Two-photon exchange

Marc Vanderhaeghen
Johannes Gutenberg Universität, Mainz

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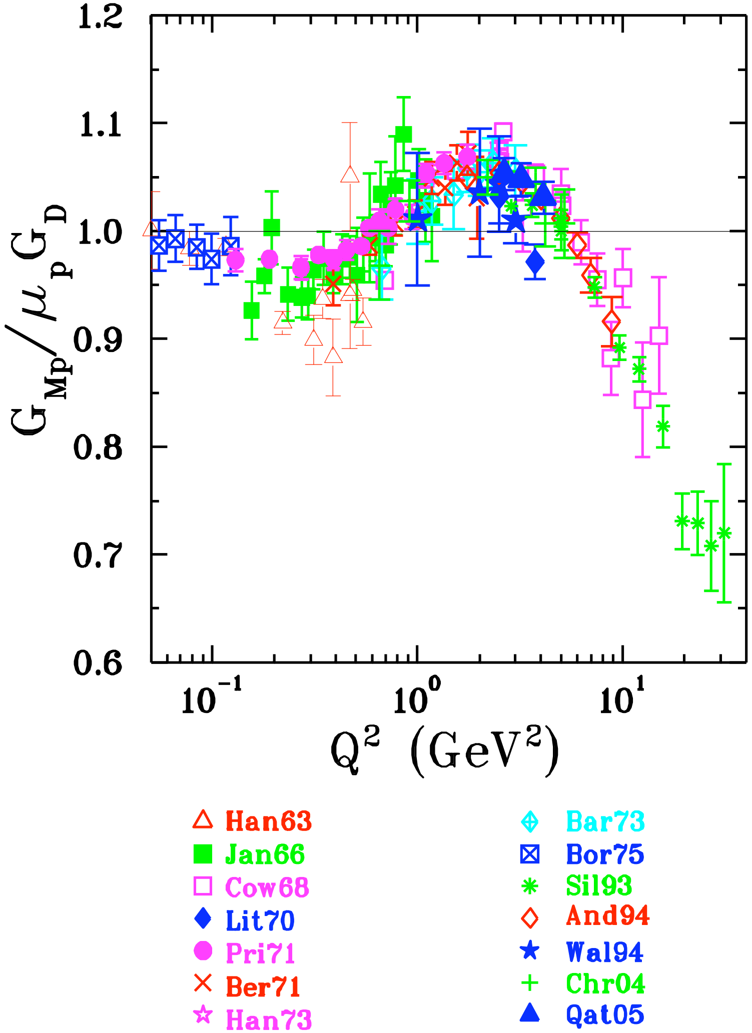
Outline

- Nucleon form factors : motivation
- Puzzle of different results extracted for G_E / G_M in Rosenbluth vs polarization experiments
- Elastic eN scattering beyond the one-photon exchange approximation
  two-photon exchange processes
- Leading pQCD analysis of two-photon exchange amplitude
  in coll. with N. Kivel : PRL 103, 092004 (2009)
- Comparison with experiments

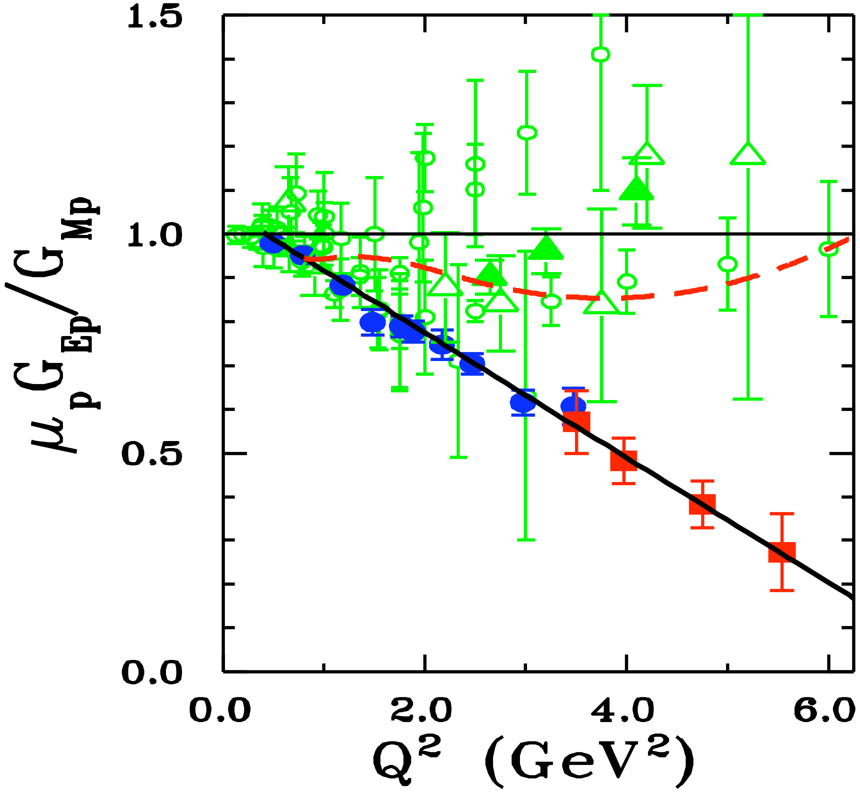
in coll. with : A.Afanasev, S. Brodsky, C. Carlson, Y.C. Chen, M. Gorchtein, P.A.M. Guichon, N. Kivel, V. Pascalutsa, B. Pasquini

review : C. Carlson and M. Vdh, Ann. Rev. Nucl. Part. Sci. 57 (2007) 171 - 204

proton e.m. form factor : status



new MAMI/A1 data up to $Q^2 \approx 0.7$ GeV^2

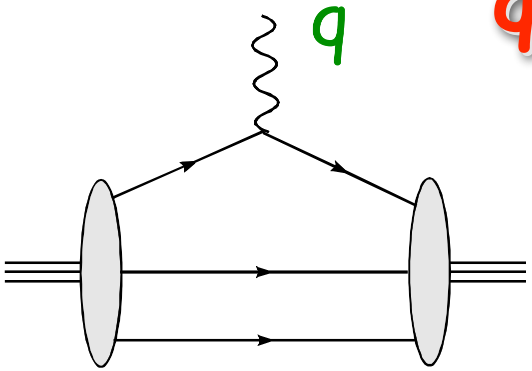


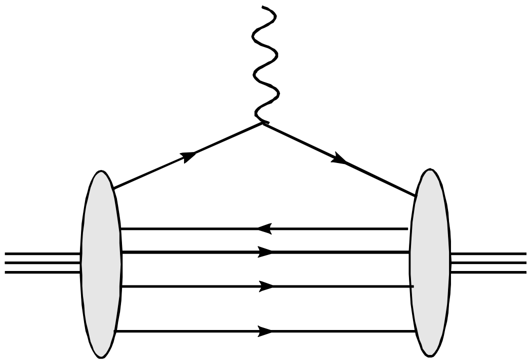
green : Rosenbluth data (SLAC, JLab)

Pun05, Gay02 } JLab/HallA recoil pol. data

new JLab/HallC recoil pol. exp. (spring 2008) : extension up to $Q^2 \approx 8.5$ GeV^2

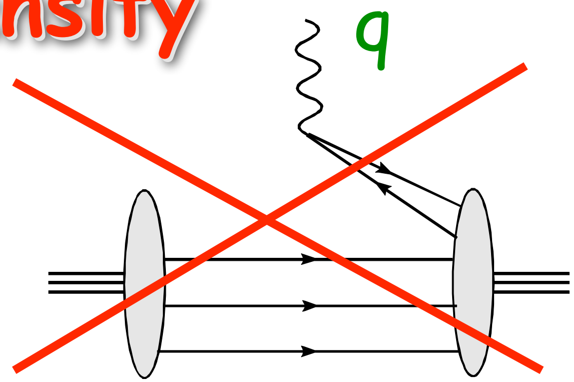
interpretation of Form Factor as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

absent in a LIGHT-FRONT frame !

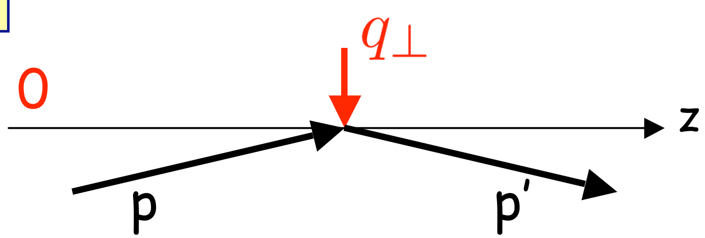
$$q^+ = q^0 + q^3 = 0$$

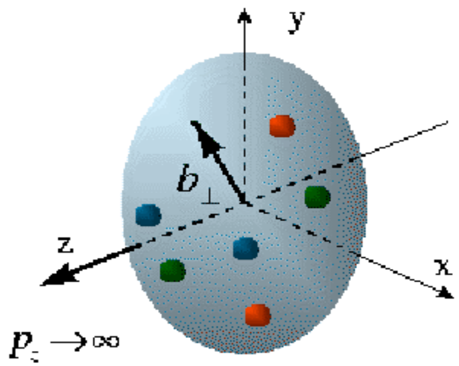
quark transverse charge densities in nucleon (I)

light-front

- $q^+ = q^0 + q^3 = 0$

  $Q^2 \equiv \vec{q}_\perp^{\,2}$

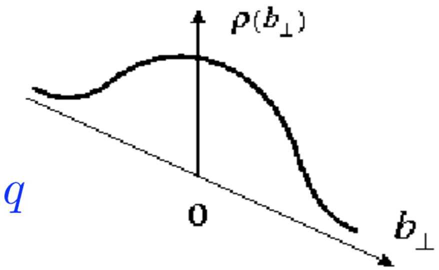




photon only couples to forward moving quarks

- quark charge density operator

$$J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+ q = 2\, q_+^\dagger\, q_+, \quad \text{with} \quad q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$$



☆ longitudinally polarized nucleon

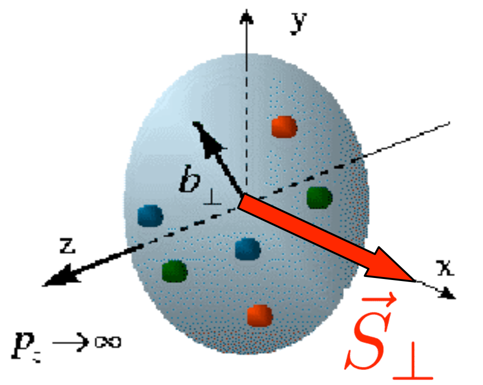
$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda\,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

Miller (2007)

quark transverse charge densities in nucleon (II)

