Review of TPE in electron scattering

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Radiative Corrections Workshop at MIT

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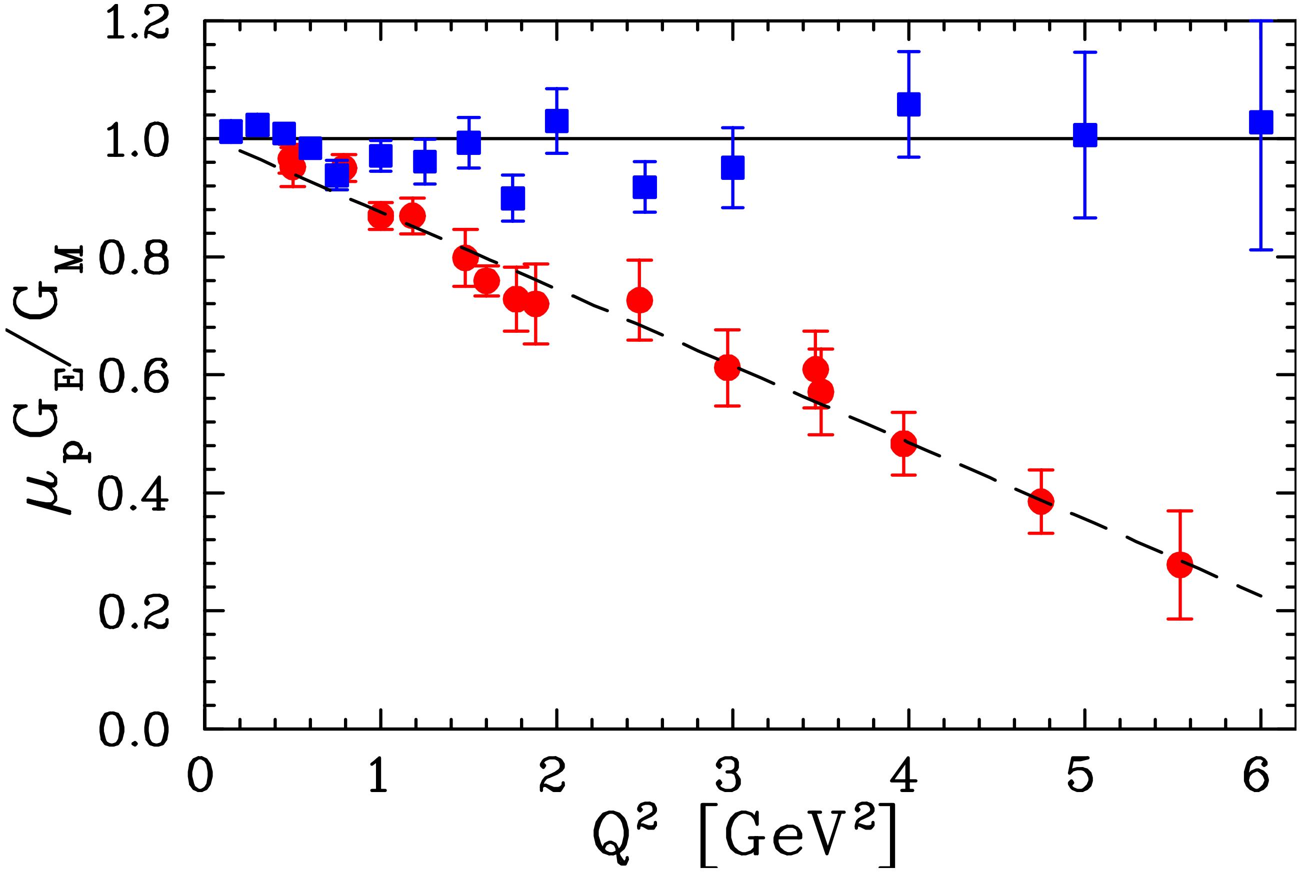
Review: Arrington, PGB, Melnitchouk, arXiv:1105.0951

(to be published in Prog. Nucl. Part. Phys.)

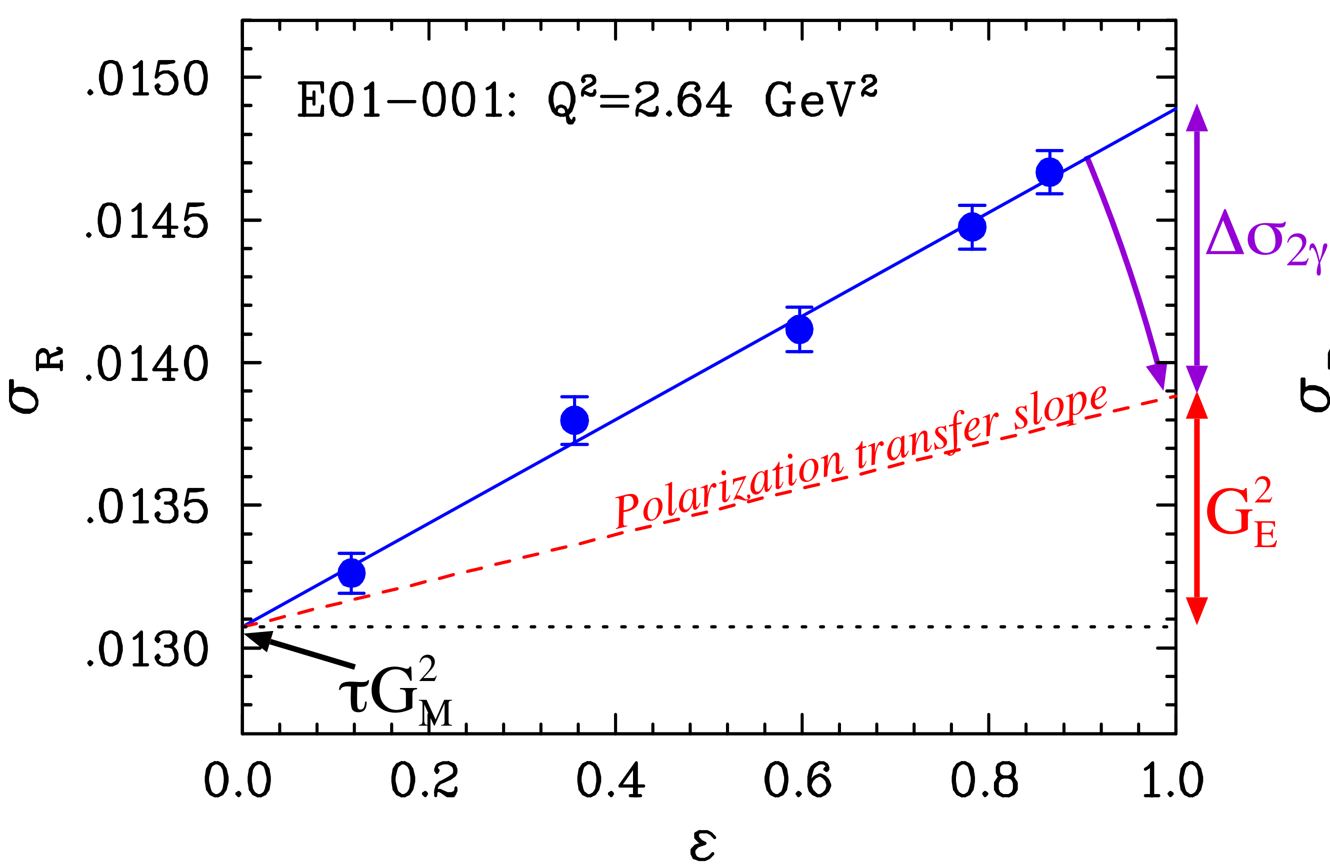
Outline

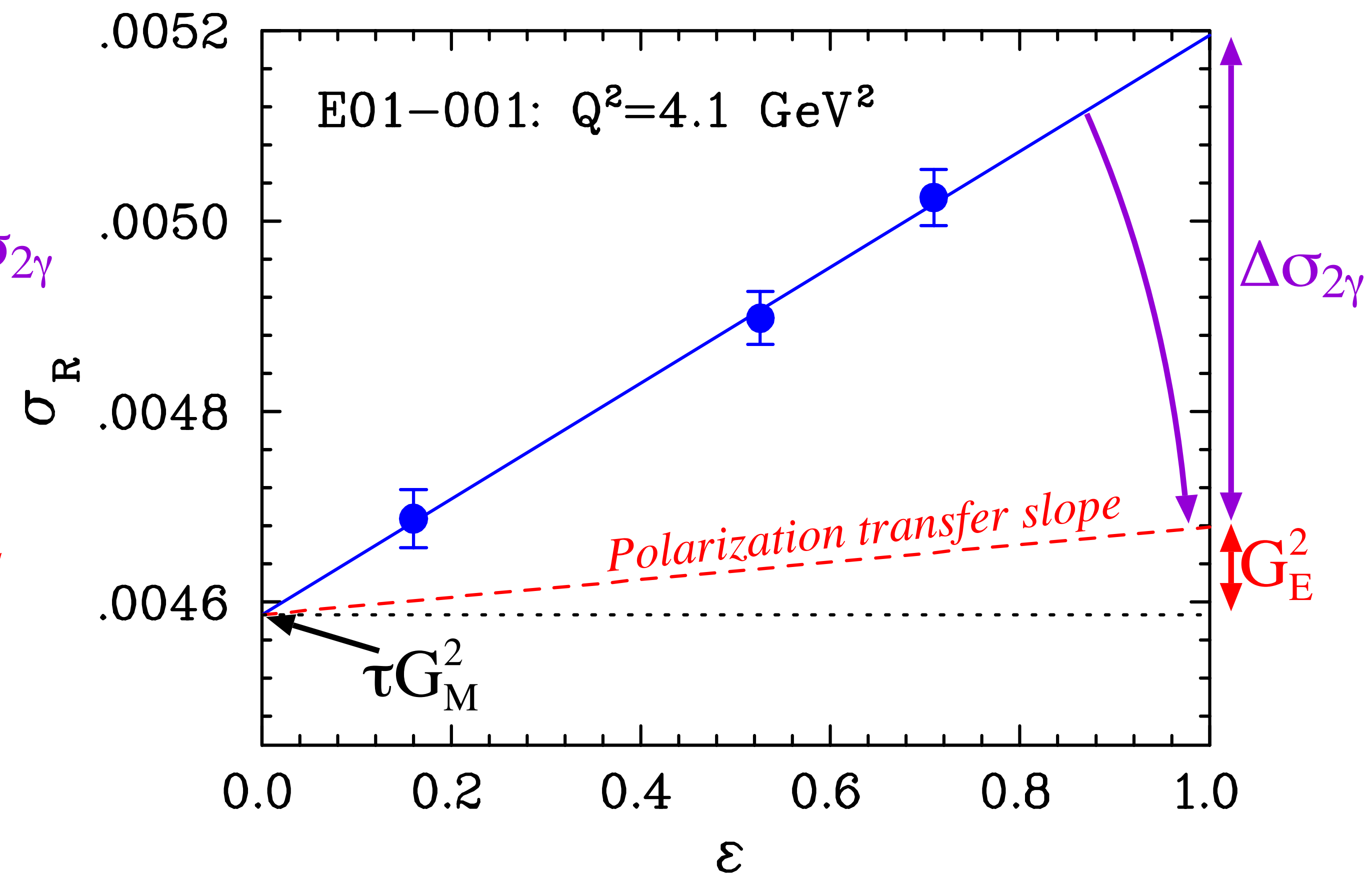
- Summary of key results
- Questions:
  - what is connection to 2nd Born approximation?
  - how do resonances and DIS region enter as Q^2 increases?
- Parity violating asymmetry A_{PV} ($\gamma\gamma$ and γZ)
  - relation to atomic PV (Marciano-Sirlin calculation) and Q_{weak}



about 50% TPE + ??

about 80% TPE + ??





Various Approaches

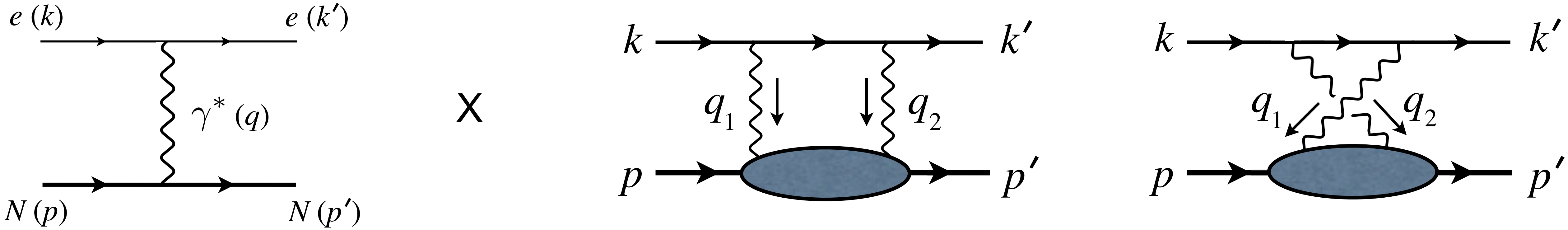
Model driven

- Low to moderate Q^2: hadronic: N + Δ + N* etc.
  - more and more parameters, less and less reliable
- Moderate to high Q^2:
  - GPD approach: assumption of 1 active quark
    - Valid only in certain kinematic range
  - pQCD: recent work indicates 2 active quarks dominate

Data driven

- Use dispersion integrals to relate Real and Imaginary parts. Imaginary parts fixed by cross section data
  - Valid at forward angles: must use models to extrapolate
  - Incomplete: not all data is available (e.g. axial hadron coupling and isospin dependence in γZ diagrams

Hadronic approach: N, Δ, … intermediate states



$$\delta_{2\gamma} = \frac{2\Re\left\{M_\gamma^\dagger M_{2\gamma}\right\}}{|M_\gamma|^2}$$

