Two-photon Corrections using GPDs

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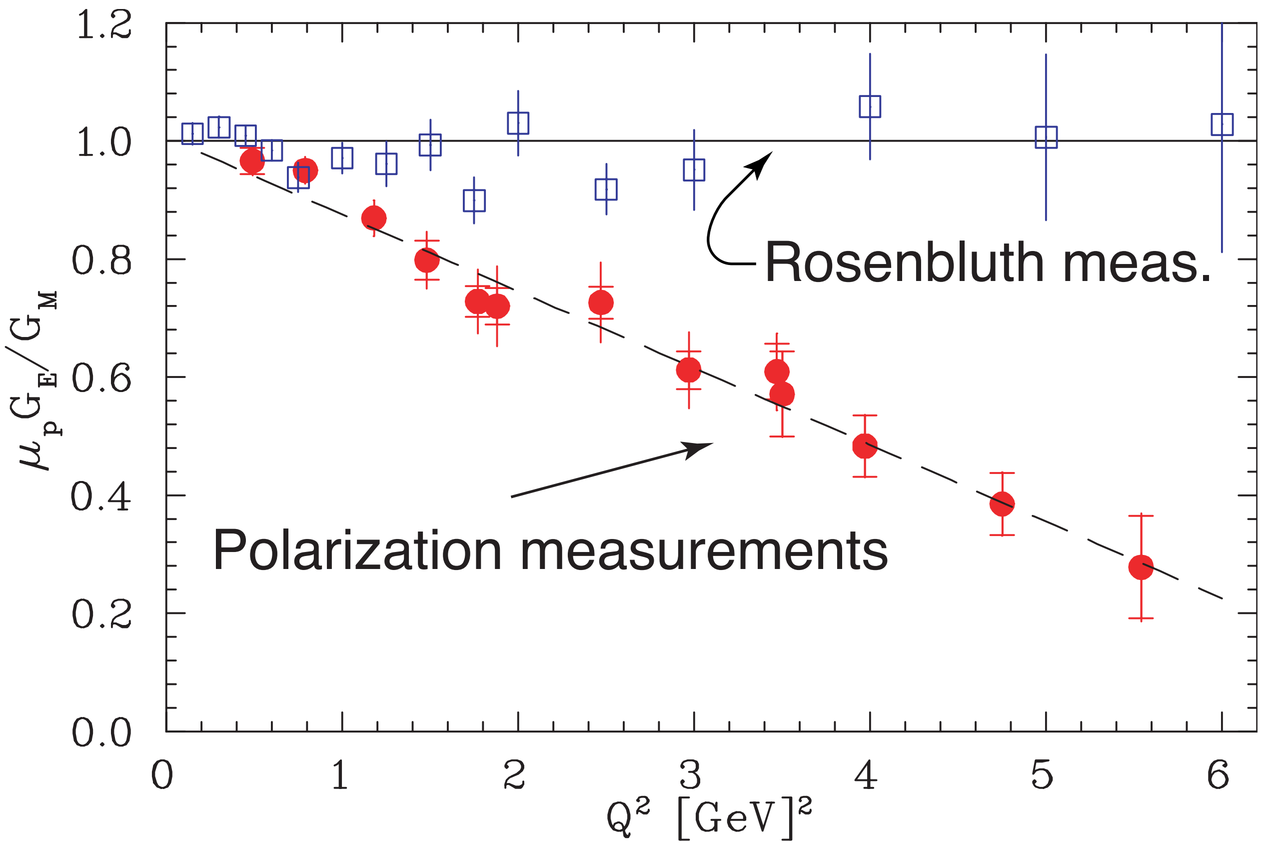
Goals for talk

- Re-present the theory
  - the problem
  - what was absent in the "old days"
  - three attempts to complete the box calculations
    - single bound hadrons
    - partonic with GPDs
    - pQCD
- Analysis of manifestations of two photon processes
  - Rosenbluth corrected
  - polarizations
  - positron/electron ratio
- Appreciation of new experimental results

The problem

- There were two ways of measuring G_E/G_M (proton), and they gave different answers
- As we saw it in about 2003,