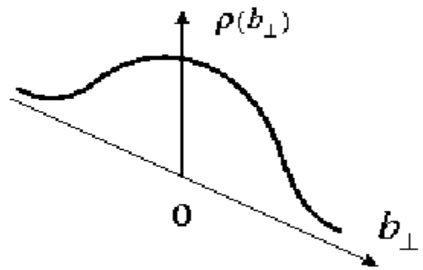
☆ transversely polarized nucleon



transverse spin $\quad \vec{S}_\perp = \cos\phi_S \, \hat{e}_x + \sin\phi_S \, \hat{e}_y$

e.g. along x-axis : $\quad \phi_S = 0$

$$\vec{b} = b \, (\cos\phi_b \, \hat{e}_x + \sin\phi_b \, \hat{e}_y)$$



$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\,\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle$$

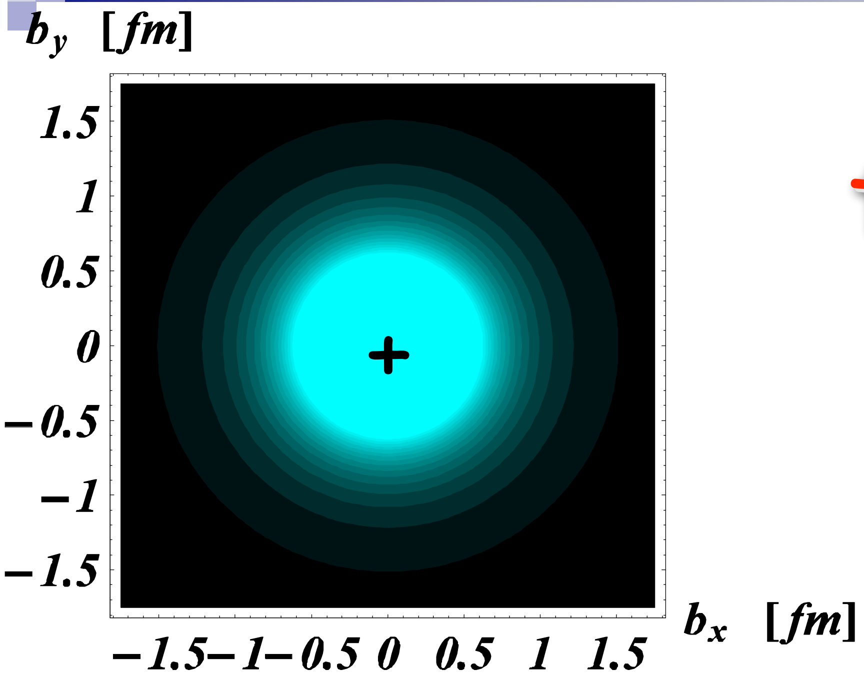
$$= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M_N} J_1(b\,Q) F_2(Q^2)$$

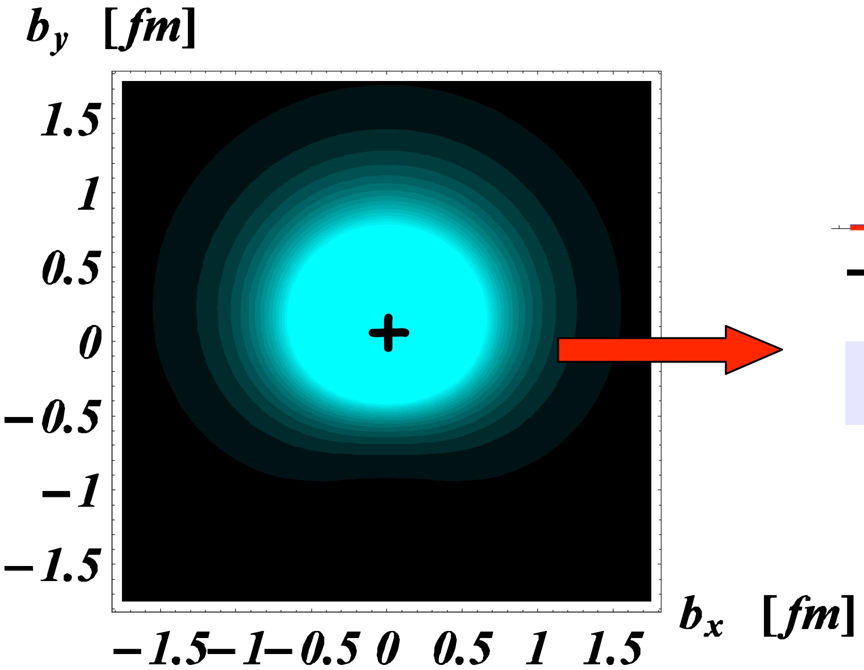
↓

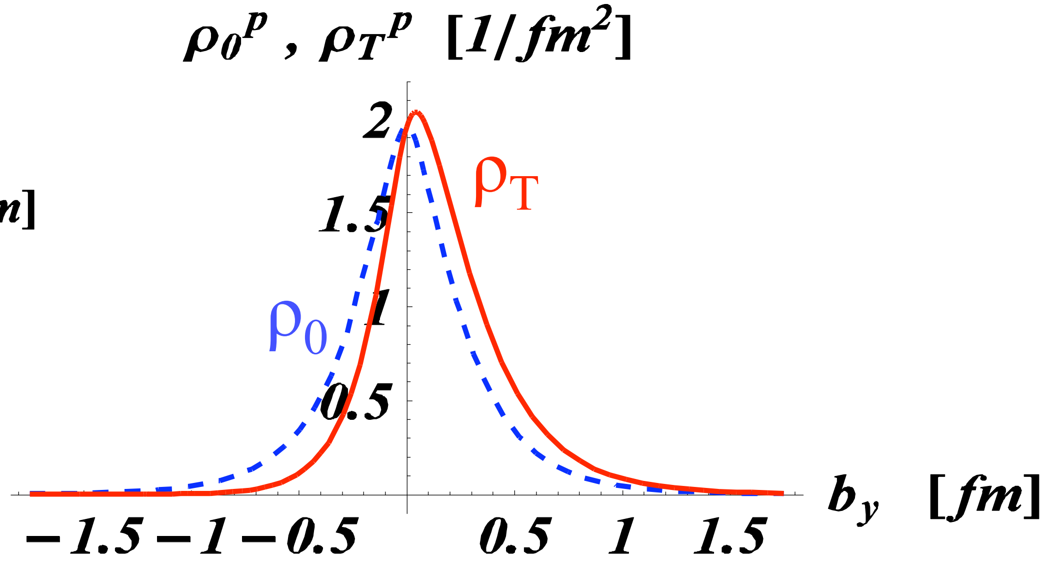
dipole field pattern

Carlson, Vdh (2007)

empirical quark transverse densities in proton





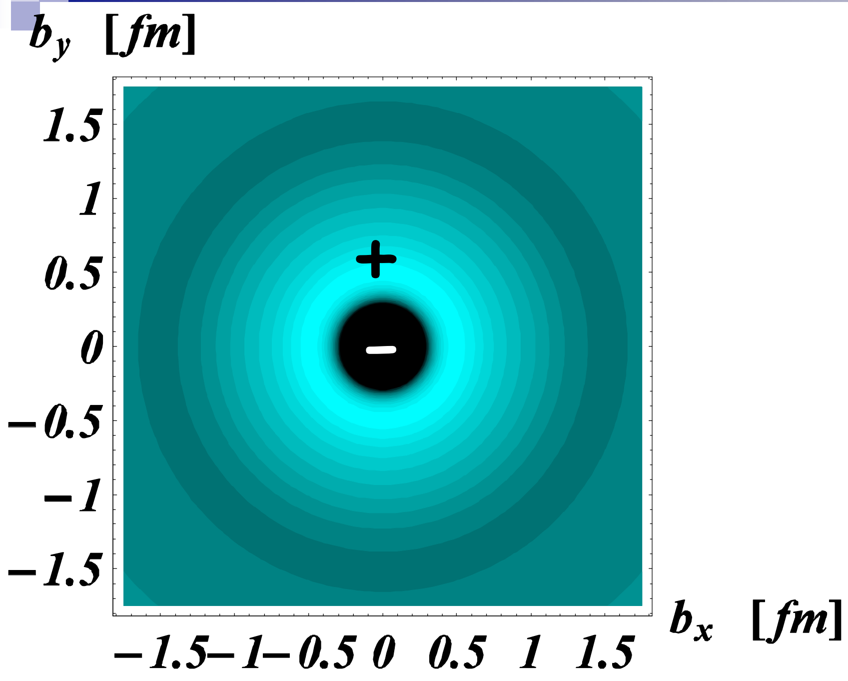


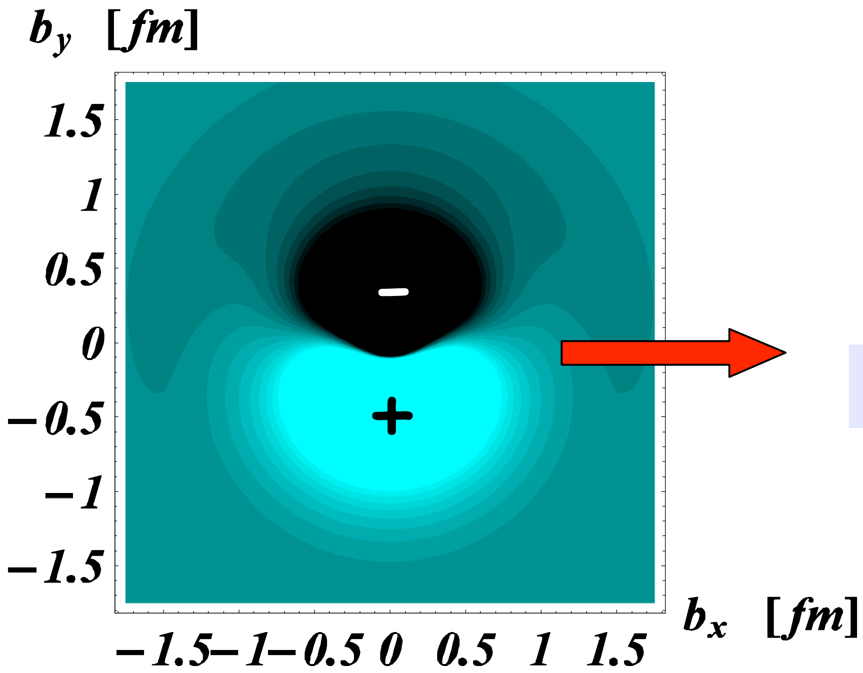
induced EDM : $d_y = F_{2p}(0) . e / (2 M_N)$