Consider $\Delta = \delta_{2\gamma} - \delta_{IR}(MT)$

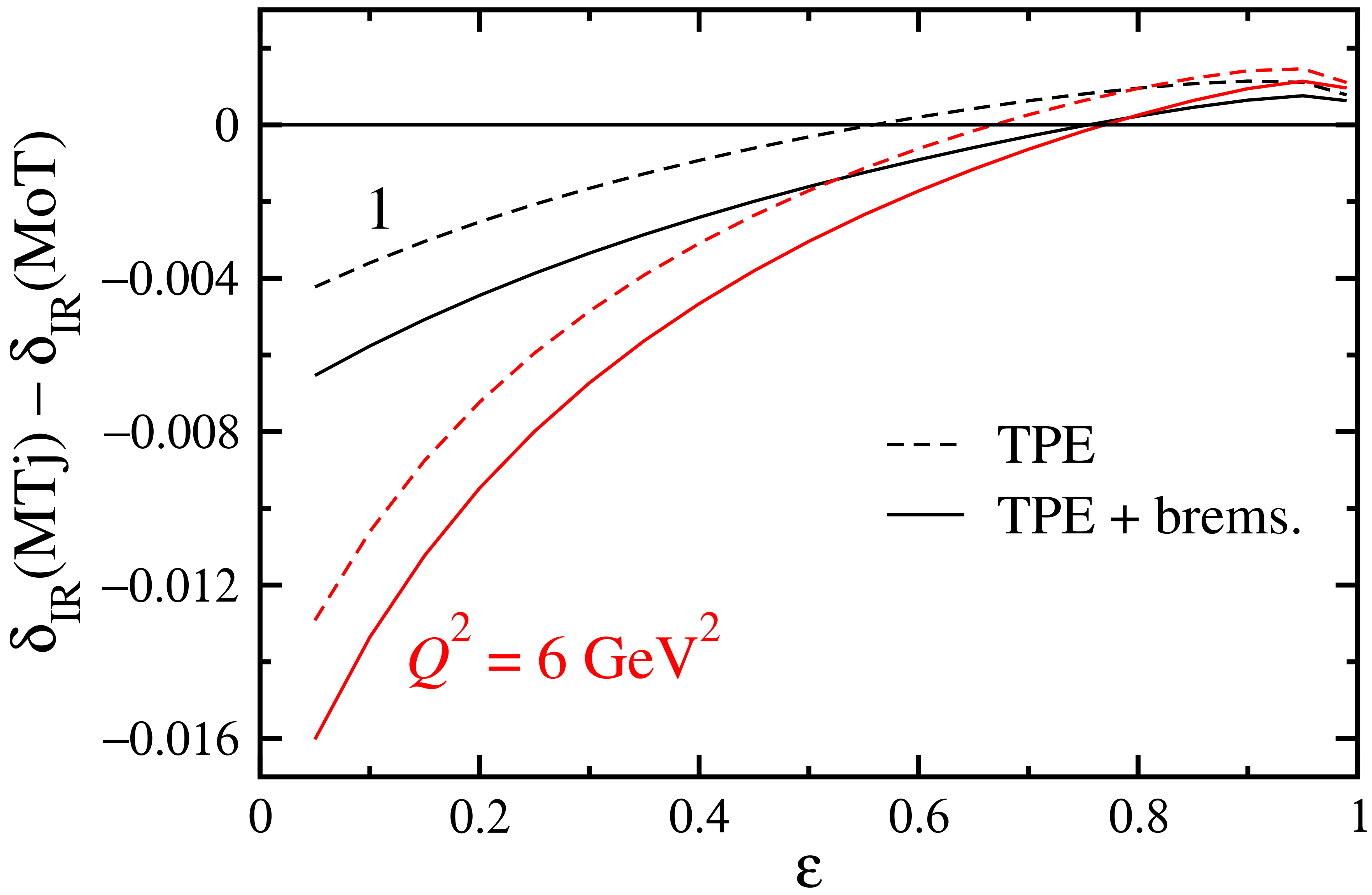
$\delta_{IR}(MT)$ is standard Mo & Tsai correction (soft photon exchange), which is ε-dependent & IR divergent

Poles at $q_1=0$ and $q_1=q$ give factorizable contribution, so

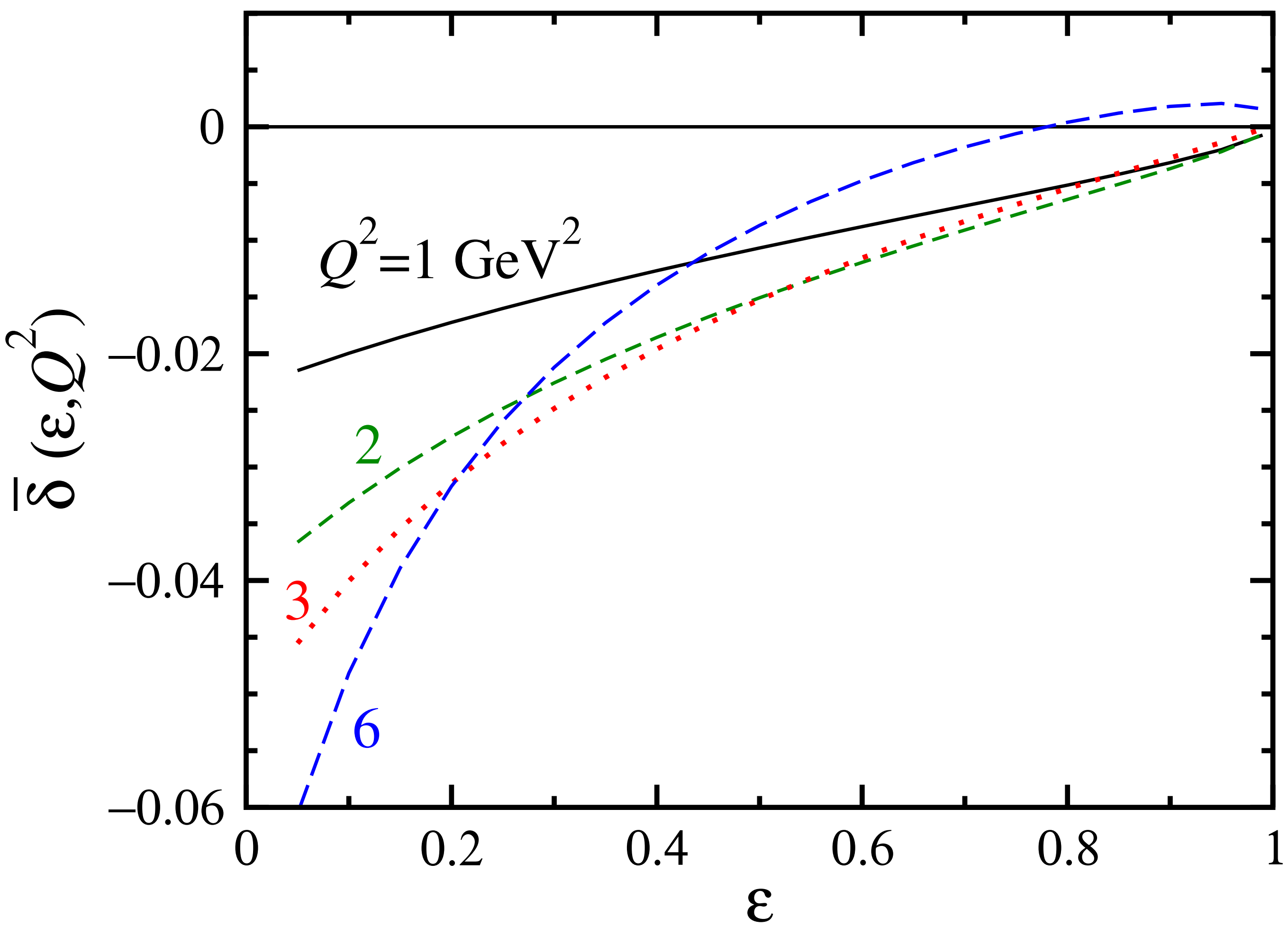
$$\mathcal{M}_{\rm IR} = \frac{1}{2}\delta_{\rm IR}\mathcal{M}_\gamma(q)$$

IR divergent terms ~log(λ) cancel in Δ

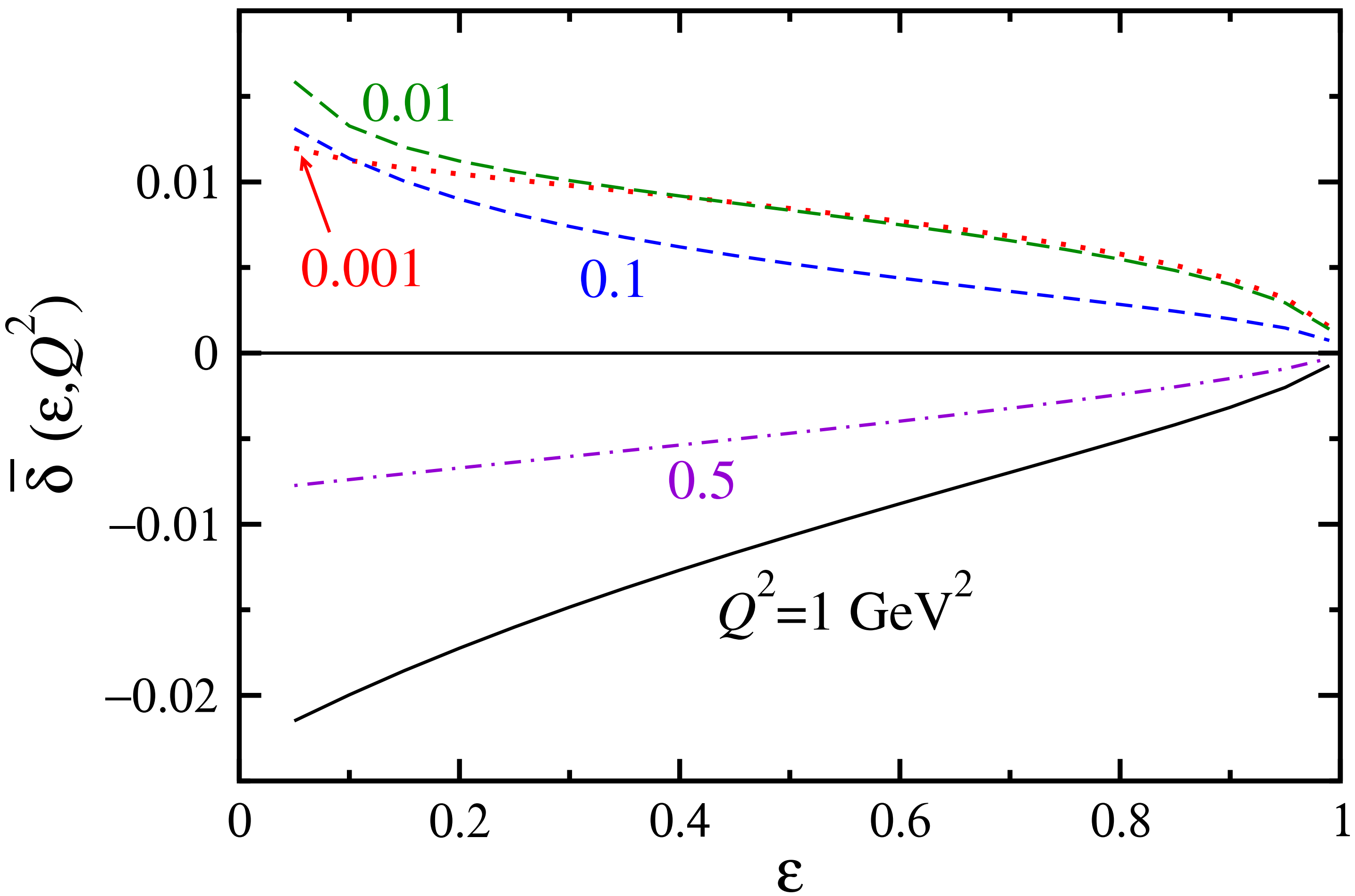
Comparison of Mo-Tsai with Maximon-Tjon



Difference in bremsstrahlung corrections smaller than difference in IR-divergent terms (~0.5 to 1%)



- positive slope
- vanishes as $\epsilon \longrightarrow 1$
- nonlinearity grows with increasing Q^2
- G_M dominates in loop integral



- changes sign at low Q^2
- agrees with static limit
- G_E dominates in loop integral

Pointlike limit (e.g. e⁻ μ⁺)

$$\delta_{\gamma\gamma} = -\frac{2\alpha}{\pi} \ln\eta \ln\frac{Q^2}{\lambda^2} + \delta_{\rm hard}$$

Static limit:

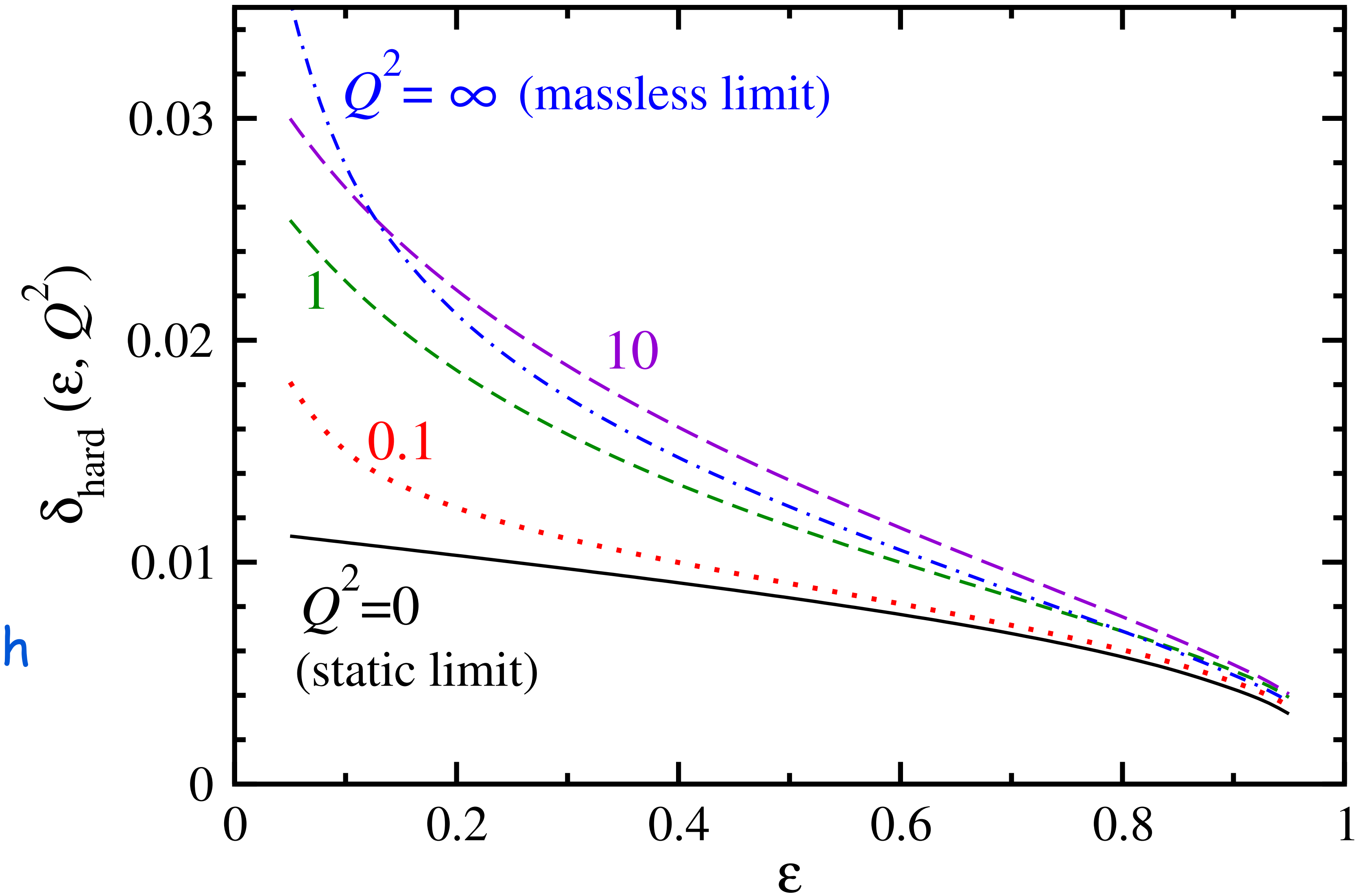
$$Q^2 \to 0 \text{ or } M \to \infty$$

$$\delta_{\rm hard} = \frac{\alpha\pi}{1+x}$$

$$x = \sqrt{(1+\epsilon)/(1-\epsilon)}$$

Agrees with McKinley & Feshbach (1948) 2nd Born result with

$$x = 1/\sin(\theta/2)$$

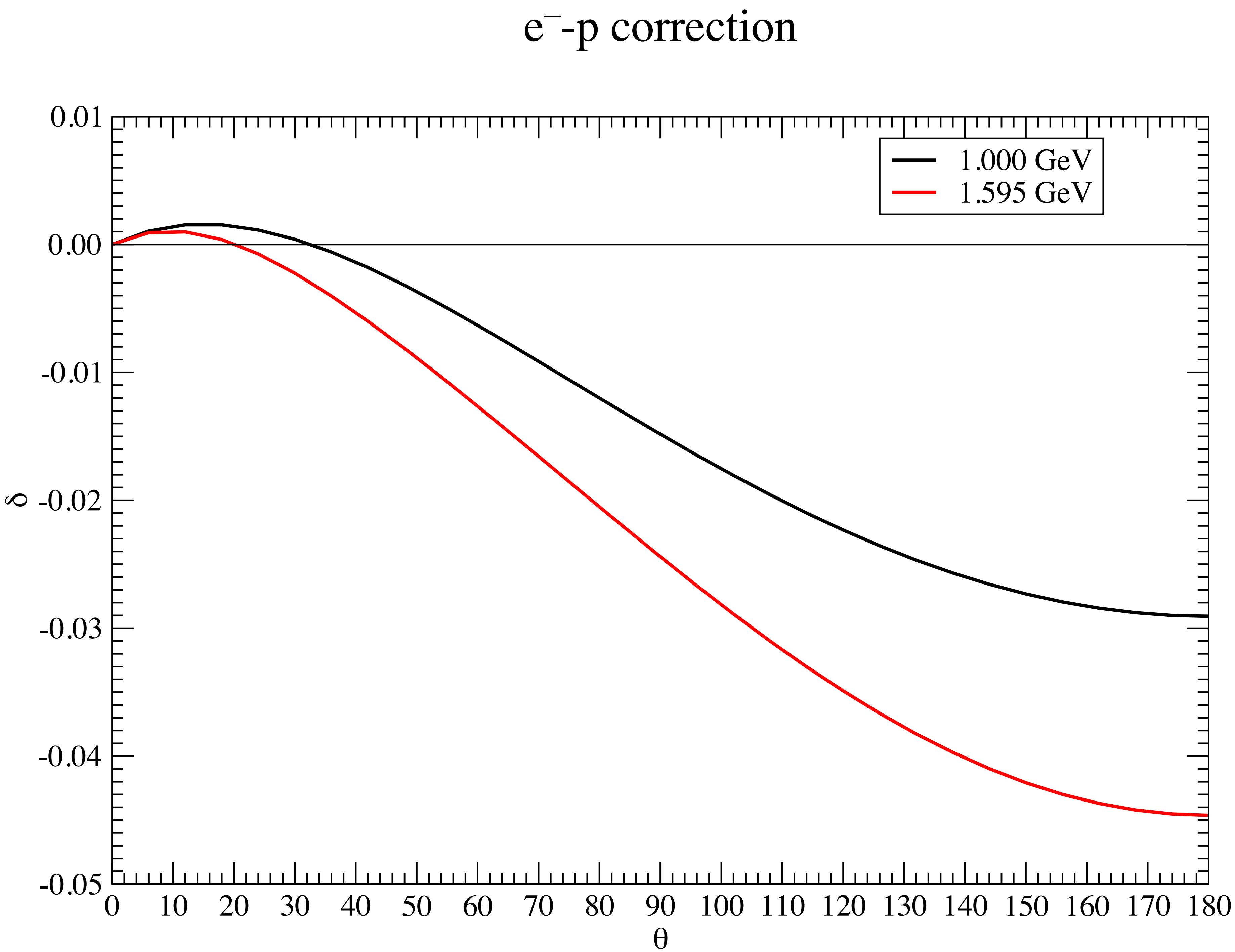


Massless limit: $Q^2 \to \infty$ or $M \to 0$

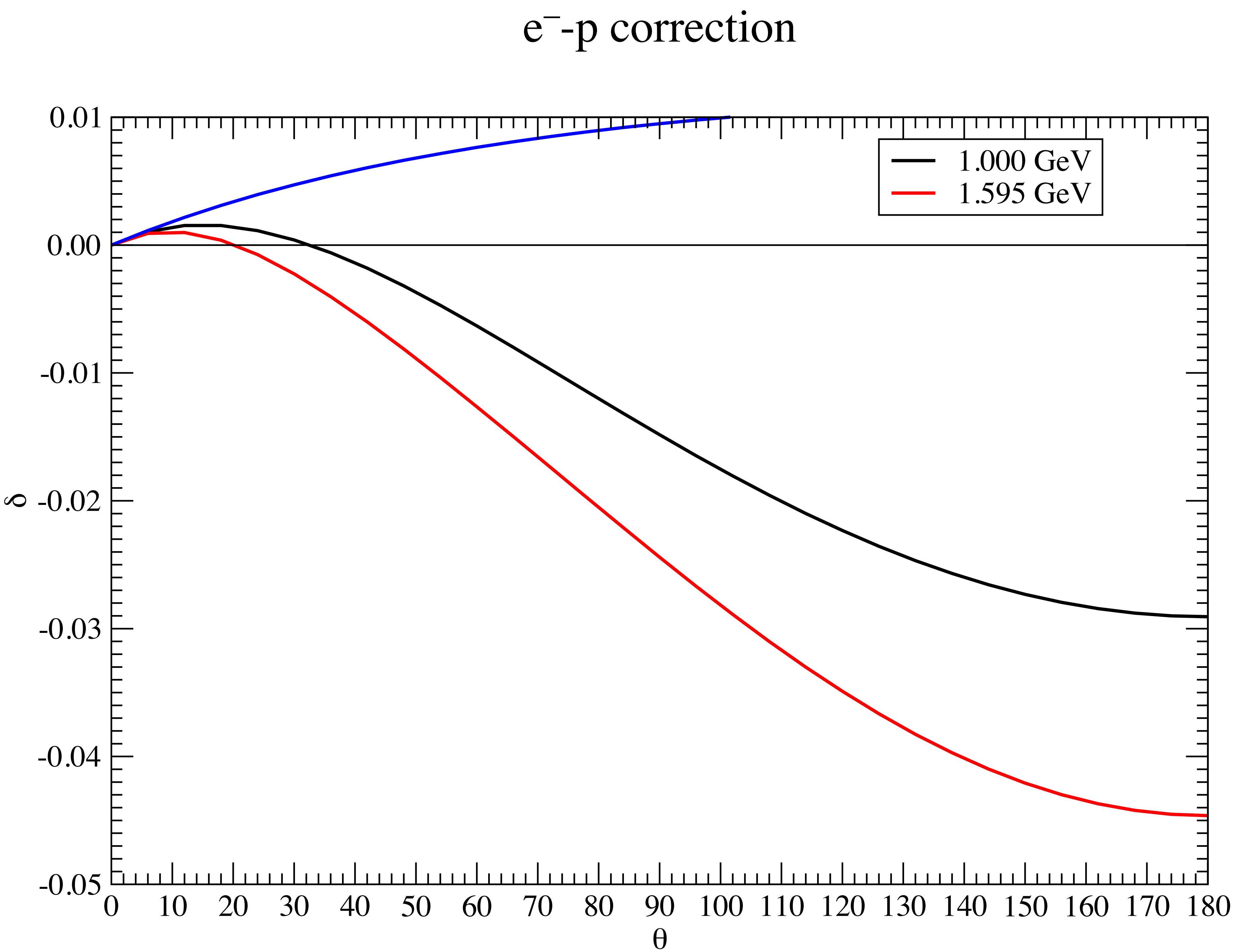
$$\delta_{\rm hard} = \frac{\alpha}{\pi(x^2+1)}\left\{\ln\left(\frac{x+1}{x-1}\right) + x\left[\pi^2 + \ln^2\left(\frac{x+1}{2}\right) + \ln^2\left(\frac{x-1}{2}\right) - \ln\left(\frac{x^2-1}{4}\right)\right]\right\}$$

Agrees with Nieuwenhiuzen (1971) and Afanasev et al. pQCD expression

Fixed E (Novosibirsk kinematics)

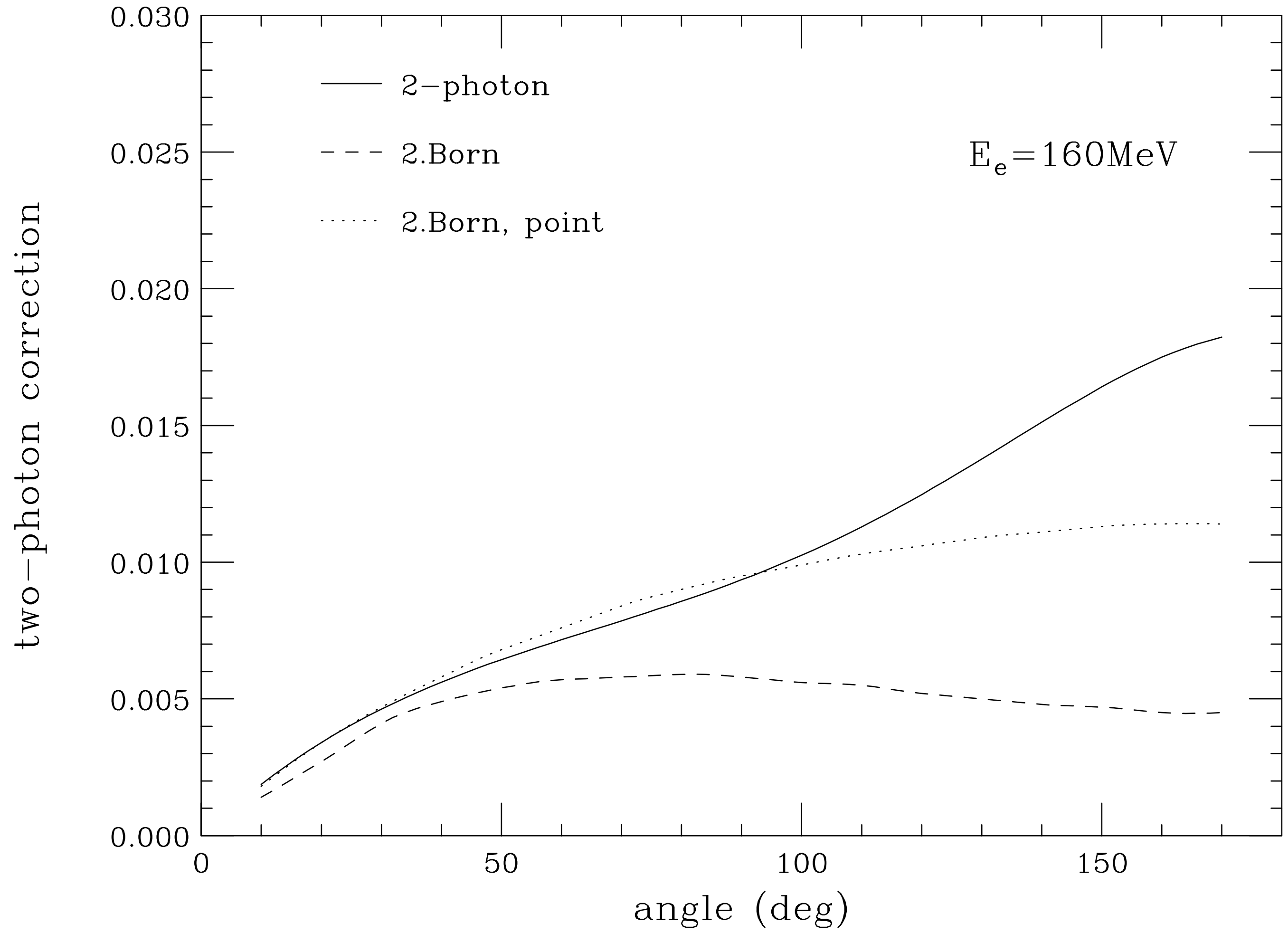


Fixed E (Novosibirsk kinematics)



Agrees with 2nd Born expression at small angles

Comparison with 2nd Born approximation at low E (PGB & Sick, 2005)