The problem

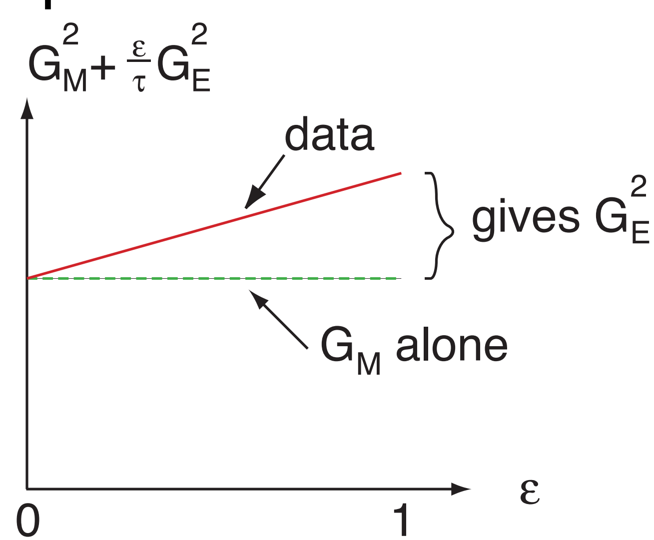
- Rosenbluth means measure the differential cross section
- In a one-photon exchange approximation,

$$\frac{d\sigma}{d\Omega_{Lab}} = \frac{d\sigma_{NS}}{d\Omega_{Lab}} \times \frac{\tau}{\epsilon(1+\tau)} \left(|G_M(Q^2)|^2 + \frac{\epsilon}{\tau} |G_E(Q^2)|^2 \right)$$

where

$$\tau \equiv \frac{Q^2}{4M^2}\,, \; \frac{1}{\epsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

- distinguish G_M and G_E by different angular dependences
- Typically plot $|G_M|^2 + \frac{\epsilon}{\tau}|G_E|^2$ vs. ϵ , at fixed Q^2



The problem

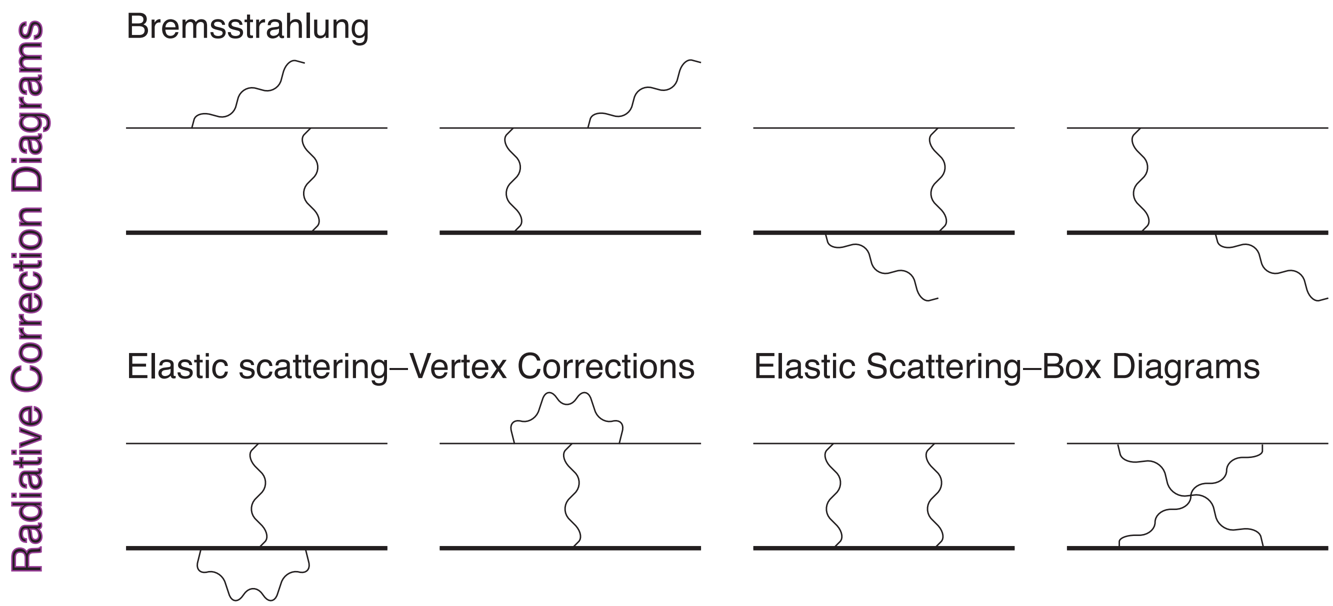
- Problem with the Rosenbluth separation for high Q^2 is that the G_E contribution is small compared to the G_M contribution. Hence small corrections to the G_M term can seriously affect the G_E term.
- Polarization technique uses $\overrightarrow{e} + p \rightarrow e + \overrightarrow{p}$
- Ratio of transverse-in-plane polarization and longitudinal polarization is