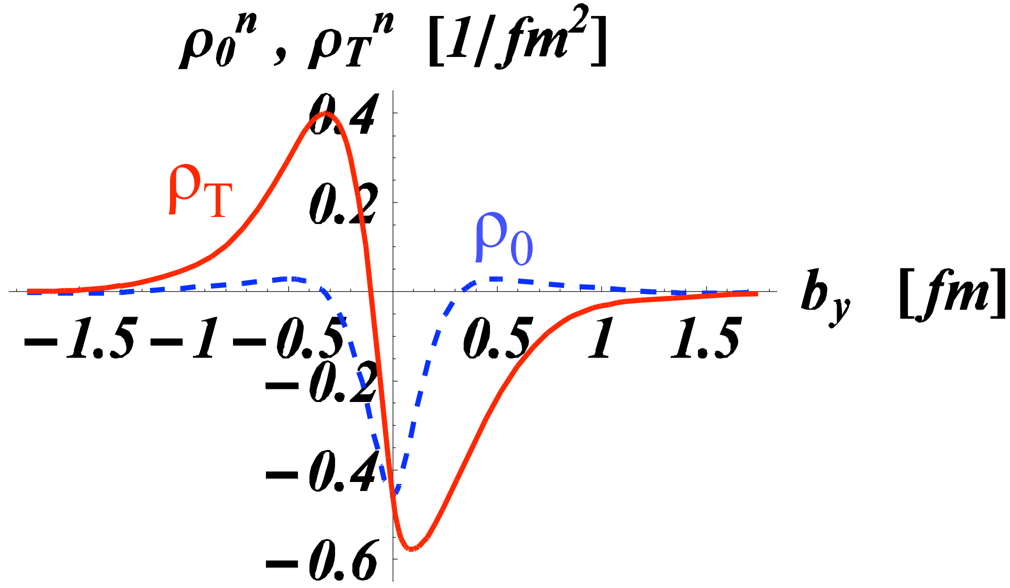
data : Arrington, Melnitchouk, Tjon (2007)

densities : Miller (2007); Carlson, Vdh (2007)

empirical quark transverse densities in neutron







induced **EDM** : $d_y = F_{2n}(0) . e / (2 M_N)$

data: Bradford, Bodek, Budd, Arrington (2006)

densities : Miller (2007); Carlson, Vdh (2007)

Rosenbluth separation method

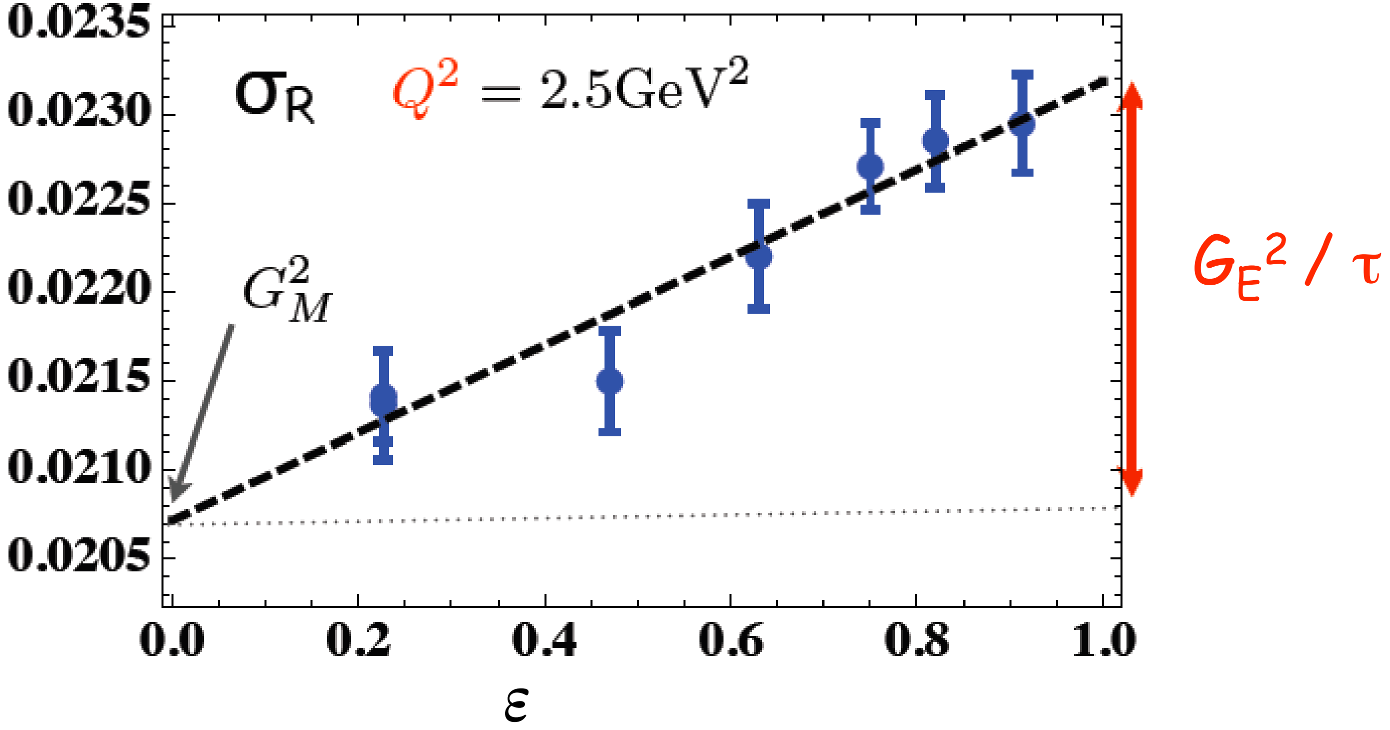
One-photon exchange elastic electron-nucleon cross section

$$\sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

$$\tau \equiv \frac{Q^2}{4M^2} \qquad \frac{1}{\varepsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

SLAC :

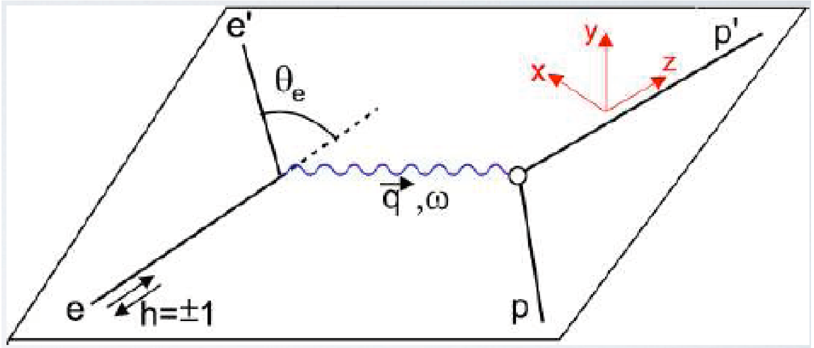
Andivahis et al. (1994)



Polarization transfer method

$$\vec{e} + p \rightarrow e + \vec{p}$$

Akhiezer, Rekalo (1974)



$$d\sigma_{pol} = d\sigma_{unpol}(1 + h\, S_x\, P_t\ + h\, S_z P_l)$$

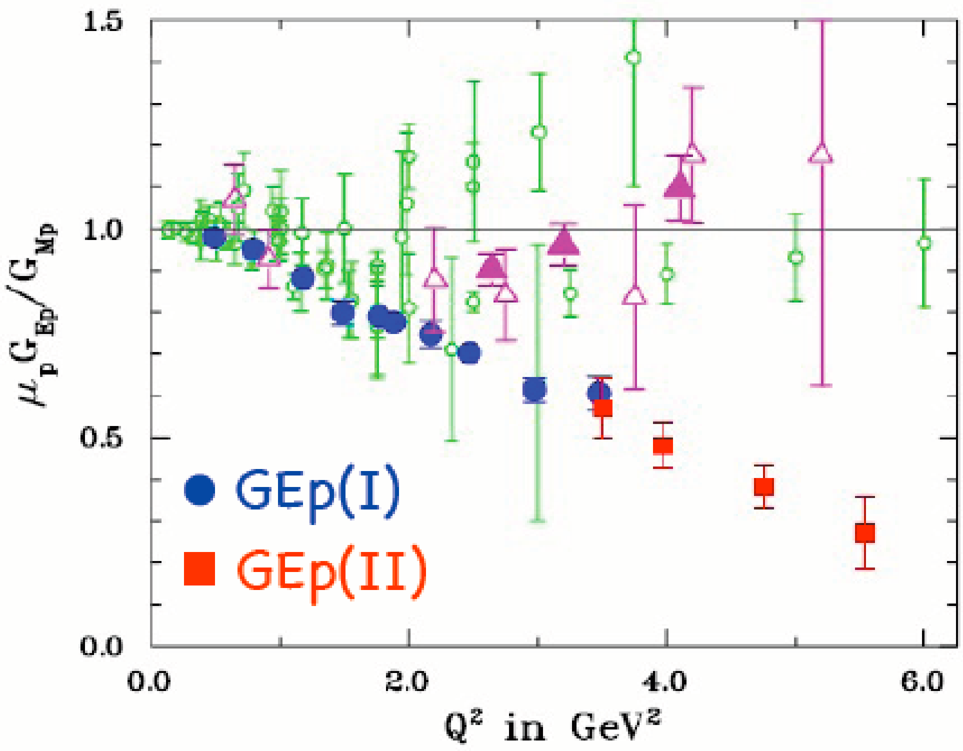
in one-photon exchange approximation :

$$P_t = -\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\frac{G_E G_M}{\tau\,\sigma_R}$$

$$P_l = \sqrt{1-\varepsilon^2}\,\frac{G_M^2}{\tau\,\sigma_R}$$

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\frac{G_E}{G_M}$$

Rosenbluth vs polarization transfer measurements of G_E/G_M of proton



SLAC,

Jlab (Hall A, Hall C)

Rosenbluth data

Jlab/Hall A

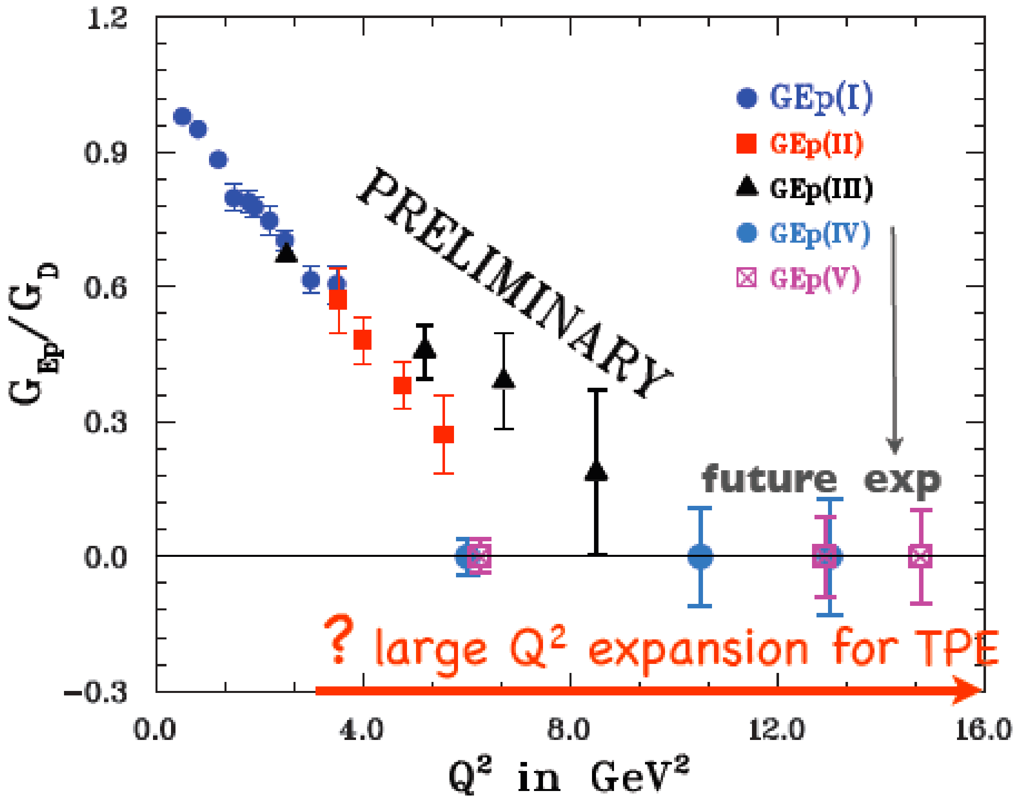
Polarization data

Jones et al. (2000)

Gayou et al. (2002)

Two methods, two different results !