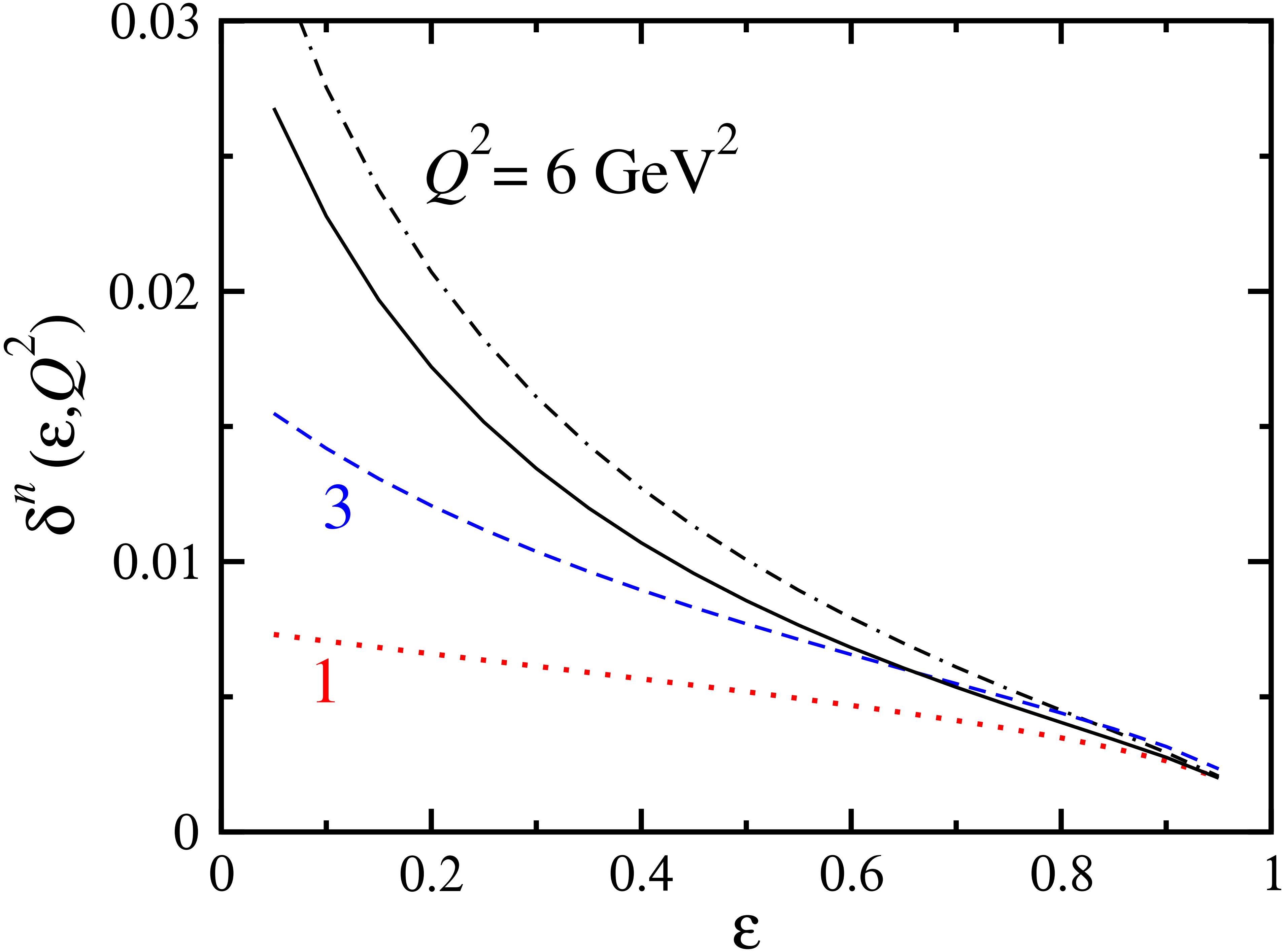


Kinematics of proton radius extraction

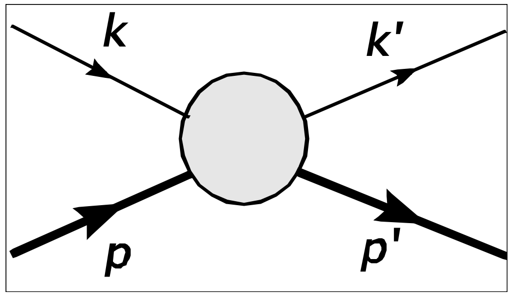
- At forward angles TPE dominated by Coulomb distortion, while at backward angles exchange of 2 hard photons contributes
- 2nd Born explanation:
  - second interaction provides "focussing" effect -- accelerating electrons towards target; should increase scattering at backward angles
  - BUT at increased Q^2 there is reduction in cross section due to form factors
  - Second process wins out. Driven by magnetic interaction.
  - Opposite effect for positrons.

Neutron

- No infrared divergences
- Positive and about 2-3 times smaller than proton (dominance of magnetic form factor)
- Some model dependence due to choice of form factors (blue curve)



Phenomenology: Generalized form factors



$$P \equiv \frac{p+p'}{2}, \qquad K \equiv \frac{k+k'}{2}$$

Kinematical invariants :

$$\begin{aligned} q^2 &= (p'-p)^2 \equiv -Q^2 \\ \nu &= K \cdot P = p \cdot k + q^2/4 \end{aligned}$$

In limit $m_e \to 0$ (helicity conservation) general amplitude can be put in form

$$T = (\gamma_\mu)^{(e)} \otimes \left(\tilde{F}_1 \gamma^\mu + i \frac{\tilde{F}_2}{2M} \sigma^{\mu\nu} q_\nu + \frac{F_3}{M^2} \gamma \cdot K P^\mu \right)^{(p)}$$

In general, 16 independent amplitudes:

parity 16 $\to$ 8; time reversal 8 $\to$ 6; helicity conservation (m_e=0) 6 $\to$ 3

Generalized (complex) form factors

$$\begin{aligned} \tilde{F}_1(\nu, Q^2) &= F_1(Q^2) + \delta F_1 \\ \tilde{F}_2(\nu, Q^2) &= F_2(Q^2) + \delta F_2 \end{aligned}$$

$$\begin{aligned} \tilde{G}_M &= \tilde{F}_1 + \tilde{F}_2 \\ \tilde{G}_E &= \tilde{F}_1 - \tau \tilde{F}_2 \\ Y_2 &= \frac{\nu}{M^2} \frac{F_3}{G_M} \end{aligned}$$

Observables including two-photon exchange

$$\frac{\delta\sigma}{\sigma_0} = 2\frac{\{\epsilon G_E \delta G_E + \tau G_M \delta G_M + \epsilon Y_2(\tau G_M^2 + G_M G_E)\}}{\epsilon G_E^2 + \tau G_M^2}$$

$$\frac{\delta P_L}{P_L} = 2\frac{\delta G_M}{G_M} + 2\frac{\epsilon}{1+\epsilon} Y_2 - \frac{\delta\sigma}{\sigma_0}$$

$$\frac{\delta P_T}{P_T} = \frac{\delta G_M}{G_M} + \frac{\delta G_E}{G_E} + \frac{G_M}{G_E} Y_2 - \frac{\delta\sigma}{\sigma_0}$$

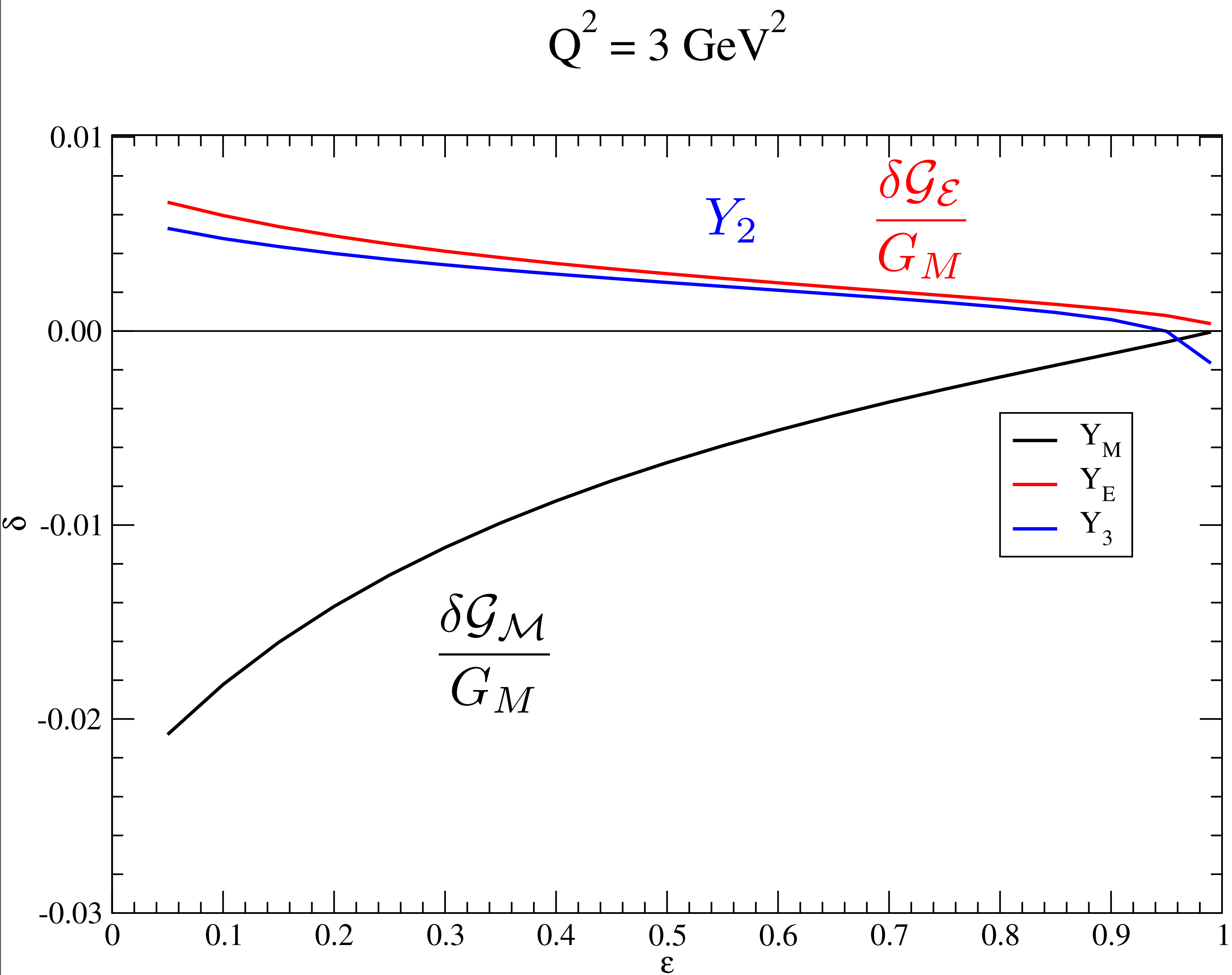
A more "convenient" parametrization (Borisyuk & Kobushkin)

$$\frac{\delta\mathcal{G}_\mathcal{M}}{G_M} = \frac{\delta G_M}{G_M} + \epsilon Y_2 \qquad \frac{\delta\mathcal{G}_\mathcal{E}}{G_M} = \frac{\delta G_E}{G_M} + Y_2$$

$$\frac{\delta\sigma}{\sigma_0} = 2\frac{\{\epsilon G_E \delta\mathcal{G}_\mathcal{E} + \tau G_M \delta\mathcal{G}_\mathcal{M}\}}{\epsilon G_E^2 + \tau G_M^2} \approx 2\frac{\delta\mathcal{G}_\mathcal{M}}{G_M} \qquad \frac{\delta P_L}{P_L} \approx -2\frac{\epsilon^2 Y_2}{1+\epsilon}$$

Forward dispersion relation implies $\delta\sigma/\sigma_0 \to 0$ as $\epsilon \to 1$

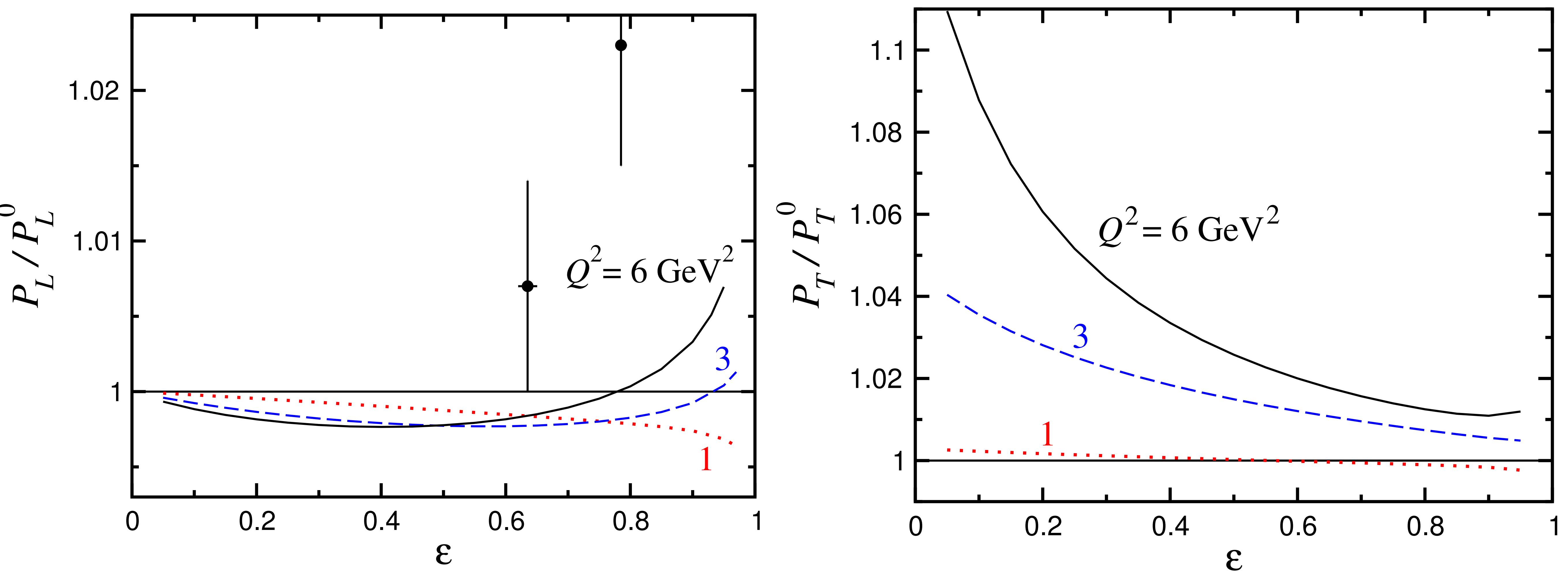
$\therefore \mathcal{G}_\mathcal{M} \to 0$ and $\mathcal{G}_\mathcal{E} \to 0$