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\,\frac{G_E}{G_M}$$

for one-photon exchange, and gives polarization ratio directly

The problem

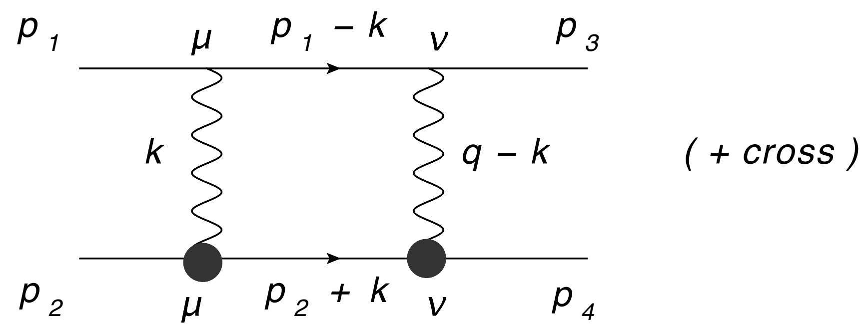
- Since the Rosenbluth separation involves a small term, need to consider the corrections, specifically radiative corrections



- Mostly well done in past,
    - Meister and Yennie (1963)
    - Mo and Tsai (1961 and 1969)
    - Maximon and Tjon (2000)
- But clear incompleteness in box or 2-photon exchange diagrams

Box diagrams in "old days"

- Couldn't be neglected: they have IR divergences.
- For elastic intermediate state,



$$\mathcal{M}_{Box} = (Ze^2)^2 \int (d^4k)(k^2 - \lambda^2 + i\epsilon)^{-1}((k-q)^2 - \lambda^2 + i\epsilon)^{-1}$$

$$\times \quad \bar{u}(p_3)\gamma_\nu \frac{\not{p}_1 - \not{k} + m}{(p_1 - k)^2 - m^2 + i\epsilon}\gamma_\mu u(p_1)$$

$$\times \quad \bar{u}(p_4)\Gamma^\nu(q-k)\frac{\not{p}_2 + \not{k} + M}{(p_2 + k)^2 - M^2 + i\epsilon}\Gamma_\mu(k)u(p_2)$$

$$\left(\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2(q^2)\right)$$

IR divergence comes from photons almost on shell at k = 0 or k = q.

“Old” boxes

- Approximation I: Set k = 0 or k = q (two separate possibilities) in the two proton vertex functions. Remember that $\Gamma^\mu(k=0) = \gamma^\mu$.

$$
\begin{aligned}
\mathcal{M}_{Box} &= (Ze^2)^2 \int (d^4k)(k^2-\lambda^2+i\epsilon)^{-1}((k-q)^2-\lambda^2+i\epsilon)^{-1} \\
&\times \quad \bar{u}(p_3)\gamma_\nu \frac{\not{p}_1 - \not{k} + m}{(p_1-k)^2 - m^2 + i\epsilon}\gamma_\mu u(p_1) \\
&\times \quad \bar{u}(p_4)\Gamma^\nu(q) \frac{\not{p}_2 + \not{k} + M}{(p_2+k)^2 - M^2 + i\epsilon}\gamma^\mu u(p_2)
\end{aligned}
$$

- Manipulation I:

$$
(\not{p}_2 + \not{k} + M)\gamma^\mu u(p_2) = \left(2p_2^\mu + \not{k}\gamma^\mu\right) u(p_2)
$$

"Old" boxes

- Approximation II: drop the k terms in the numerator

$$\mathcal{M}_{Box} = Ze^2 \;\; 4p_1 \cdot p_2 \;\; \mathcal{M}_{Lowest\ order} \;\times\; q^2 \times \text{ Integral}$$

$$\begin{aligned} \text{Integral} \;\; &= \;\; \int (d^4k)\,(k^2-\lambda^2+i\epsilon)^{-1}\,((k-q)^2-\lambda^2+i\epsilon)^{-1} \\ &\times \qquad ((p_1-k)^2-m^2+i\epsilon)^{-1}\;\;((p_2+k)^2-M^2+i\epsilon)^{-1} \end{aligned}$$

- Virtues
  - get IR divergences correctly
  - can do integral analytically

“Old” boxes

- Vices:
  - Wrong overall
  - Omits proton structure dependent contributions
  - Corrections bigger numerically than anticipated
- But the proton structure dependent corrections could not have been done before the 2000’s (at least not with GPDs, the focus of this talk)