Future experiments : large Q^2 behavior of FFs



Future experiments:

larger Q^2, better accuracy

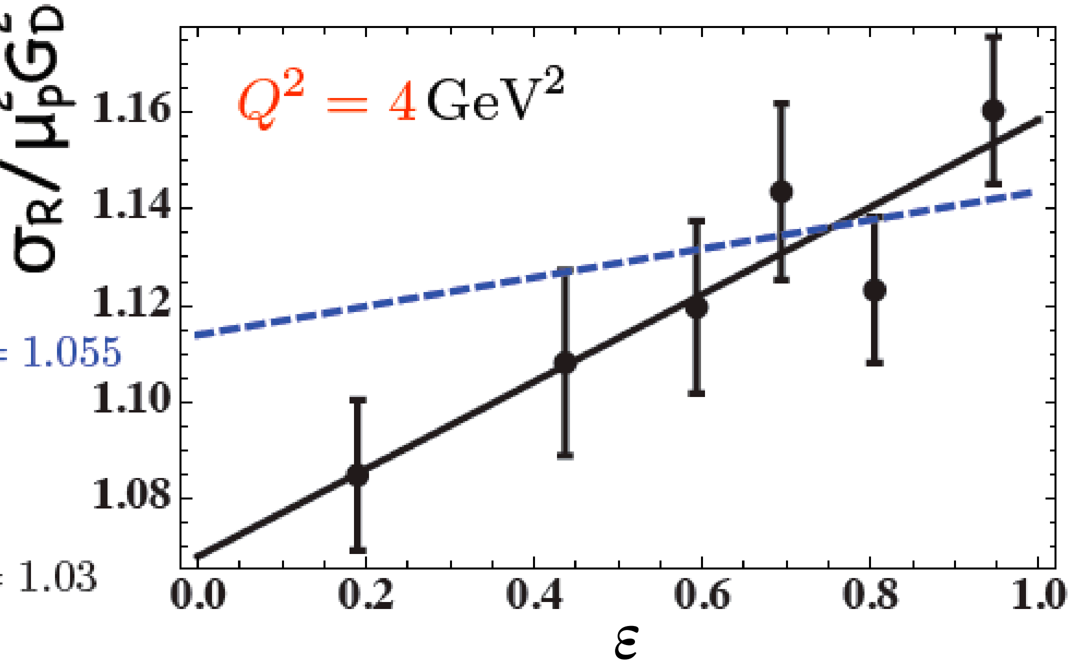
- JLab Hall A E1207-109 G_{Ep}/G_{Mp} recoil pol Q^2=6-14.8GeV^2
- JLab Hall C E1209-001 G_{Ep}/G_{Mp} recoil pol Q^2=6-13GeV^2
- JLab Hall A E1207-108 σ_R unpol Q^2=7-17.5GeV^2, total err. < 2%

Discrepancy between Rosenbluth and polarization data

SLAC :

Andivahis et al. (1994)

$$\sigma_R = G_M^2 \left(1 + \frac{\varepsilon}{\tau} \left(\frac{G_E}{G_M} \right)^2_{exp} \right)$$



$$\left(\frac{\mu_p G_E}{G_M} \right)_R = 0.89$$

$$\left(\frac{\mu_p G_E}{G_M} \right)_{exp} = 0.48$$

diff 46%

$$\left(\frac{G_M}{\mu_p G_D} \right)_{exp} = 1.055$$

diff 2.4%

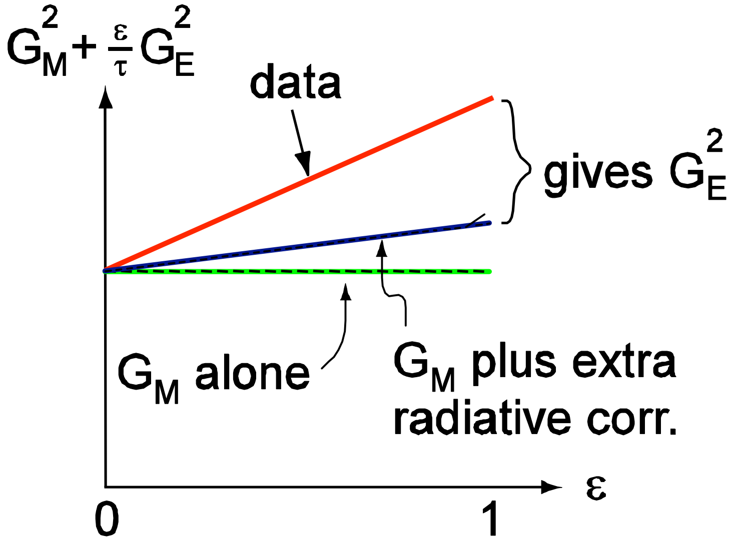
$$\left(\frac{G_M}{\mu_p G_D} \right)_R = 1.03$$

Difference cannot be explained by experimental uncertainties :
requires few % ε-dependence, linear in ε

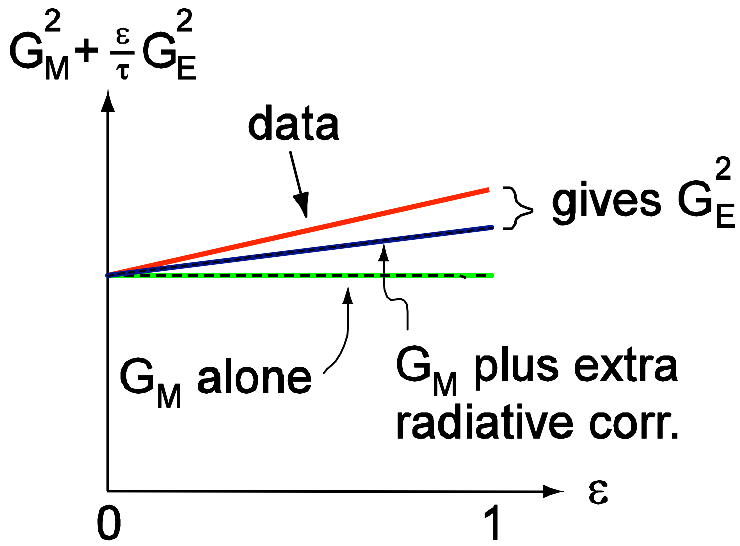
Speculation : missing radiative corrections

Speculation : there are radiative corrections to Rosenbluth experiments that are important and are not included

missing correction : linear in ε, not strongly Q^2 dependent



Low τ (Low Q^2)



High τ (High Q^2)

$Q^2 = 6\ GeV^2$

G_E term is proportionally smaller at large Q^2

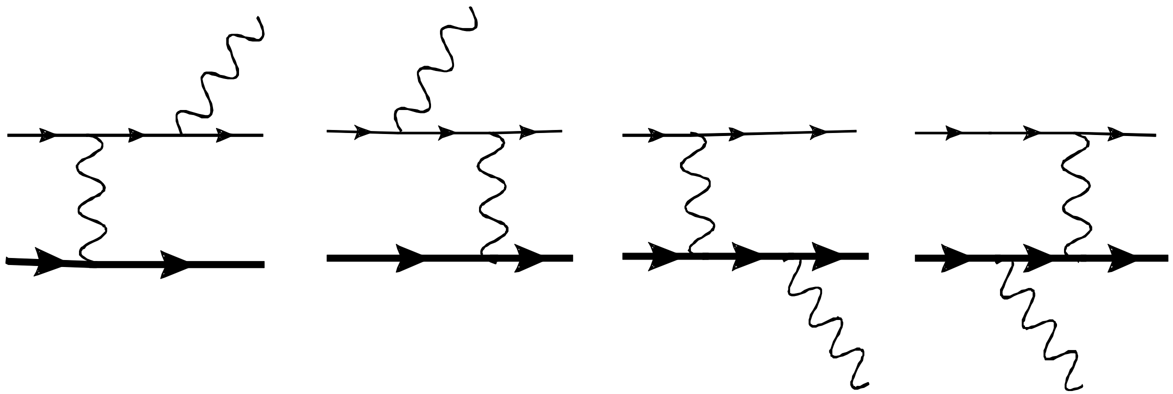
$$\frac{G_E^2}{\tau\, G_M^2} = \frac{4\, M^2}{Q^2\, \mu_p^2} = 7.5\%$$

effect more visible at large Q^2

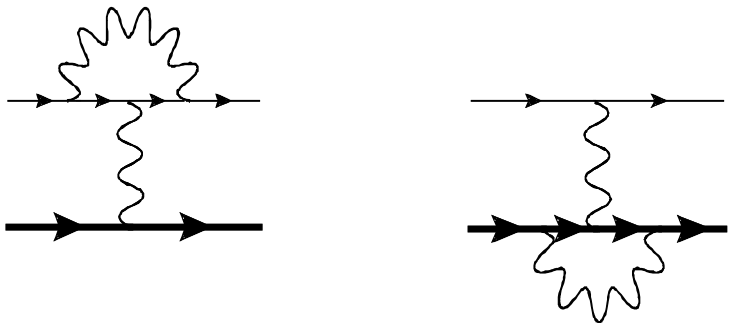
if both FF scale in same way

Radiative correction diagrams

bremsstrahlung



vertex corrections



2 photon exchange
box diagrams

Comments on radiative corrections

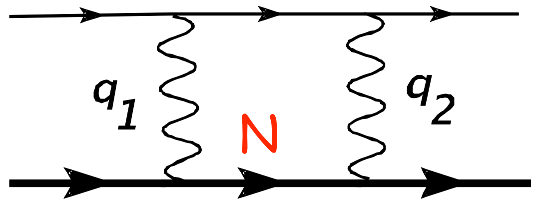
- Radiative corrections at electron side,
  ⇨ well understood and taken care of