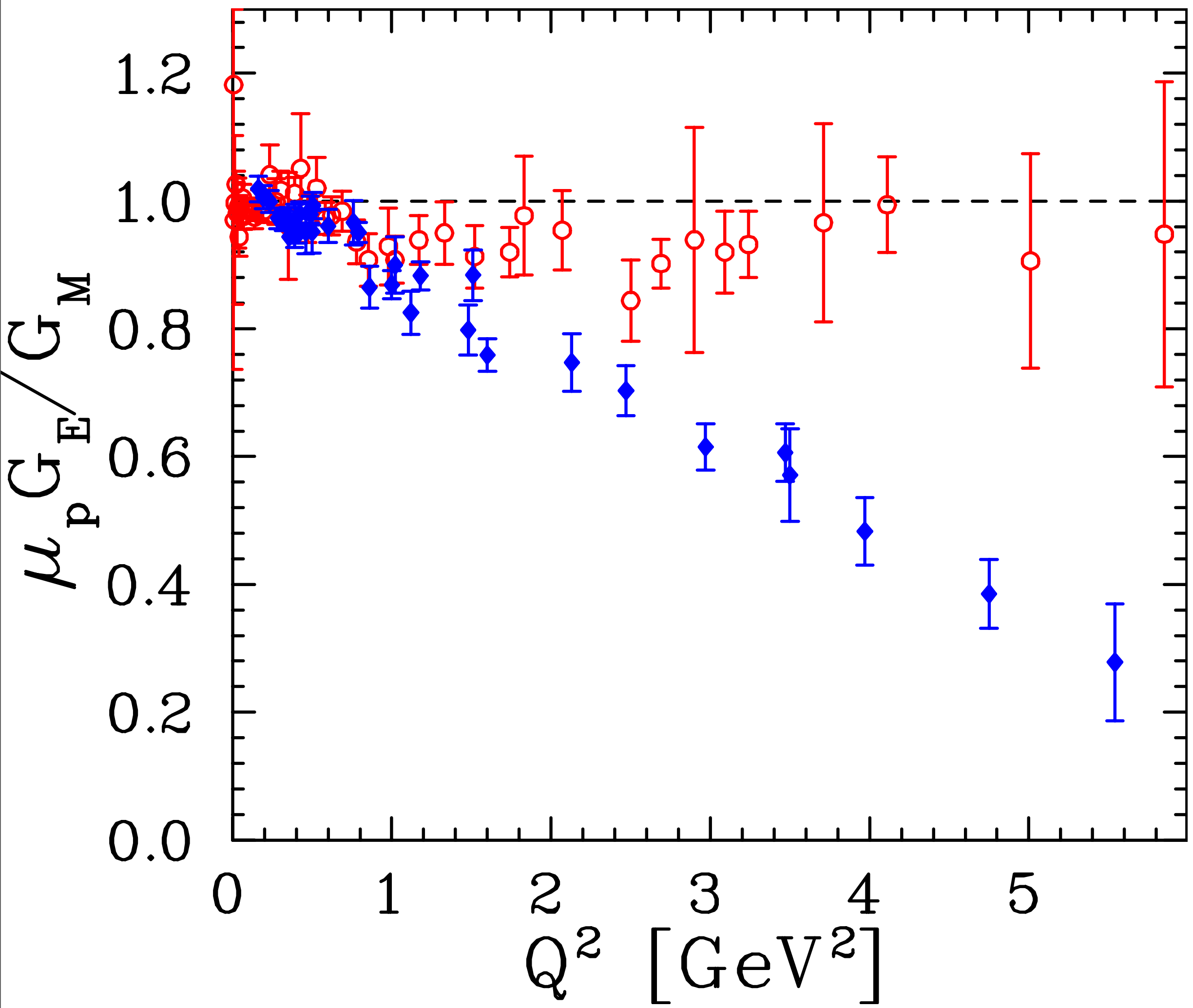
Parameterize $\mathcal{G}_\mathcal{M}, \mathcal{G}_\mathcal{E} \sim (1-\epsilon)$ (Borisyuk & Kobushkin)

$\delta \approx 2\frac{\delta \mathcal{G}_\mathcal{M}}{G_M}$ to within few percent

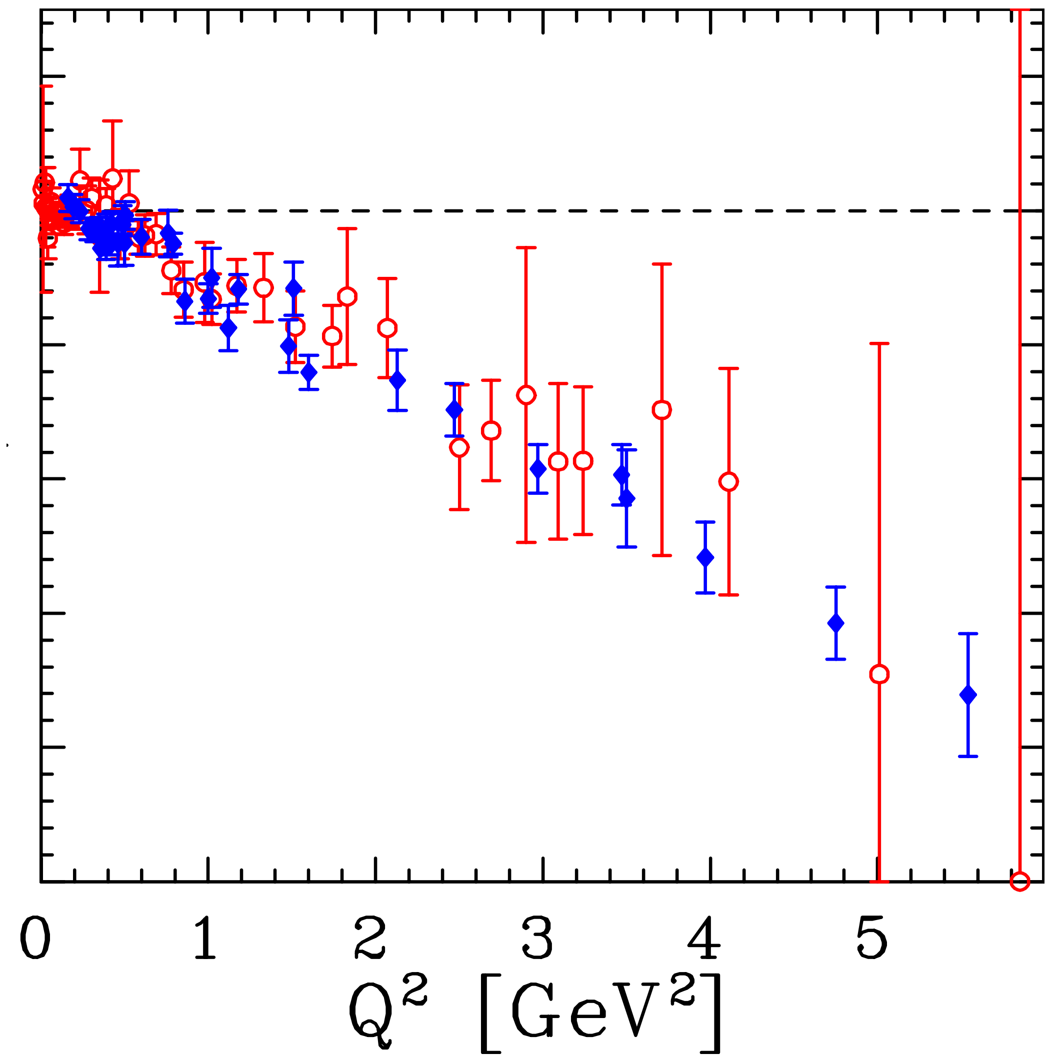
$$\vec{e} + p \to e + \vec{p}$$



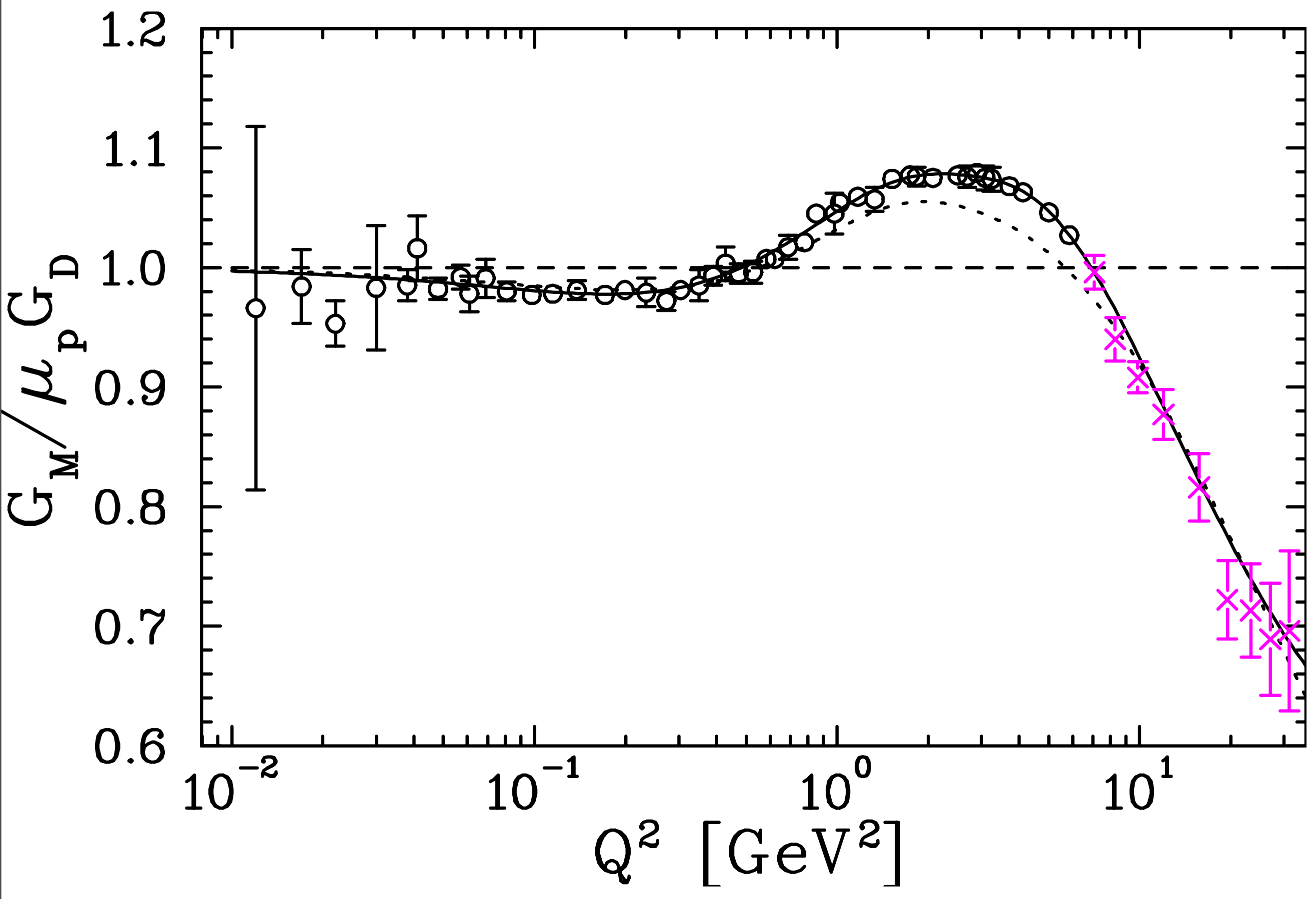
Note different scales!