“Old” boxes

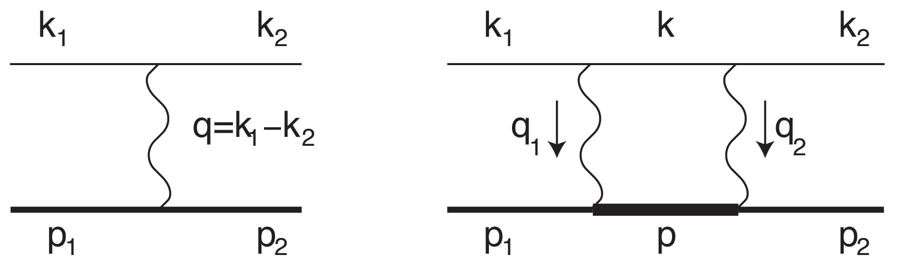
- Why show this much detail?
  - Emphasize that some boxes are included in Mo-Tsai or Maximon-Tjon calculations.
  - Note that they had to do something. The boxes have IR divergences that cancel IR divergences from bremsstrahlung. Omitting the boxes leaves IR infinities elsewhere.
  - Note that these “soft” boxes give interference terms that are C-odd. Hence note that some signs have to be changed in the MT (old or new) formulas when doing radiative corrections. (A.A.)
  - Our coming results are for two-photon corrections---but we know about the soft boxes already done and have subtracted them out of our results.
  - Also remember that bremsstrahlung corrections depend on which particle is observed. (Also A.A.)

2-photon calculations

- Hadronic method: include full integrand in box diagrams—including form factors [Blunden et al. 2003]
  - Later included other resonances.
  - Explain half or more of discrepancy
  - Peter will talk about it later
- Partonic method (GPD method)---coming here
- pQCD, for highest Q^2. Nikolai may say something

Partonic Evaluation of Box Diagrams.

First: extra structure in *ep* → *ep* when multiple photon exchange



$$\mathfrak{M}^N_{h,\lambda_2,\lambda_1} = \frac{Ze^2}{Q^2}\Big\{\bar{u}_h(k_2)\gamma_\mu u_h(k_1)\times\bar{u}_{\lambda_2}(p_2)\left[\gamma^\mu G'_M - \frac{(p_1+p_2)^\mu}{2M}F'_2\right]u_{\lambda_1}(p_1)$$

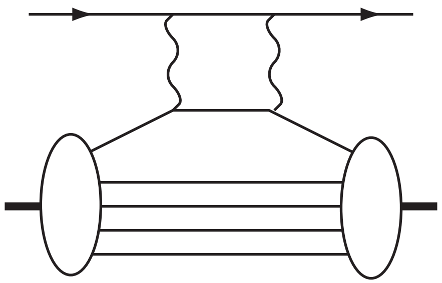
$$+\,\bar{u}_h(k_2)\gamma_\mu\gamma_5\, u_h(k_1)\times\bar{u}_{\lambda_2}(p_2)\gamma^\mu\gamma^5 G'_A u_{\lambda_1}(p_1)\Big\}$$

For 1-γ exchange, G_M' and G_E' become standard G_M and G_E. G_A' new.

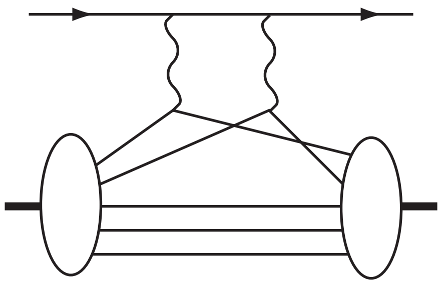
$$G'_M = G_M^{(1\gamma)} + G_M^{\prime(2\gamma)} = G_M + \delta G'_M \quad \text{etc.}$$

Strategy

Calculate box (and crossed box) diagram at quark level, then embed in proton using generalized parton distribution (GPD).



"Handbag"

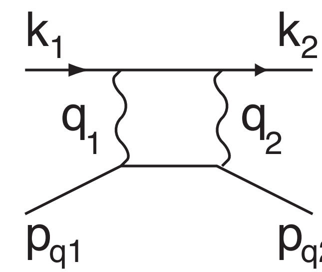


"Cat’s Ears"

- ``Cat's ears'' diagrams, where photons interact with different quarks, mostly neglected. Important for getting overall IR divergence correct. However, contributions when both photons are hard is suppressed at higher Q^2.

Box diagrams for *eq* → *eq*, with massless quarks

$$\mathfrak{M}^q_{h,\lambda} = \frac{(e\,e_q)^2}{Q^2}\Big\{g_M\,\bar{u}(k',h)\gamma_\mu u(k,h)\cdot\bar{u}(p'_q,\lambda)\gamma^\mu u(p_q,\lambda)$$
$$+\,g_A^{(2\gamma)}\,\bar{u}(k',h)\gamma_\mu\gamma_5 u(k,h)\cdot\bar{u}(p'_q,\lambda)\gamma^\mu\gamma^5 u(p_q,\lambda)\Big\}$$



- Calculation same as *e*μ → *e*μ, which has been done analytically, can be found in the literature (e.g., van Nieuwenhuizen [1971]), and has been verified by us.
- Boxes have IR divergence, which cancel or disappear in end. Separate soft (IR divergent) and hard parts by criterion of Grammer and Yennie.

g_M and $g_M^{(2\gamma)}$ have both real and imaginary parts.