- Soft bremsstrahlung
  involves long-wavelength photons
  ⇨ compositeness of nucleon only enters through on-shell form factors

- Box diagrams involve photons of all wavelengths
  ⇨ long wavelength (soft photon) part is included in radiative correction (IR divergence is cancelled with electron proton bremsstrahlung interference)

  ⇨ short wavelength contributions : not done in "old" days

Status of radiative corrections



- **Tsai (1961), Mo & Tsai (1968)**

  box diagram calculated using only nucleon intermediate state and using $q_1 \approx 0$ or $q_2 \approx 0$ in both numerator and denominator (calculate 3-point function) -> gives correct IR divergent terms

- **Maximon & Tjon (2000)**

  same as above, but make the above approximation only in numerator (calculate 4-point function)

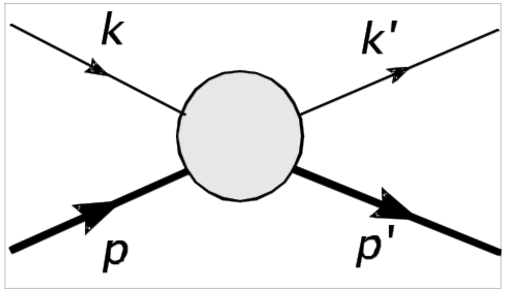
  + use on-shell nucleon form factors in loop integral

- **Blunden, Melnitchouk, Tjon (2003)**

  further improvement by keeping the full numerator

Formalism
of
2-photon exchange

Elastic eN scattering beyond one-photon exchange approximation



$$P \equiv \frac{p+p'}{2}, \qquad K \equiv \frac{k+k'}{2}$$

Kinematical invariants :

$$\begin{aligned} Q^2 &= -(p-p')^2 \\ \nu &= K \cdot P = (s-u)/4 \end{aligned}$$

for $m_e = 0$

$$\begin{aligned} T^{non-flip}_{h'\lambda'_N, h\lambda_N} &= \frac{e^2}{Q^2}\bar{u}(k',h')\gamma_\mu u(k,h) \\ &\times \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2\frac{P^\mu}{M} + \tilde{F}_3\frac{\gamma.K P^\mu}{M^2}\right)u(p,\lambda_N) \end{aligned}$$

$$\begin{aligned} \tilde{G}_M(\nu,Q^2) &= G_M(Q^2) + \delta\tilde{G}_M \\ \tilde{F}_2(\nu,Q^2) &= F_2(Q^2) + \delta\tilde{F}_2 \\ \tilde{F}_3(\nu,Q^2) &= 0 + \delta\tilde{F}_3 \end{aligned}$$

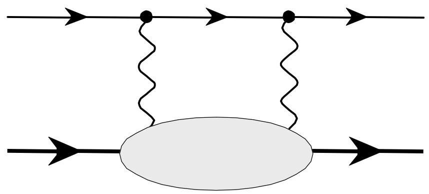
equivalently, introduce

$$\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\,\tilde{F}_2$$

$$\tilde{G}_E(\nu,Q^2) = G_E(Q^2) + \delta\tilde{G}_E$$

Guichon, Vdh (2003)

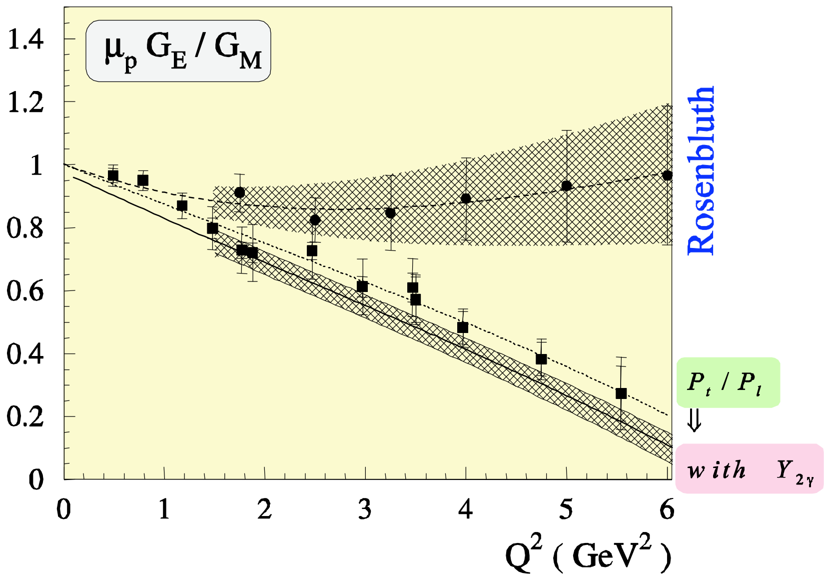
Observables including two-photon exchange
Real parts of two-photon amplitudes



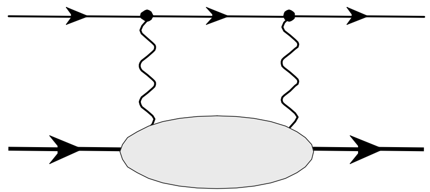
$$\sigma_R = G_M^2 \left(1 + 2\frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \varepsilon\left\{\frac{1}{\tau}G_E^2\left(1 + 2\frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_E}\right) + 2G_M^2\left(1 + \frac{1}{\tau}\frac{G_E}{G_M}\right)\underbrace{\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}}_{Y_{2\gamma}}\right\} + \mathcal{O}(e^4)$$

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\left\{\frac{G_E}{G_M}\left(1 - \frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_M} + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)\underbrace{\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}}_{Y_{2\gamma}}\right\} + \mathcal{O}(e^4)$$

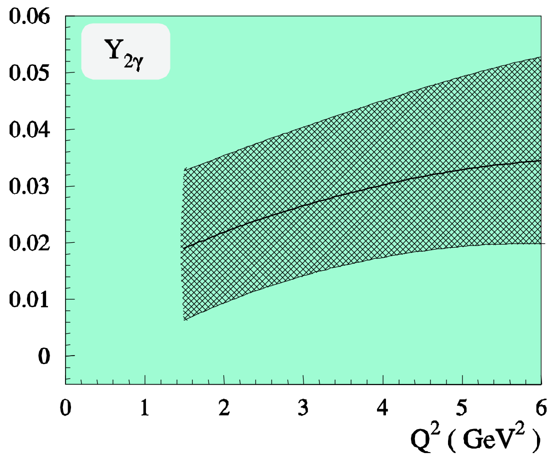
Phenomenological analysis



Guichon, Vdh (2003)



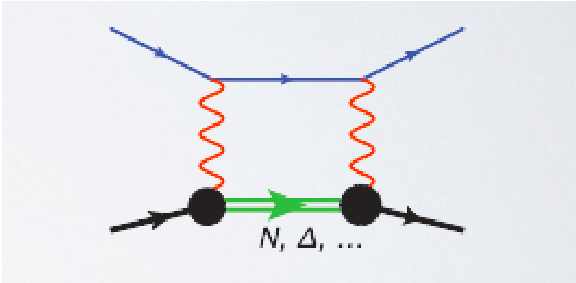
2-photon exchange corrections can become large on the Rosenbluth extraction,and are of different size for both observables

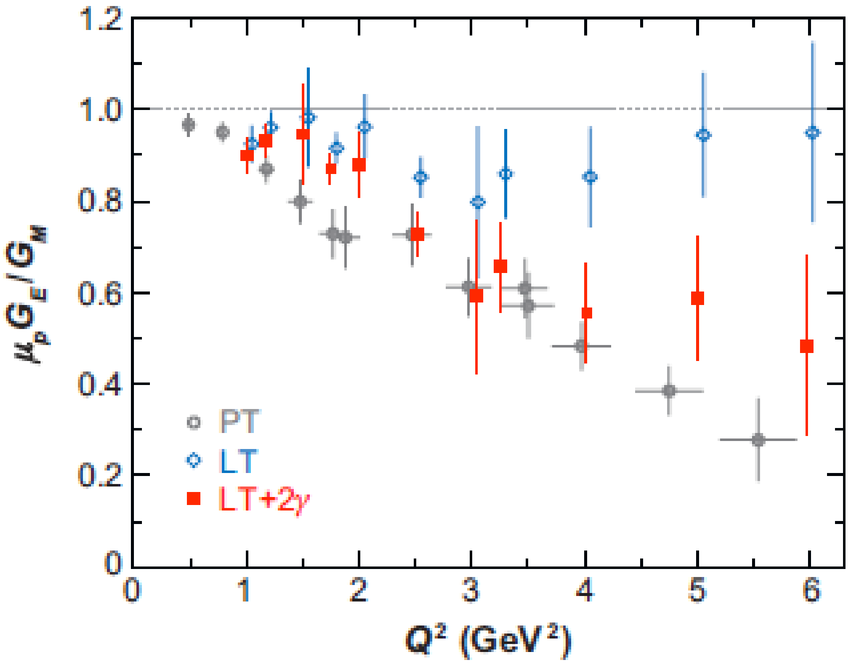


➡ relevance when extracting form factors at large Q^2

2-photon exchange calculations

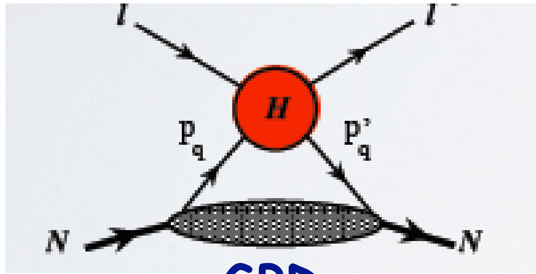
hadronic calculation



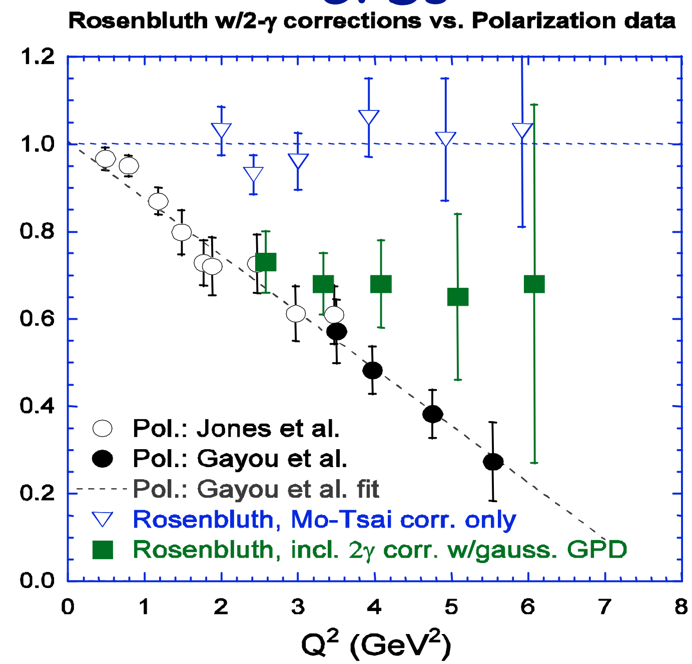


Blunden, Tjon, Melnitchouk (2003, 2005)

partonic calculation



GPDs



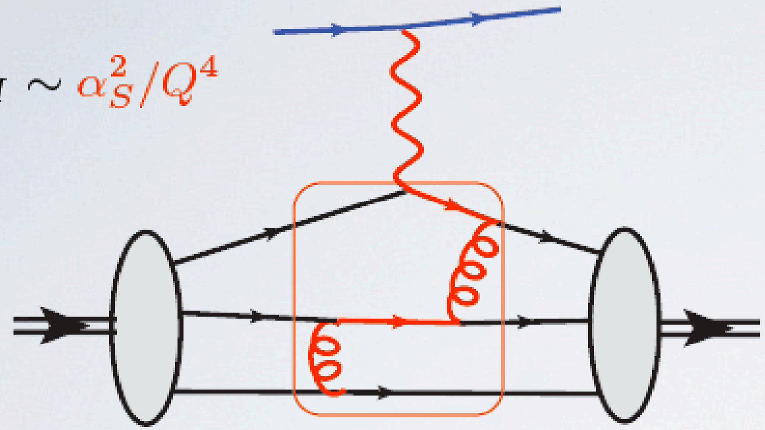
Chen, Afanasev, Brodsky, Carlson, Vdh (2003)

