OLYMPUS Collaboration, *OLYMPUS Collaboration April Meeting, 2009*, April 2009.