Imaginary parts,

$$\Im\left(g_M^{soft}\right) = \alpha \ln\left(\frac{\hat{s}}{\lambda^2}\right),$$

$$\Im\left(g_M^{hard}\right) = -\alpha\left\{\frac{\hat{s}^2+3\hat{u}^2}{4\hat{u}^2}\ln\left(\frac{\hat{s}}{-t}\right) - \frac{t}{4\hat{u}}\right\},$$

$$\Im\left(g_A^{(2\gamma)}\right) = -\alpha\frac{t}{4\hat{u}}\left\{\frac{\hat{s}-\hat{u}}{\hat{u}}\ln\left(\frac{\hat{s}}{-t}\right)+1\right\}.$$

($\hat{s}$ and $\hat{u}$ are Mandelstam variables for the subprocess *eq* → *eq*).

And also

$$\Re\left(g_M^{soft}\right) = \frac{\alpha}{\pi}\left\{\ln\left(\frac{\lambda^2}{\sqrt{-\hat{s}\hat{u}}}\right)\ln\left(\frac{\hat{s}}{-\hat{u}}\right)+\frac{\pi^2}{2}\right\}$$

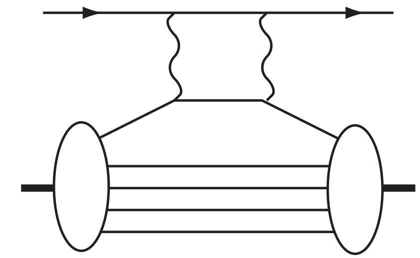
Soft contributions: mostly skip details

- The IR divergence in the box is cancelled by an IR divergence from bremsstrahlung, specifically an interference between bremsstrahlung from the electron and bremsstrahlung from the proton. Write

$$\sigma_{1\gamma+2\gamma,soft} = \sigma_{1\gamma}\left(1 + \delta_{2\gamma}^{soft} + \delta_{brems}^{ep}\right)$$

- Take bremsstrahlung (only) from Maximon and Tjon [2000]
- Compare numerically to corresponding Mo and Tsai soft correction: same, to 0.1\% level, except for OA factor $(1+\pi\alpha)$.
- (Latter has to do with using the Grammer-Yennie criterion for separating soft and hard parts.   Could use different criterion and push into hard parts. Important thing is constancy: any new ε dependence comes from hard parts.)

Embed partonic calculation in nucleon



"Handbag"

- Perfect for generalized parton distributions: remove a quark from the proton, and replace it with a quark of different momentum and possibly different helicity.

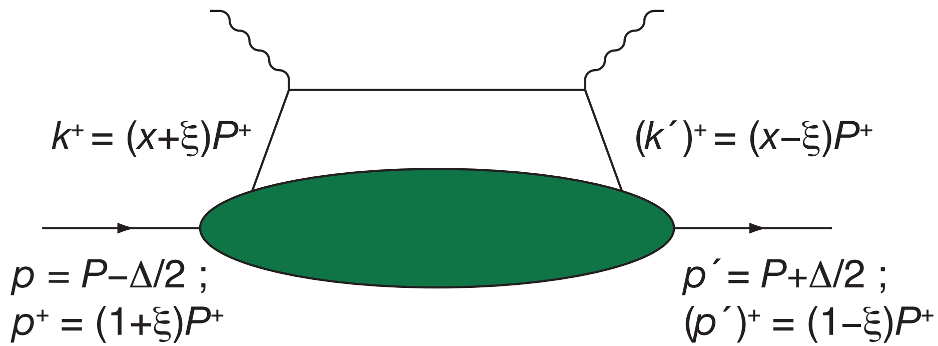
$$\mathfrak{M}^N_{h,\lambda_2,\lambda_1} = \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2P^+} \left[\mathfrak{M}^q_{h,+1/2} + \mathfrak{M}^q_{h,-1/2} \right] \bar{u}_{\lambda_2}(p_2) \left[\gamma^+ H^q + \frac{i\sigma^{+\nu} q_\nu}{2M} E^q \right] u_{\lambda_1}(p_1)$$
$$+ \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2P^+} \left[\mathfrak{M}^q_{h,+1/2} - \mathfrak{M}^q_{h,-1/2} \right] \mathrm{sgm}(x)\, \bar{u}_{\lambda_2}(p_2)\, \gamma^+ \gamma^5 \tilde{H}^q\, u_{\lambda_1}(p_1)$$

- Use light-front frame, $q^+ \propto q^0 + q^3 = 0$.

- Arguments of GPD's are $H^q(x, \xi = 0, Q^2)$, etc.

More on GPDs

- Originally designed for studies of Deeply Virtual Compton Scattering (DVCS). Variables are x, ξ, and $t = \Delta^2$.