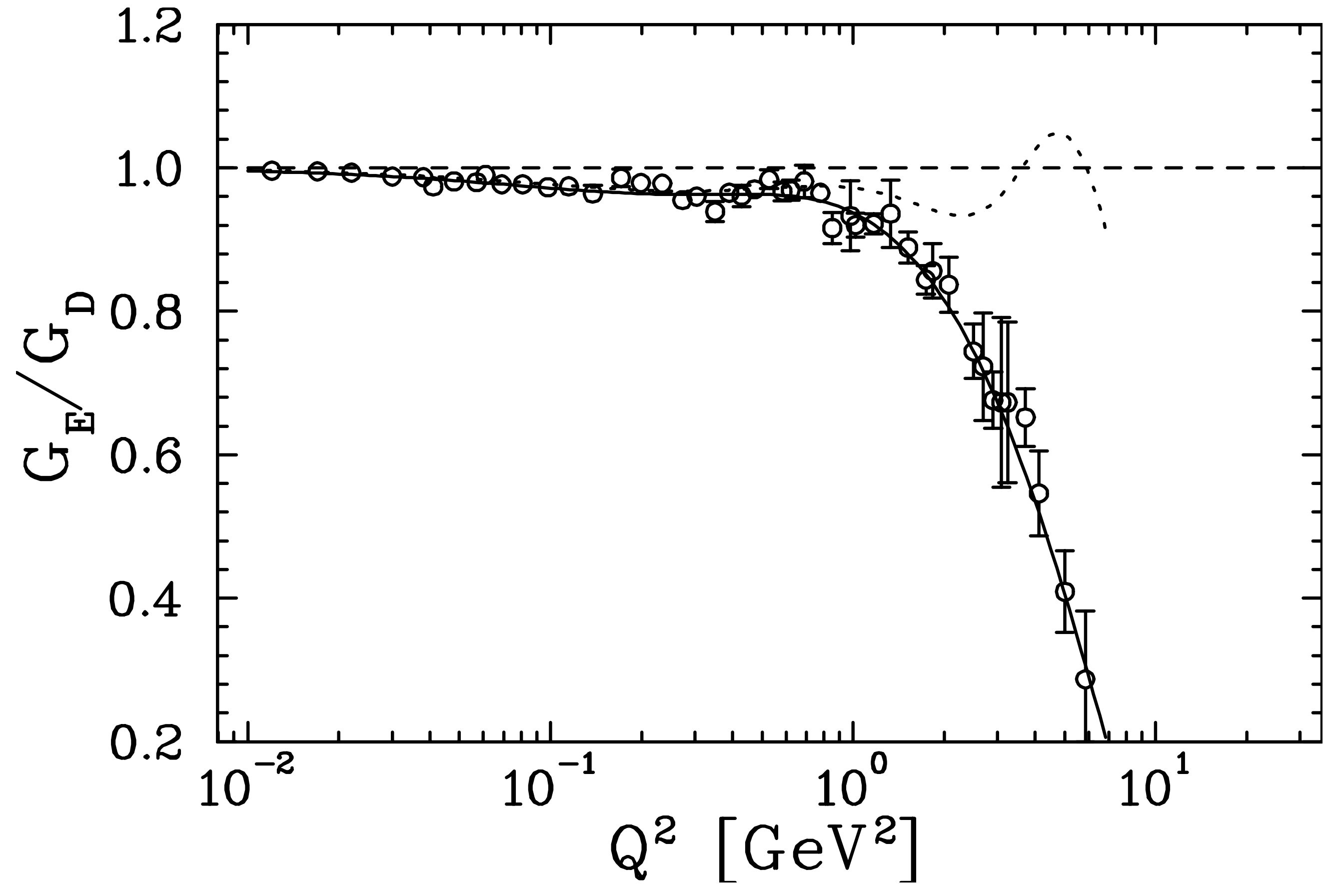


Raw results



Corrected with TPE



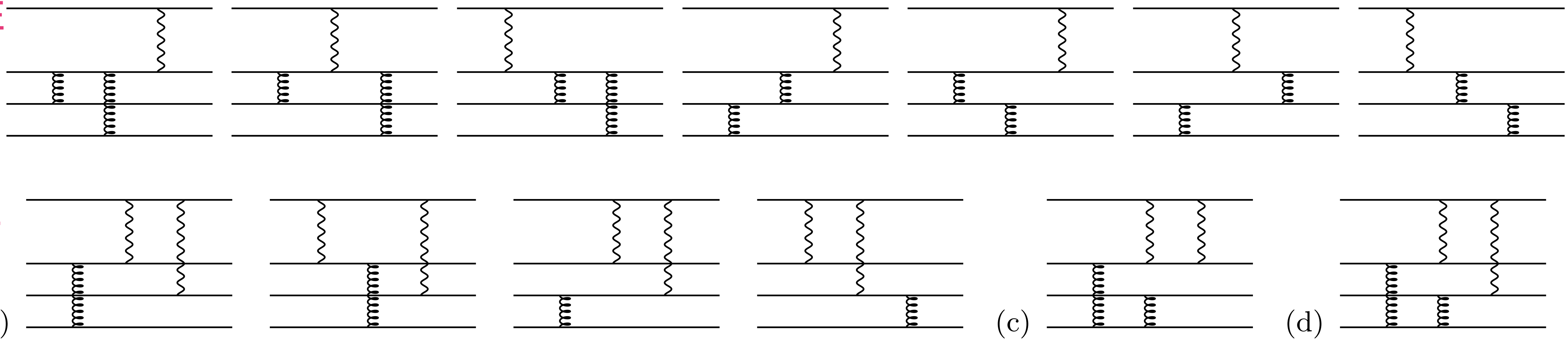


Extraction of G_M and G_E

High Q^2 partonic models

OPE

TPE



Recent pQCD calculation: Borisyuk & Kobushkin, PRD, 2009
Kivel & vanderhaeghen, PRL, 2009
see also Hoodbhoy, PRD, 2006 for TPE in lepton pair production

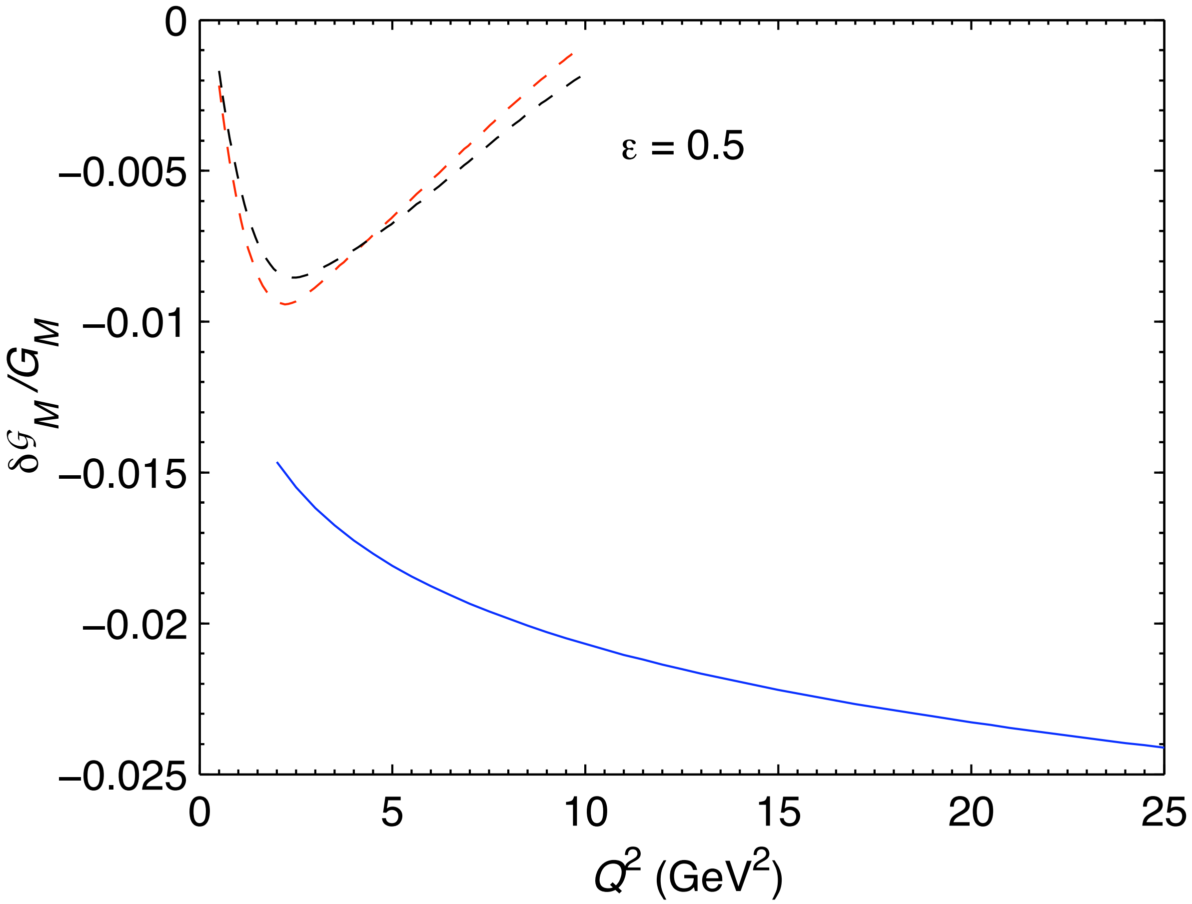
(a) one-photon exchange: need 2 hard gluons to turn momentum of all 3 quarks

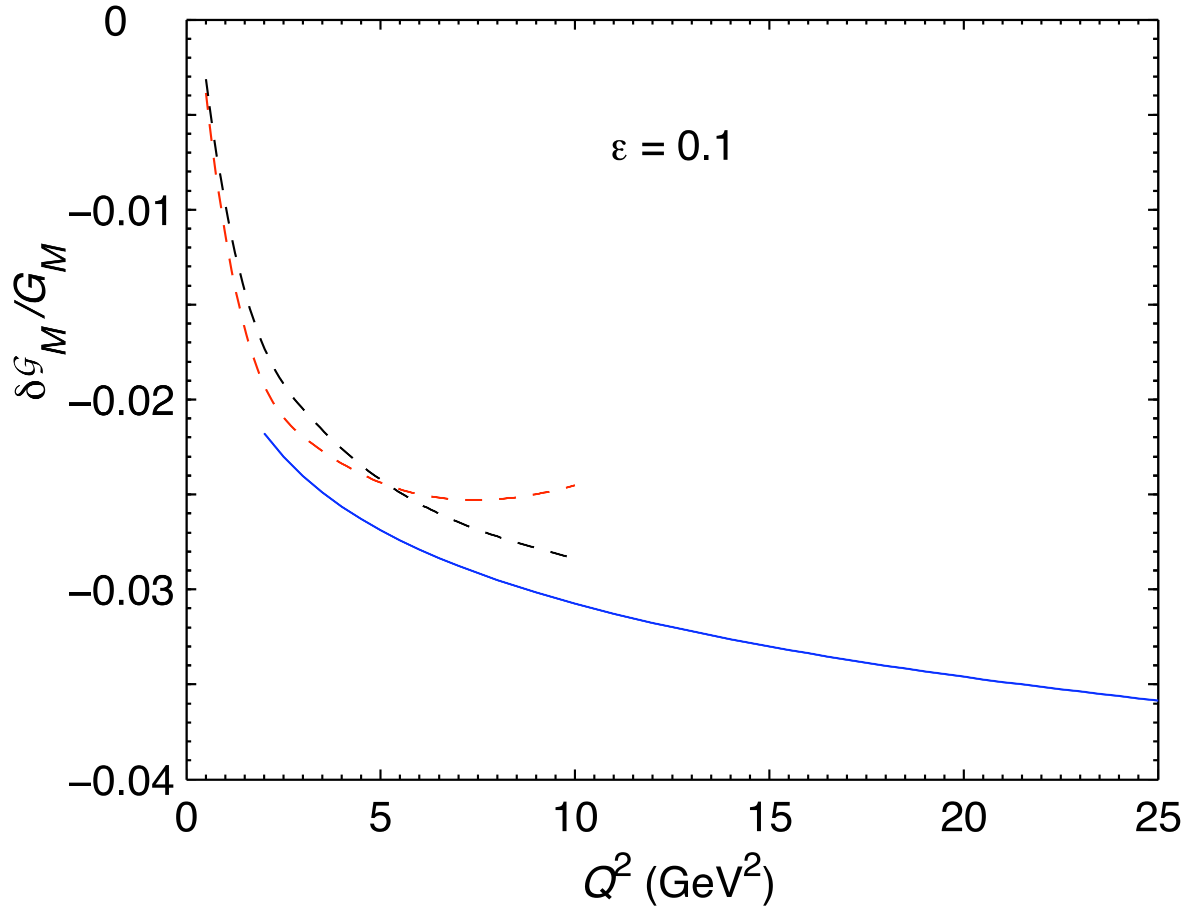
$$\alpha\alpha_s^2/Q^6$$

(b) two-photon exchange:
leading order needs 1 hard gluon

$$\alpha^2\alpha_s/Q^6 \qquad \text{TPE/OPE} \sim \alpha/\alpha^s$$

subleading order (both photons on one quark) requires 2 hard gluons





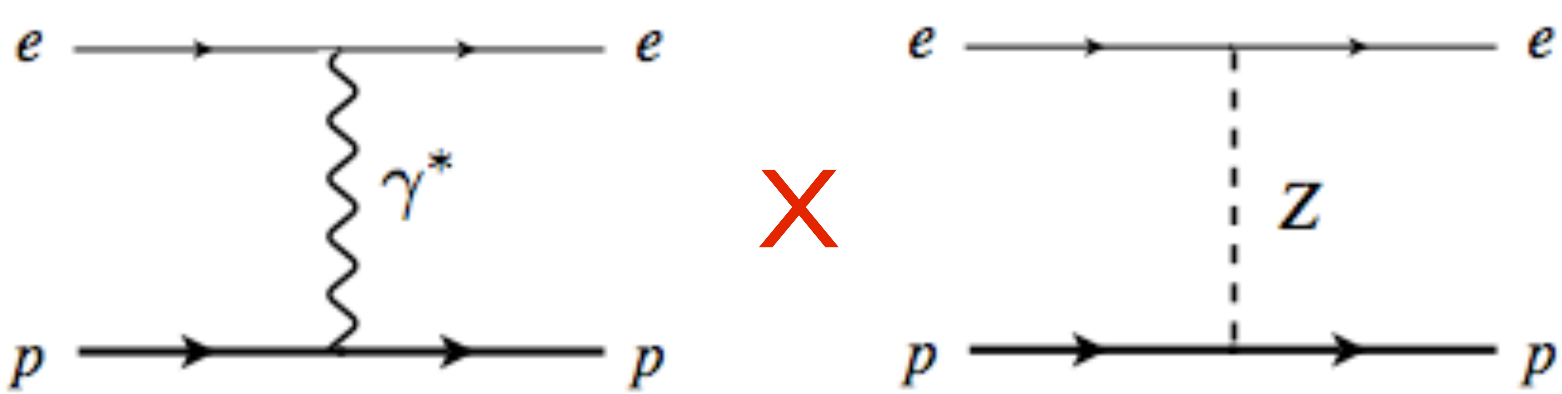
Comparison of hadronic and pQCD results

Connect smoothly around Q^2 = 3 GeV^2 ??