Leading pQCD analysis of 2-photon exchange amplitude

Proton FFs at large Q^2

Chernyak, Zhitnizky (1977) ; Brodsky, Lepage (1979); Efremov, Radyushkin (1980)

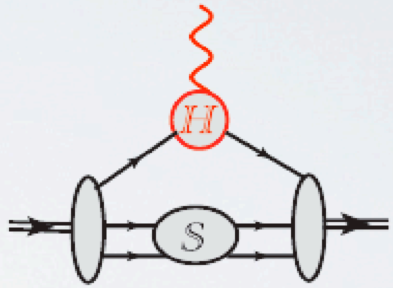
$G_M \sim \alpha_S^2/Q^4$



$\varphi_N(x_i) * T_H(x_i, x_i', Q^2) * \varphi_N(x_i')$

$G_E = G_M - (1+\tau)F_2 \sim 1/Q^4$

$F_2 \sim 1/Q^6$

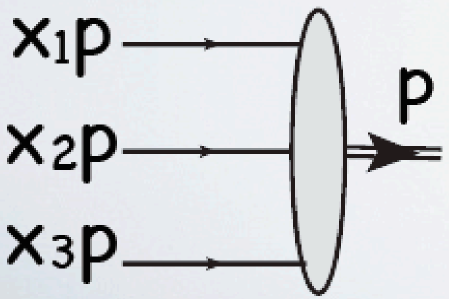


helicity flip ff suppressed, not factorized

$\sim 1/Q^6$

large soft overlap contribution

Distribution amplitude



$$\varphi_N(x_1, x_2, x_3) \sim \int dk_{i\perp} \Psi_P(x_1, x_2, x_3, k_{i\perp})$$

$\varphi_N(x_i)$ describes how the longitudinal momentum is shared between the constituents

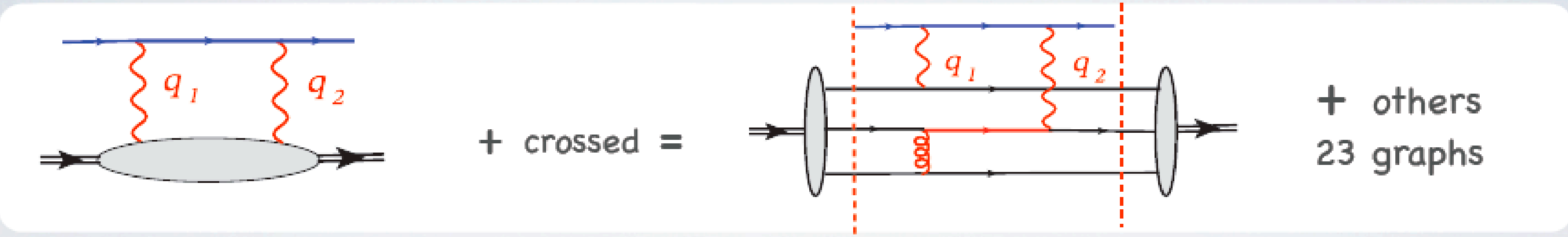
Leading pQCD analysis of 2-photon exchange amplitude

Kivel, Vdh (2009);

Borisyuk, Kobushkin (2009)

Dominant region : both photons are highly virtual

$$q_1^2 \sim q_2^2 \sim q^2 = (p' - p)^2 \equiv -Q^2$$



$$\varphi_N(x_i) * T_H(x, x', \varepsilon, Q^2) * \varphi_N(x_i')$$

helicity conserving amplitudes

1-γ exchange

$$G_M \sim \alpha_S^2 / Q^4$$

2-γ exchange (2 amplitudes)

$$\delta \tilde{G}_M \sim \tau \tilde{F}_3 \sim \alpha_{em} \alpha_S / Q^4$$

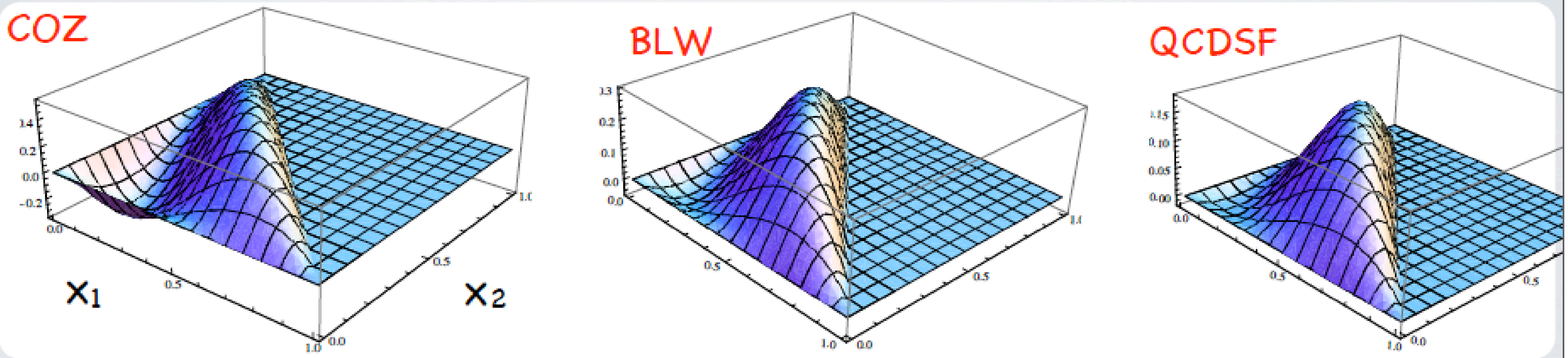
- all integrals are IR-finite with asymptotic DA

Proton Distribution Amplitude

Typical model $\varphi_N(x_i) \simeq 120\, f_N\, x_1 x_2 x_3 (1 + r_-(x_1 - x_2) + r_+(1 - 3x_3) + ...)$

includes 3 parameters $f_N,\, r_-,\, r_+ \qquad \mu^2 = 1\mathrm{GeV}^2$

	f_N $(10^{-3}$ GeV$^2)$	r_-	r_+
COZ	5.0 ± 0.5	4.0 ± 1.5	1.1 ± 0.3
BLW	5.0 ± 0.5	1.37	0.35
QCDSF	3.23	1.06	0.33



largest TPE effect — QCD sum rules — Chernyak, Ogloblin, Zhitnitsky (1988)

intermediate — QCD light-cone sum rules — Braun, Lenz, Wittmann (2006)

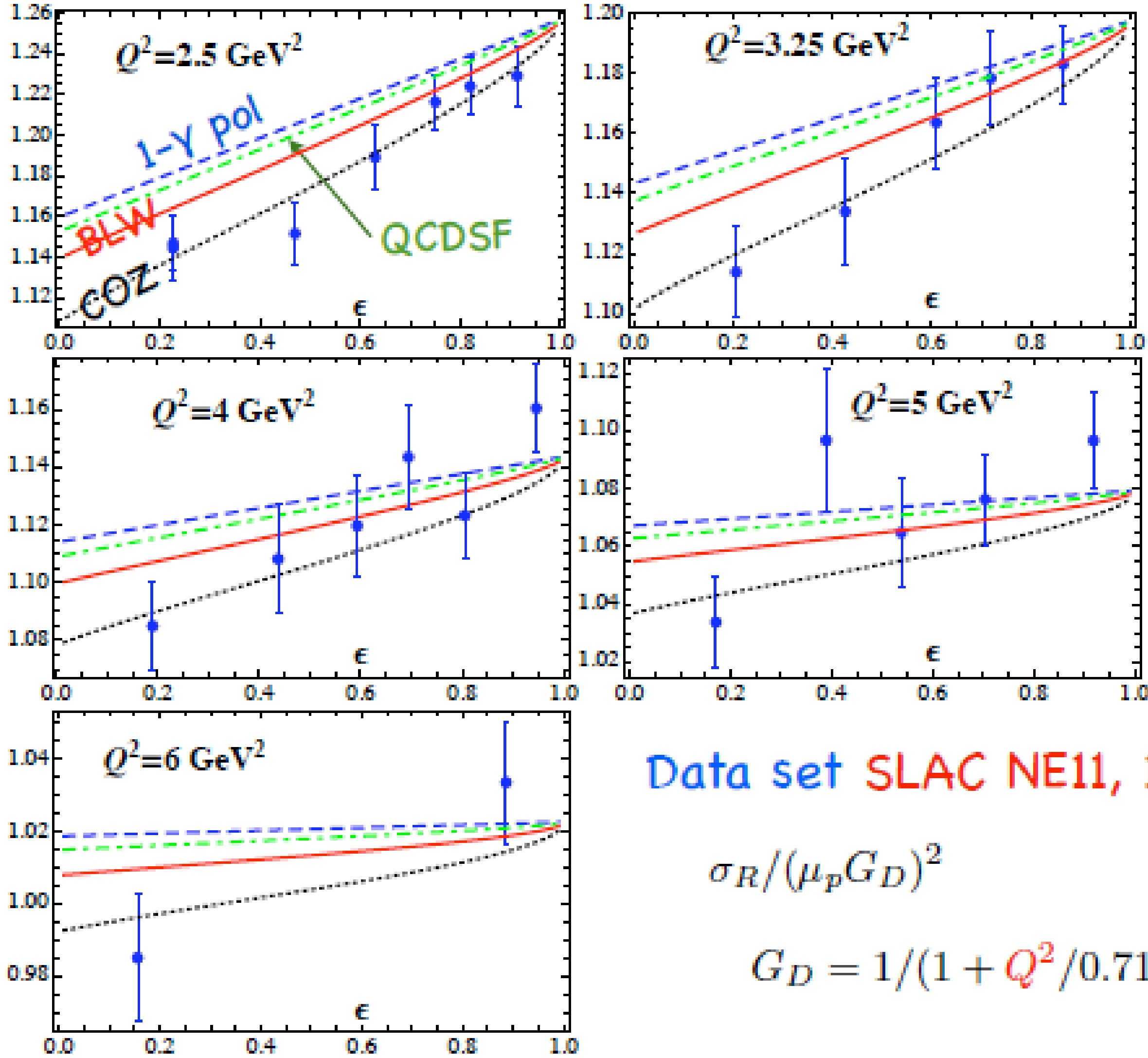
small TPE effect — Lattice QCDSF — Gockeler et al (2008)