- Algebraic definition

$$p^+ \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p's'|\bar{q}(-\frac{z}{2})\gamma^\mu q(\frac{z}{2})|ps\rangle\Big|_{z^+=0, z_\perp=0} =$$

$$H_q(x,\xi,t)\ \bar{u}(p',s')\gamma^\mu u(p,s) + E_q(x,\xi,t)\ \bar{u}\frac{i}{2M}\sigma^{\mu\nu}\Delta_\nu u$$

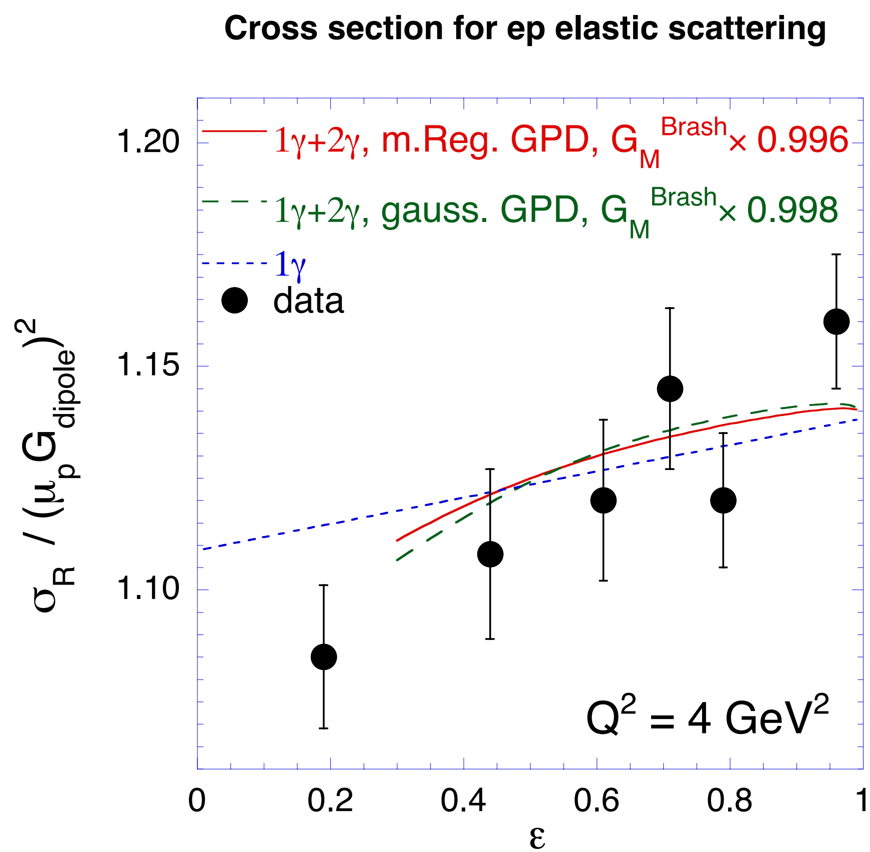
$$p^+ \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p's'|\bar{q}(-\frac{z}{2})\gamma^\mu \gamma^5 q(\frac{z}{2})|ps\rangle\Big|_{z^+=0, z_\perp=0} =$$

$$\tilde{H}_q(x,\xi,t)\ \bar{u}(p',s')\gamma^\mu\gamma^5 u(p,s) + \tilde{E}_q(x,\xi,t)\ \bar{u}\frac{i}{2M}\gamma^5\Delta^\mu u$$