Parity-violating electron scattering

right-left polarization asymmetry in $\vec{e} + p \to e + p$ scattering

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{2\Re\{\mathcal{M}_\gamma^\dagger \mathcal{M}_Z\}}{|\mathcal{M}_\gamma|^2}$$



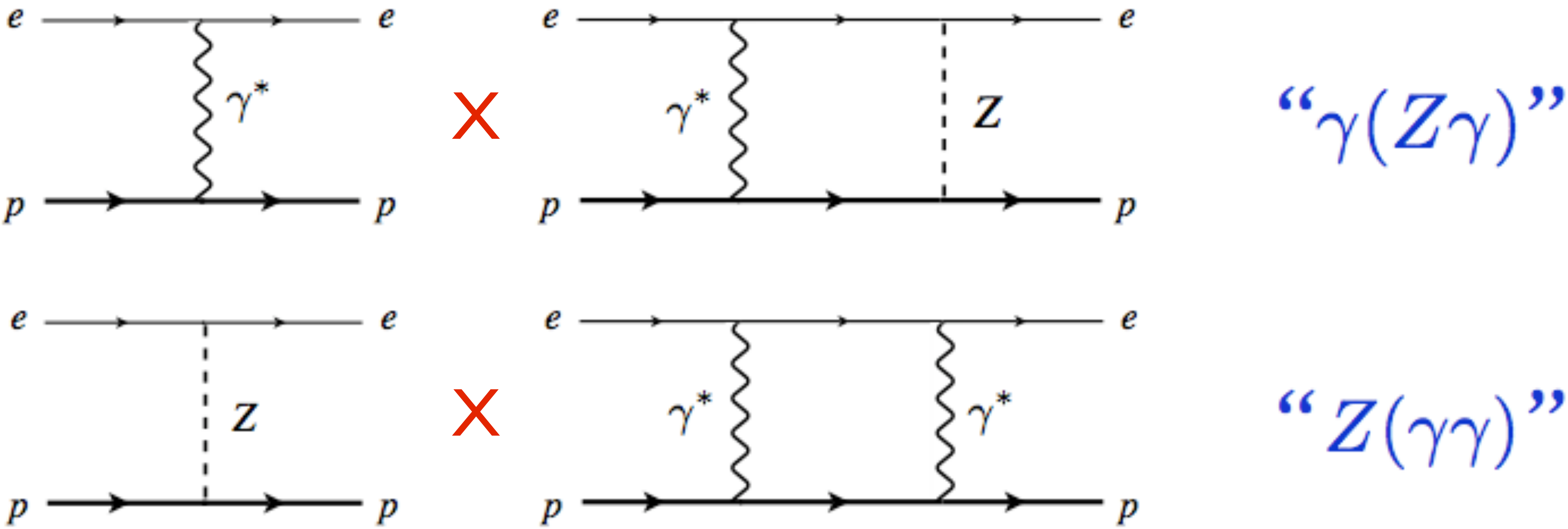
$$j_\mu^Z = \bar{u}\,(g_V^e \gamma_\mu + g_A^e \gamma_\mu \gamma_5)\,u$$

$$J_Z^\mu(q) = \bar{U}\left(F_1^Z \gamma^\mu + F_2^Z i\frac{\sigma^{\mu\nu} q_\nu}{2M} + G_A^Z \gamma^\mu \gamma_5\right) U$$

$$g_V^e = -(1 - 4s_W^2)$$
$$g_A^e = +1$$
$$F_i^Z = (1 - 4s_W^2)F_i^p - F_i^n - F_i^s$$
$$G_A^Z = -G_A \tau_3 + G_A^s$$

using relation between weak and EM form factors

Two-boson exchange corrections



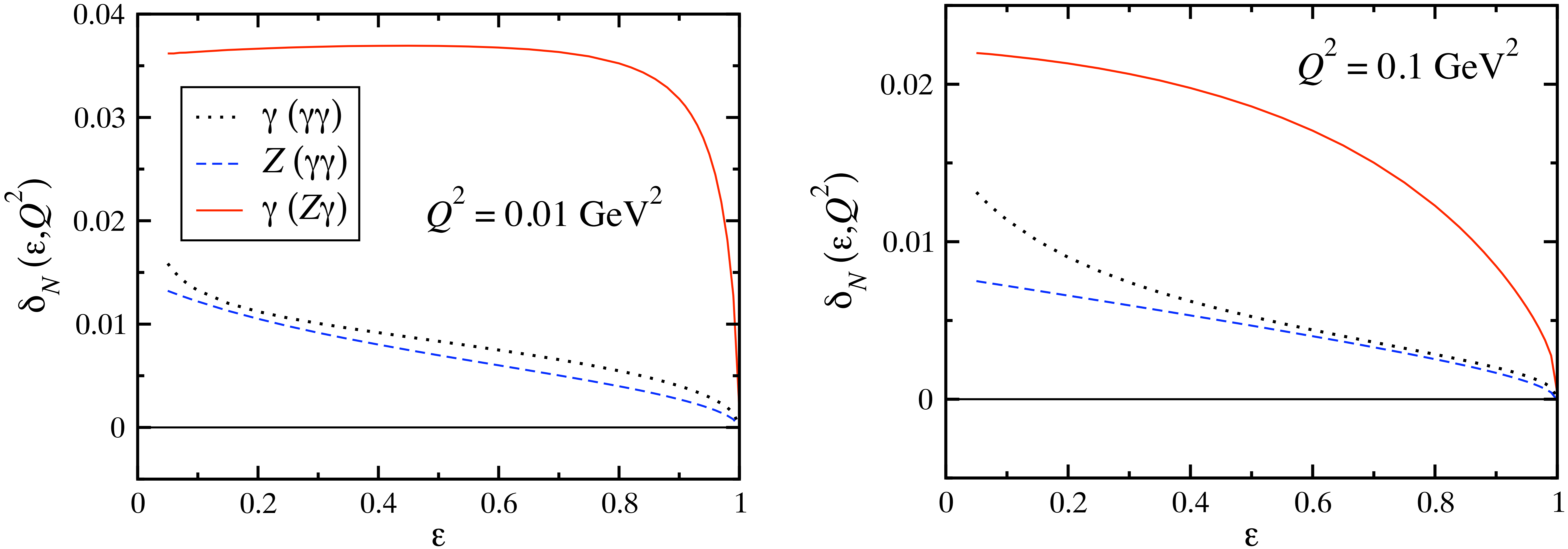
Electromagnetic radiative corrections interfere with M_Z ($M_\gamma \rightarrow M_\gamma + M_{\gamma\gamma}$)

plus weak radiative corrections interfere with M_γ ($M_Z \rightarrow M_Z + M_{\gamma Z}$)

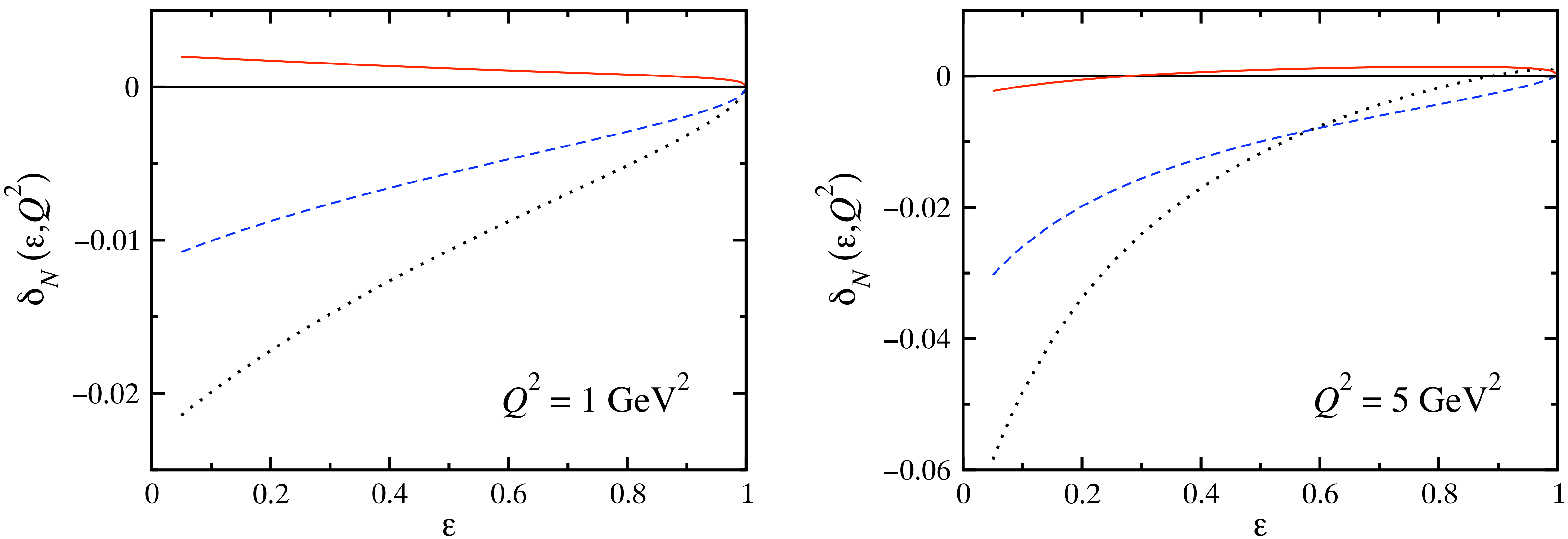
plus two-photon exchange “γ(γγ)” in denominator

Zhou, Kao & Yang, PRL 2007; Tjon & Melnitchouk, PRL 2008;
Tjon, PGB & Melnitchouk, PRC 2009

Tjon, PGB & Melnitchouk, PRC (2009)



$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$



Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p = (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e') + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$ ← box diagrams

$$= 0.0713 \pm 0.0008$$

*Erler et al., PRD **72** (2005) 073003*

→ *WW* and *ZZ* box diagrams dominated by short distances, evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions

$$\Box_{\gamma Z} = \Box_{\gamma Z}^A + \Box_{\gamma Z}^V$$

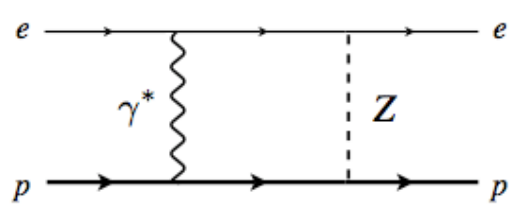
$\Box_{\gamma Z}^A$: vector e – axial h (finite at E=0)

$\Box_{\gamma Z}^V$: axial e – vector h (vanishes at E=0)

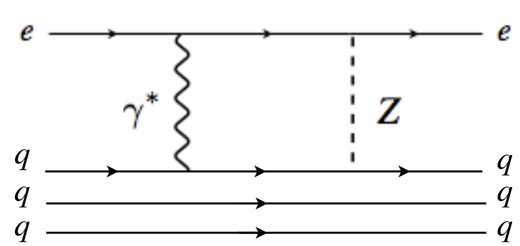
Axial h correction

- axial h correction $\Box^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

$\longrightarrow$ computed by Marciano & Sirlin as sum of two parts:



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box^{A}_{\gamma Z} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

$$\approx 0.0052 \pm 0.0005$$

short−distance (→ $\ln\frac{M_Z^2}{\Lambda^2}$)

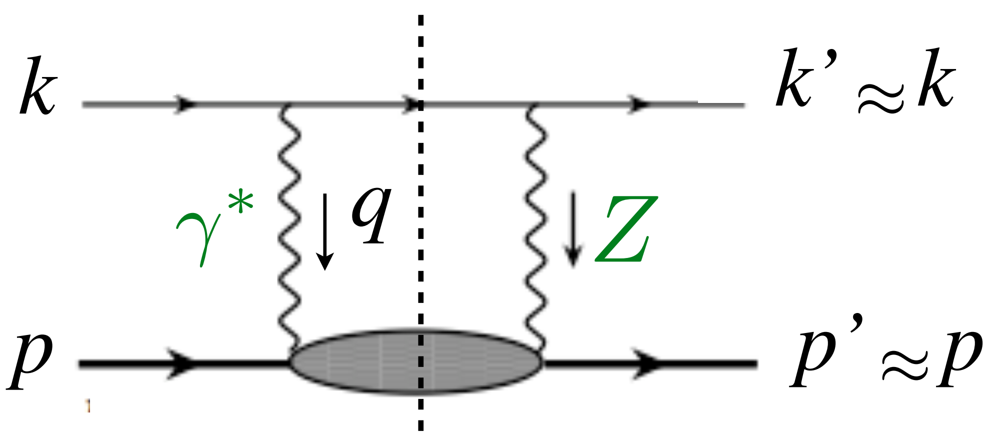
long−distance: $\frac{3}{2} \pm 1$ (→ $C_{\gamma Z}(\Lambda)$)

Marciano, Sirlin, PRD ***29*** *(1984) 75; Erler et al., PRD* ***68*** *(2003) 016006*

Axial h correction

- axial h correction $\Box^{A}_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

$\longrightarrow$ repeat calculation using forward dispersion relations with realistic (structure function) input



By optical theorem

$$2\Im m\, \mathcal{M}_{fi} = \int d\rho \sum_n \mathcal{M}^*_{nf} \mathcal{M}_{ni}.$$

vector *h*

axial *h*

☆ hadronic tensor:

$$MW^{\mu\nu}_{\gamma Z} = -g^{\mu\nu} F_1^{\gamma Z} + \frac{p^\mu p^\nu}{p \cdot q} F_2^{\gamma Z} - i\varepsilon^{\mu\nu\lambda\rho} \frac{p_\lambda q_\rho}{2p \cdot q} F_3^{\gamma Z}$$

☆ axial h contribution *anti*symmetric under $E' \longleftrightarrow -E'$:

$$\Re e\, \Box^{A}_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2}\, \Im m\, \Box^{A}_{\gamma Z}(E')$$

☆ negative energy part corresponds to crossed box (using crossing symmetry $s \rightarrow u$)

Axial h elastic + resonance correction

- elastic part: $F_3^{\gamma Z(\mathrm{el})}(Q^2) = -Q^2 G_M^p(Q^2) G_A^Z(Q^2) \delta(W^2 - M^2)$
- resonance part from parametrization of ν scattering data; includes lowest 4 spin 1/2 and 3/2 states (Lalakulich-Paschos)

Axial h correction DIS part

- change integration variable $W^2 \to x$ and switch order of integration. Energy integral can be done analytically.
- DIS part dominated by leading twist PDFs at small x (MSTW, CTEQ, Alekhin)

$$F_3^{\gamma Z(\mathrm{DIS})}(x, Q^2) = \sum_q 2\, e_q\, g_A^q \, \left(q(x, Q^2) - \bar{q}(x, Q^2)\right)$$