Comparision with experiments

Results for Rosenbluth plots

Blue :

1-photon results



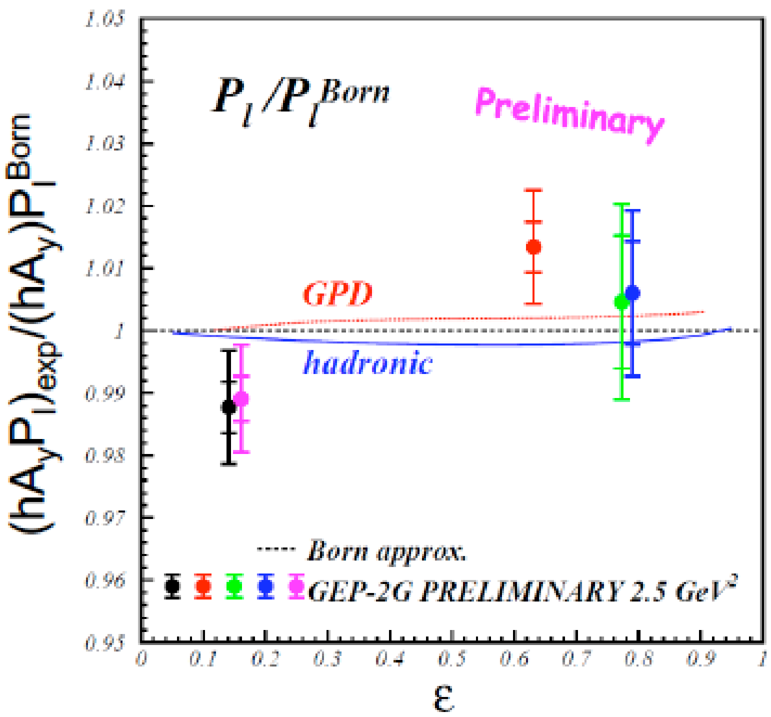
Data set SLAC NE11, 1994

$$\sigma_R/(\mu_p G_D)^2$$

$$G_D = 1/(1 + Q^2/0.71)^2$$

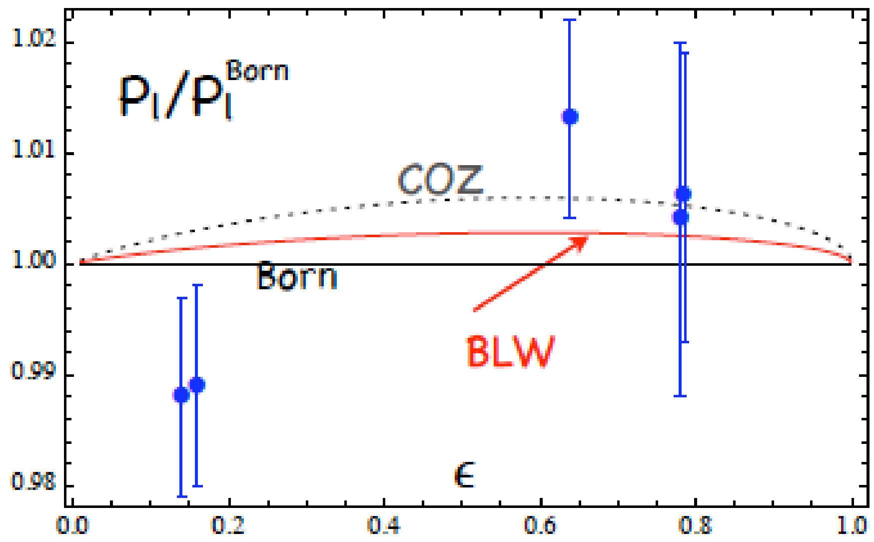
test of ε- dependence of P_l

$$P_l = \sqrt{1-\varepsilon^2}\,\frac{1}{\sigma_R}\left\{G_M^2 + 2\,G_M\,\mathcal{R}\left(\delta\tilde{G}_M + \frac{\varepsilon}{1+\varepsilon}\frac{\nu}{M^2}\tilde{F}_3\right) + \mathcal{O}(e^4)\right\}$$



JLab/Hall C data

Q^2=2.5 GeV²



2γ corrections on P_l small !

test of ε- dependence of P_t / P_l

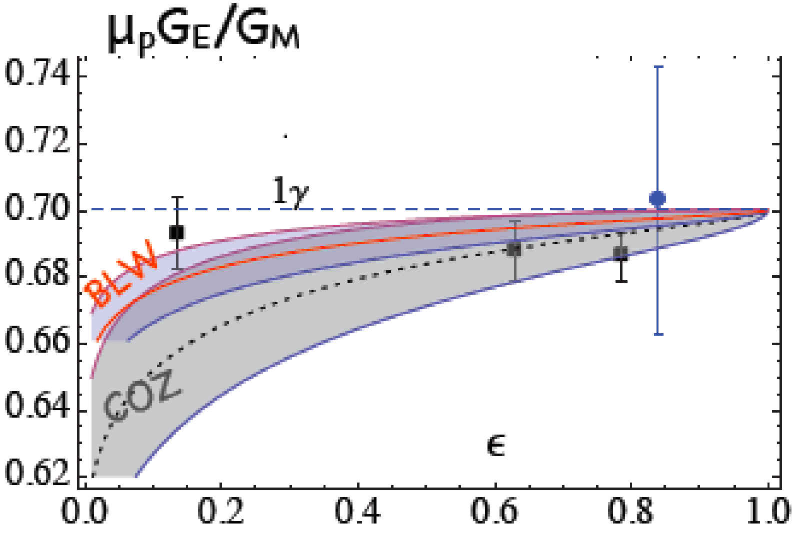
$$P_t = -\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\frac{1}{\sigma_R}\left\{G_E G_M + G_E\,\mathcal{R}\left(\delta\tilde{G}_M\right) + G_M\,\mathcal{R}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + \mathcal{O}(e^4)\right\}$$

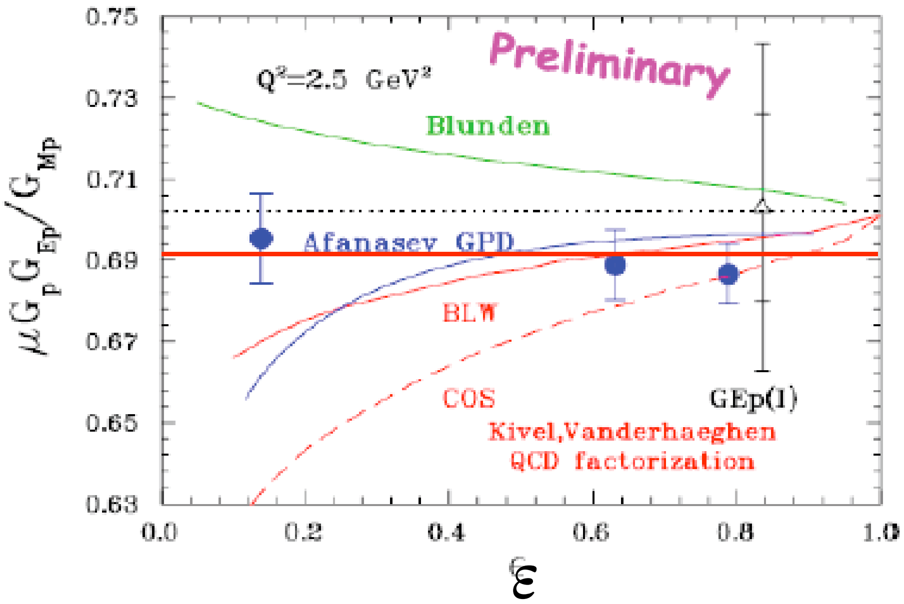
pQCD calculations

$$\delta\tilde{G}_E \sim \lambda\delta\tilde{G}_M \quad \lambda = 0,1$$

JLab/Hall C data

Q^2=2.5 GeV^2

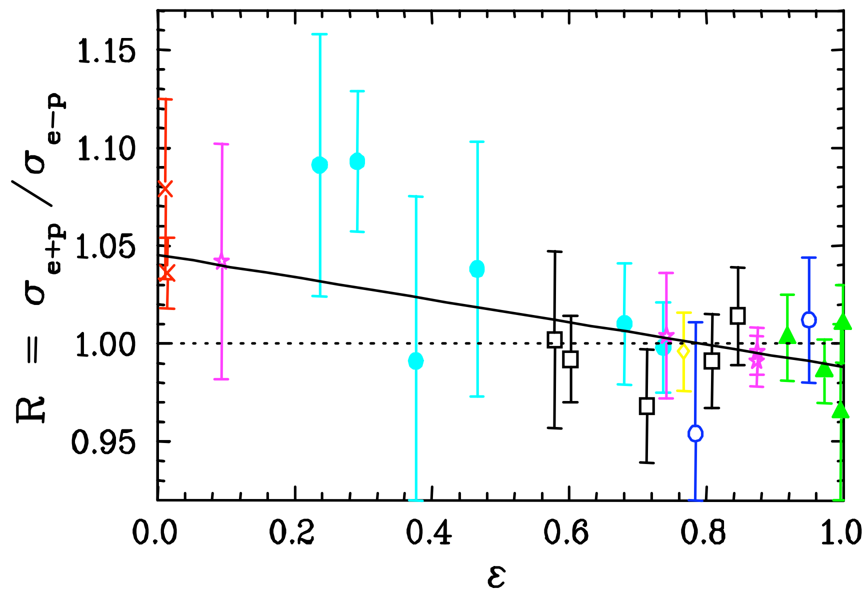




2γ corrections on P_t / P_l small !