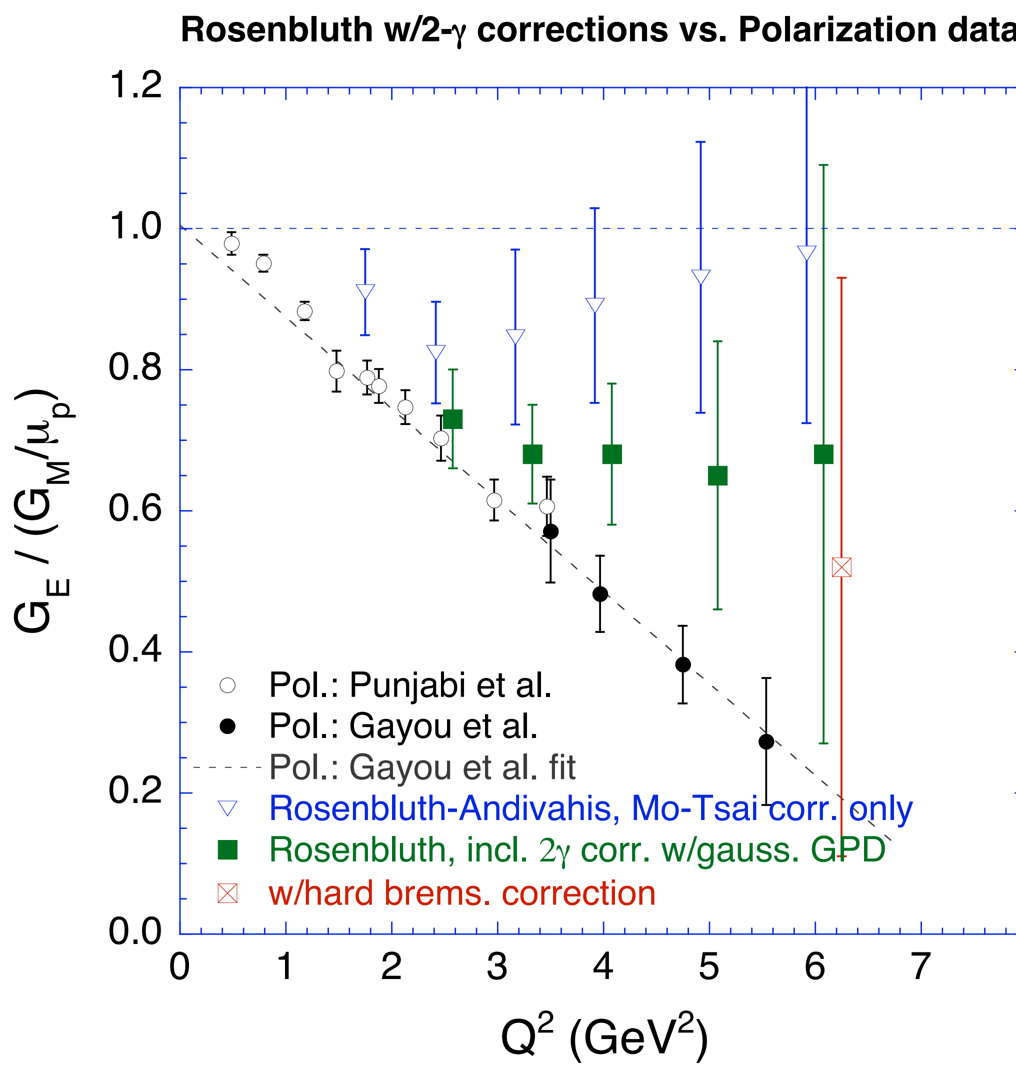
- For present case, $\xi = 0$.

Sample results

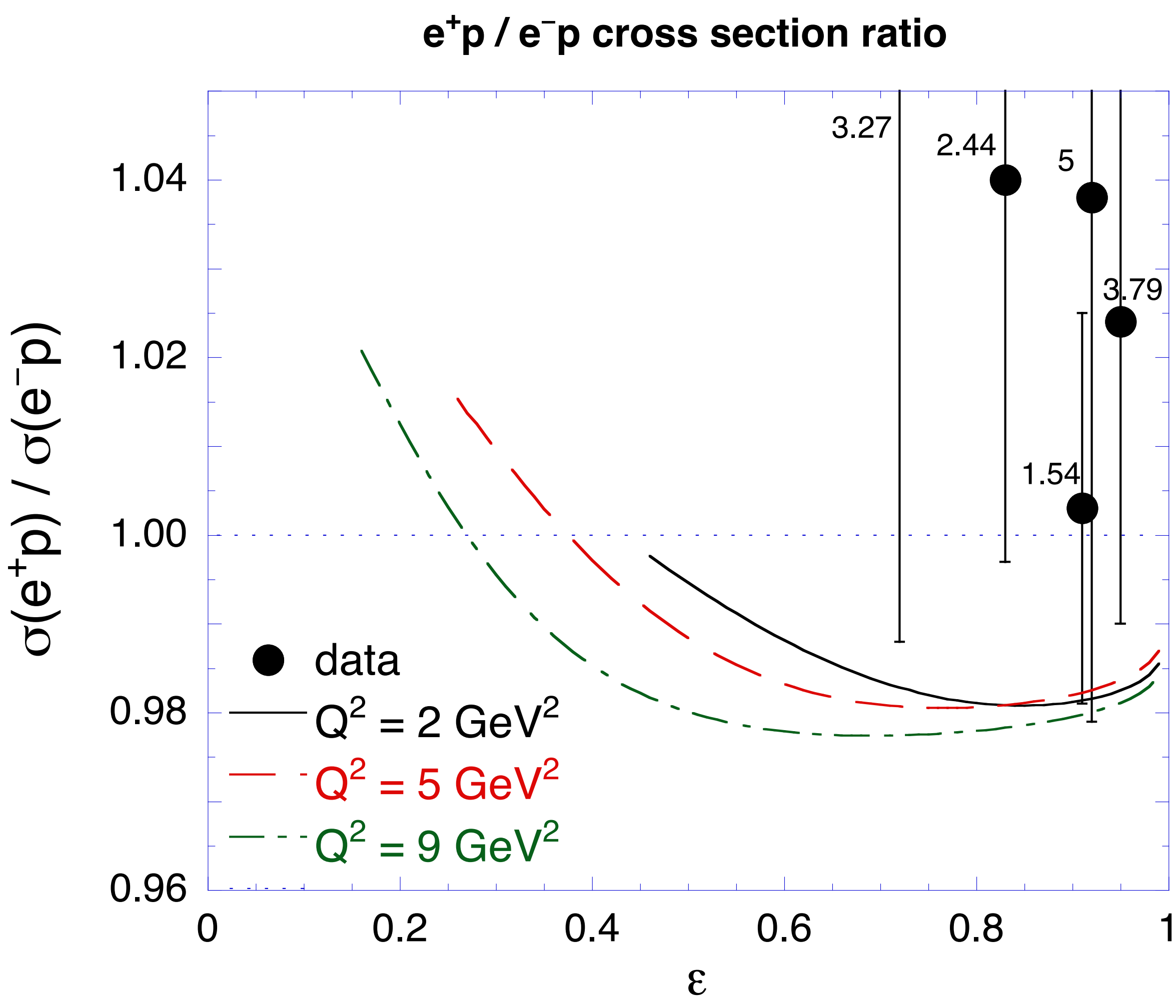
- Note: GPDs not actually measured. Proceed using models that are fit to form factors and pdf's in appropriate limits. Used two model GPDs.