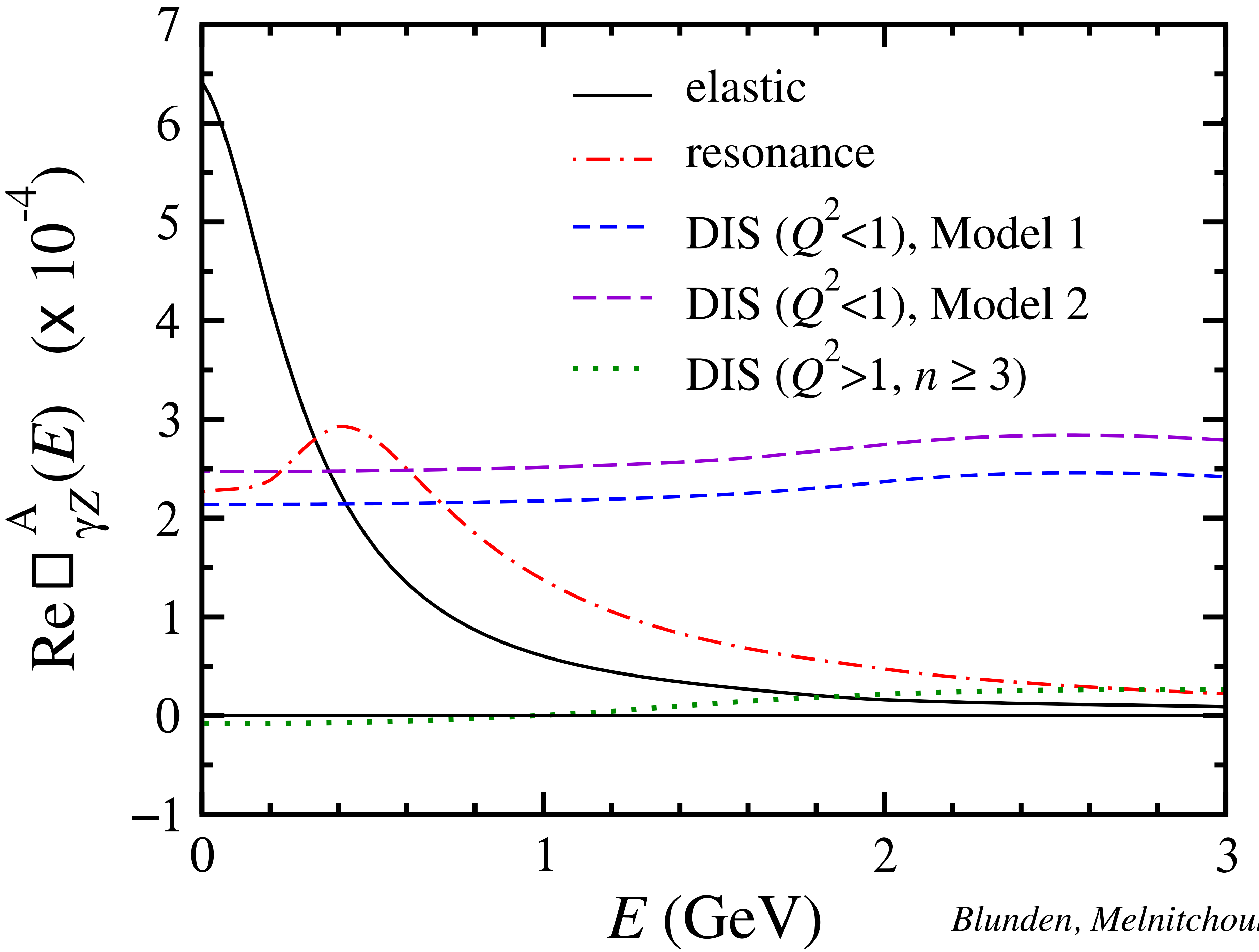
Axial h correction DIS part

→ in DIS region ($Q^2 \gtrsim 1\ \mathrm{GeV}^2$), expand integrand in $1/Q^2$

$$\Re e \Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi} \int_{Q_0^2}^{\infty} dQ^2 \frac{v_e(Q^2)\alpha(Q^2)}{Q^2(1+Q^2/M_Z^2)} \times \left[M_3^{(1)}(Q^2) + \frac{2M^2}{9Q^4}(5E^2 - 3Q^2) M_3^{(3)}(Q^2) + \ldots \right]$$

with moments $$M_3^{\gamma Z(n)} = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x, Q^2)$$

Axial h correction



Blunden, Melnitchouk, Thomas (2011)

- n = 1 moment: 32.8×10^{-4}
- dominated by DIS contribution (weak E dependence)

Axial h correction

→ correction at $E = 0$

$$\Re e \Box_{\gamma Z}^{A} = 0.00064 + 0.00023 + 0.00350 \rightarrow \underline{0.0044 \pm 0.0004}$$

↑ elastic ↑ resonance ↑ DIS

→ correction at $E = 1.165$ GeV (Qweak)

$$\Re e \Box_{\gamma Z}^{A} = 0.00005 + 0.00011 + 0.00352 \rightarrow \underline{0.0037 \pm 0.0004}$$

cf. MS value: 0.0052 ± 0.0005

Q_W^p shifts from $0.0713(8)$ to $0.0705(8)$

- Consider vector *h* before looking at E dependence

Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1$ GeV – what is energy dependence?

⟶ forward dispersion relation

★ $\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$

★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \longleftrightarrow -E'$

⟶ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

factor 2 larger than GH; confirmed by Rislow & Carlson, arXiv:1011:2397 [hep-ph]

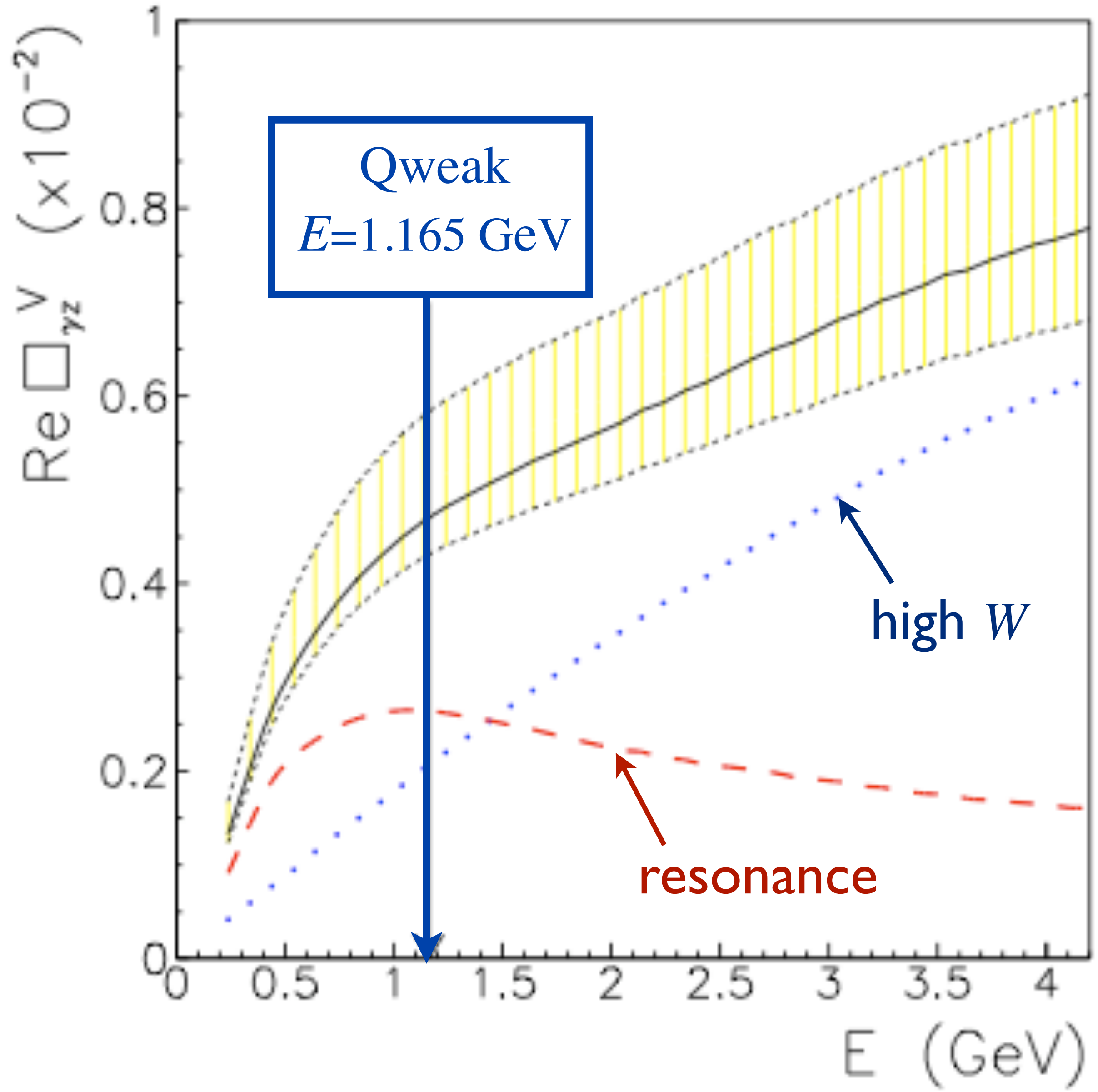
Gorchtein, Horowitz, PRL ***102*** *(2009) 091806*
Gorchtein, Horowitz, Ramsey-Musolf, arXiv:1003.4300

Vector h correction

→ total $\Box^V_{\gamma Z}$ correction:

$$\Re e\ \Box^V_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q^p_W



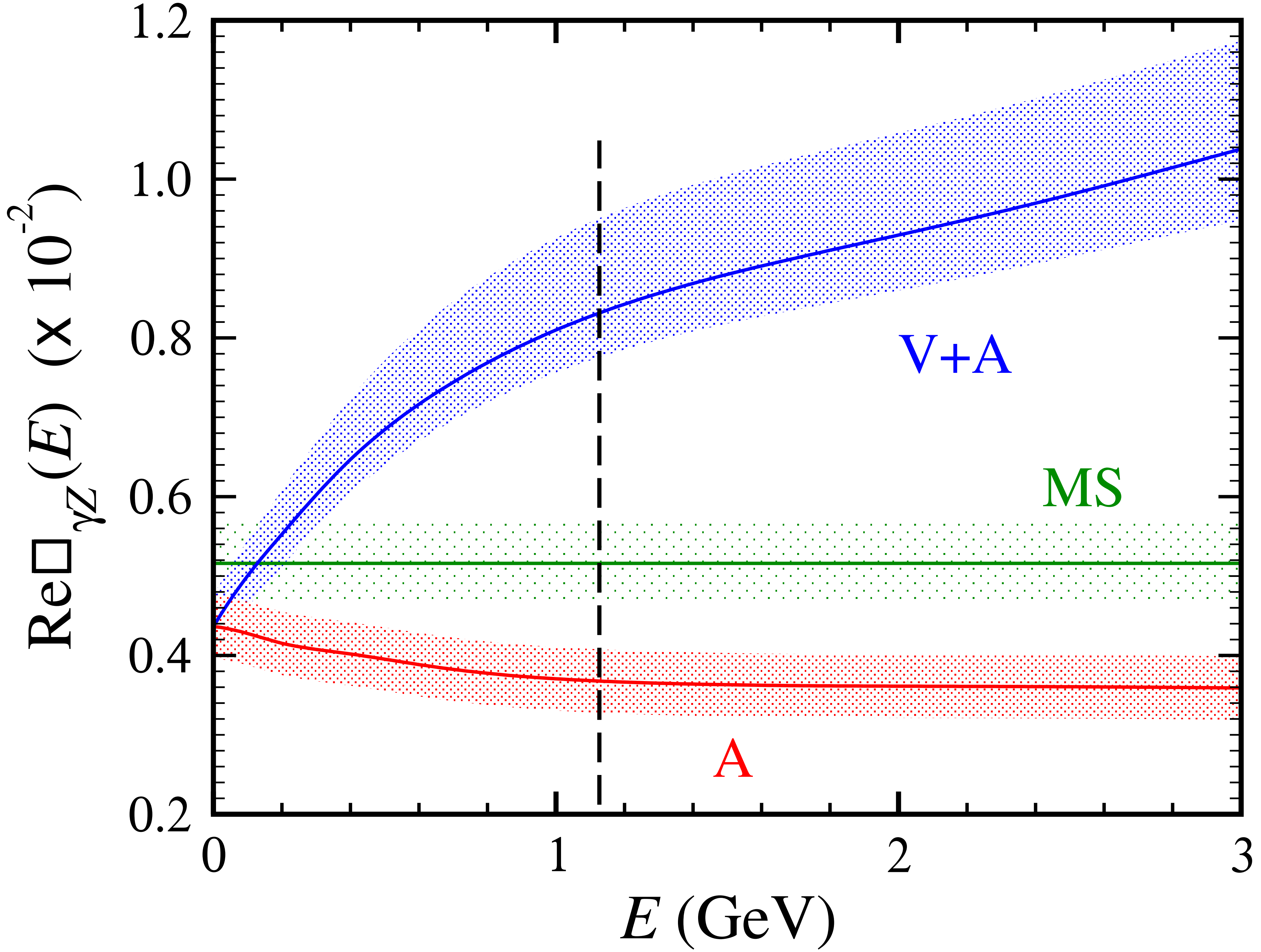
Sibirtsev, Blunden, Melnitchouk, Thomas
*PRD **82** (2010) 013011*

Combined vector and axial h correction

$$Q_W^p = 0.0713 \rightarrow 0.0705$$

At E=1.165 GeV

$$-0.0007 + 0.0047 = 0.0040$$



- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on *proton weak charge*, $\Delta Q^{p}_{\widetilde{W}}\ 6\%$, *cf*. PDG
  - $\longrightarrow$ would significantly shift extracted weak angle
  - $\longrightarrow$ better constraints from direct measurement of $F_{1,2,3}^{\gamma Z}$ (*e.g*. in PVDIS at JLab)

- New formulation in terms of *moments* of structure functions
  - $\longrightarrow$ places on firm footing earlier derivation of Marciano/Sirlin in "free quark model"
  - $\longrightarrow$ will have slight effect on atomic PV predictions (e.g. Cs, Fr)

Summary

- Qweak correction large, but uncertainty under control
- Uncertainty in Qweak may be reduced further with measurements of γZ interference structure functions in PVDIS
- Dispersion relations that use cross section data are useful at forward angles, however still need for models to extrapolate (not all data is available, e.g. γZ interference, axial part)

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