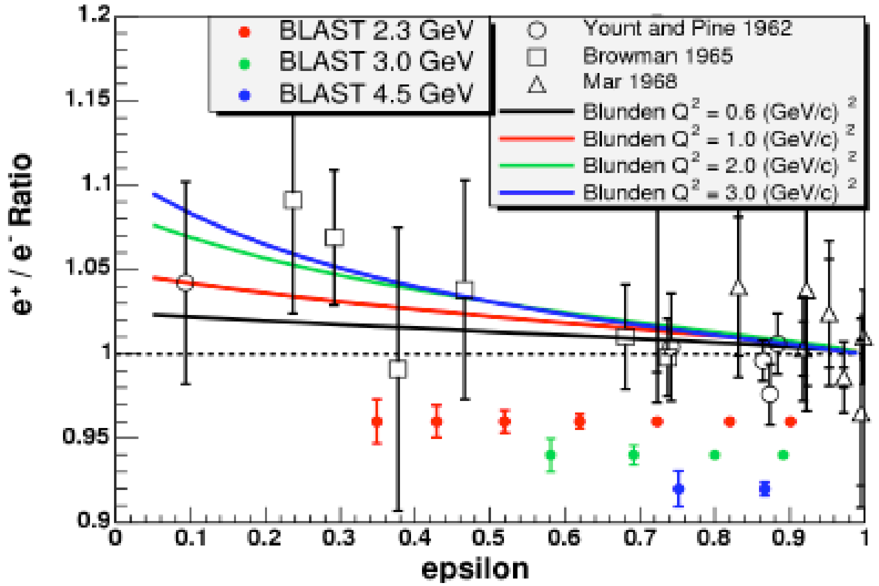
Results for e+/e- ratio

Direct test of **real part** of 2γ amplitude



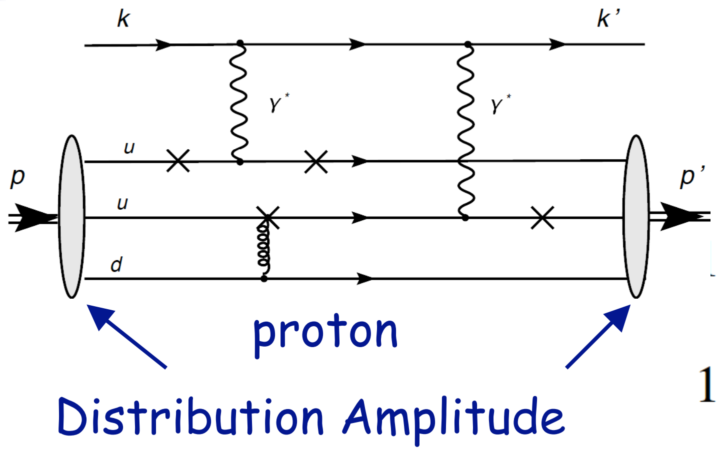
SLAC data

Arrington (2003)



Olympus projected data

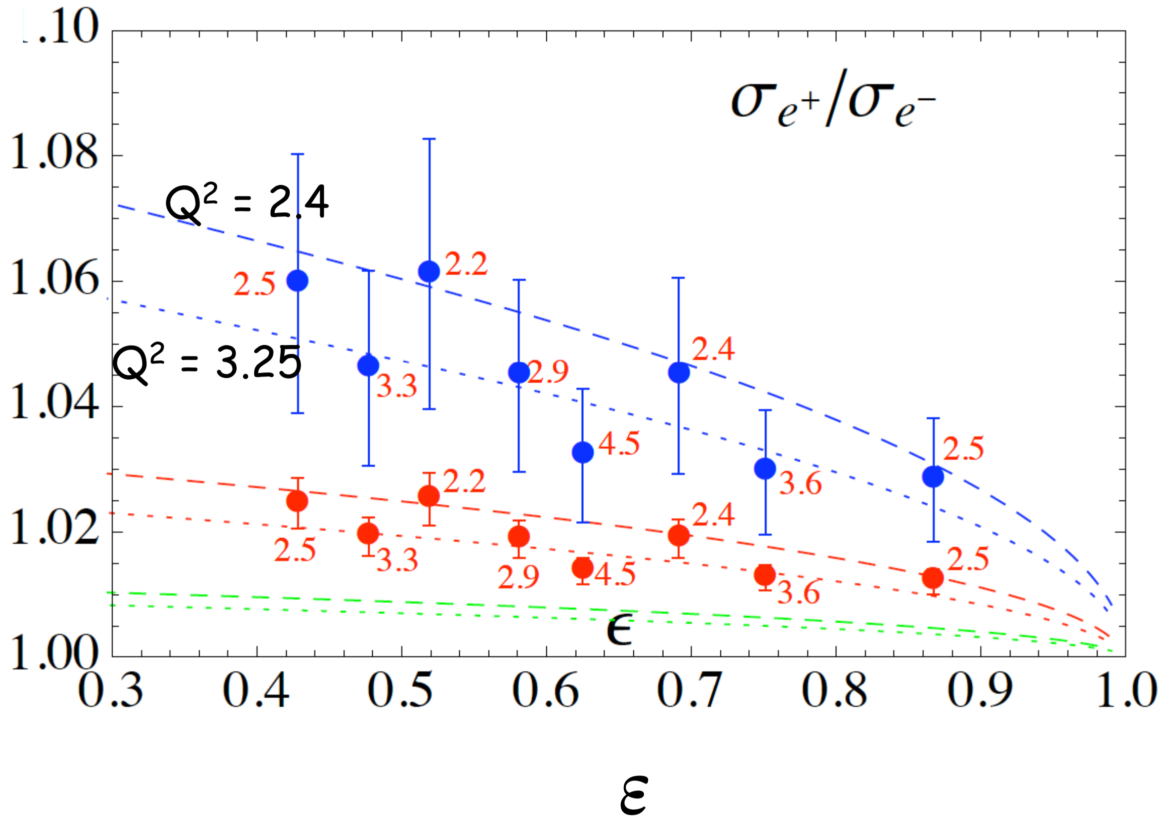
Results for e+/e- ratio



COZ DA

BLW DA

Lattice DA (QCDSF)



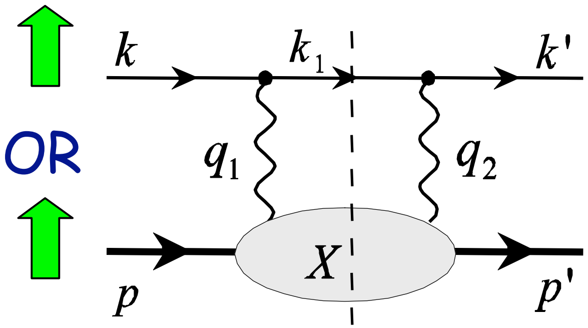
Planned experiments : **Jlab/CLAS** , **Olympus@DESY**

Calculations : Kivel, Vdh (2009)

Normal spin asymmetries in elastic eN scattering

➡ directly proportional to the **imaginary part** of 2-photon exchange amplitudes

spin of beam OR target
NORMAL to scattering plane



on-shell intermediate state

➡ order of magnitude estimates :

target : $A_n \sim \alpha_{em} \sim 10^{-2}$

beam : $B_n \sim \alpha_{em} \cdot m_e \sim 10^{-6} - 10^{-5}$

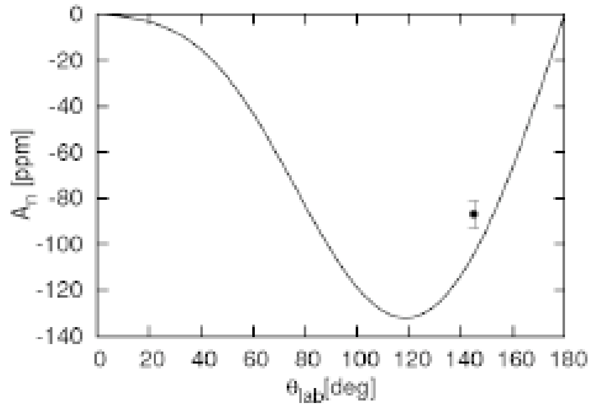
Beam normal spin asymmetry

MAMI data

A4 experiment

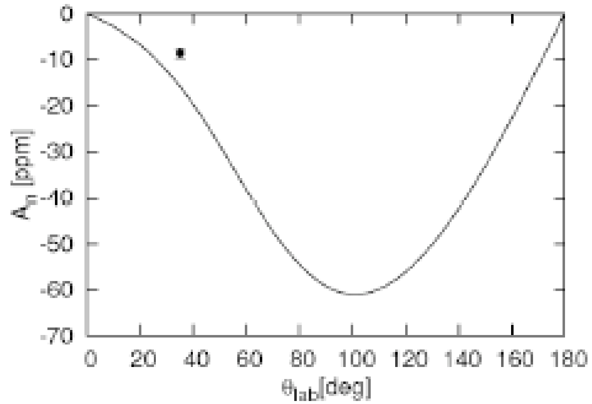
E_e = 0.300 GeV

Θ_e = 145 deg



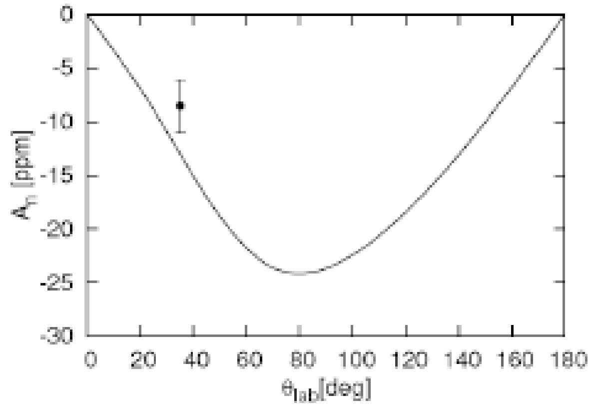
E_e = 0.570 GeV

Θ_e = 35 deg



E_e = 0.855 GeV

Θ_e = 35 deg



theory : Pasquini & Vdh (2004)

Beam normal spin asymmetry : experiments

Expt.	*E(GeV)*	Q^2 GeV^2	B_n*(ppm)*
SAMPLE	0.192	0.10	-16.4±5.9
A4	0.570	0.11	-8.59±0.89
A4	0.855	0.23	-8.52±2.31
HAPPEX	3.0	0.11	-6.7 ± 1.5
G0	3.0	0.15	-4.06 ± 1.62
G0	3.0	0.25	-4.82 ± 2.85
E-158(ep)	46.0	0.06	-3.5 -> -2.5

whether two-photon exchange is entirely responsible for the discrepancy in the FF extraction is to be determined experimentally

Real part of $Y_{2\gamma}$

1) ε-independence of G_{Ep}/G_{Mp} in recoil polarization → Hall C 04-019, completed (e^+ and e^-)

2) cross section difference in e^+ and e^- proton scattering → Hall B 07-005; Olympus/Doris with refurbished BLAST detector

3) non-linearity of Rosenbluth plot → Hall C 05-017; being analyzed

Also imaginary part

4) from induced out-of-plane polarization → by-product of 04-019/04-108?

5) single-spin target asymmetry → Hall A 05-015 (^{3}He↑)