Sample results: G_E/G_M extracted from Rosenbluth



Electron-Positron Ratio Results (from GPD calc.)



Other comments on 2-γ exch. observables

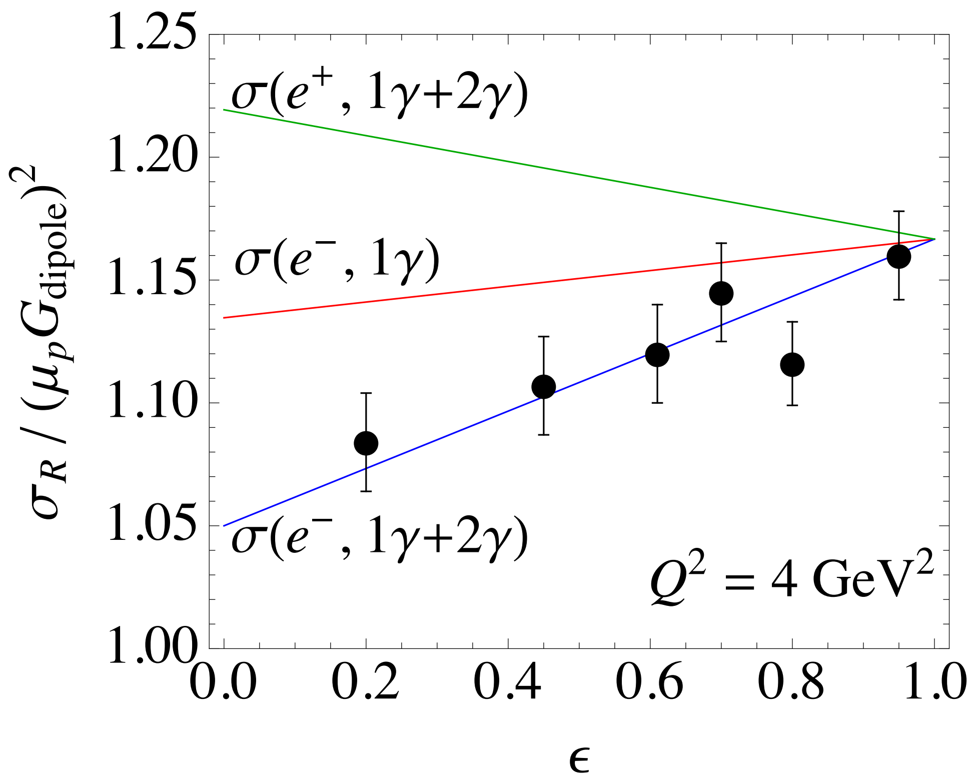
- ε-dependence of polarizations
- Normal polarization, P_y
- → positron/electron ratio (e^+p/e^-p)
  - Experiments under way at VEPP, Olympus(DESY), CLAS
- Curvature in Rosenbluth plot
  - Not seen in present data [Tvaskis et al., 2006]
  - Dedicated Hall C experiment
  - Theoretically quantified by Abidin et al.

ratio $\sigma(e^+p)/\sigma(e^-p)$

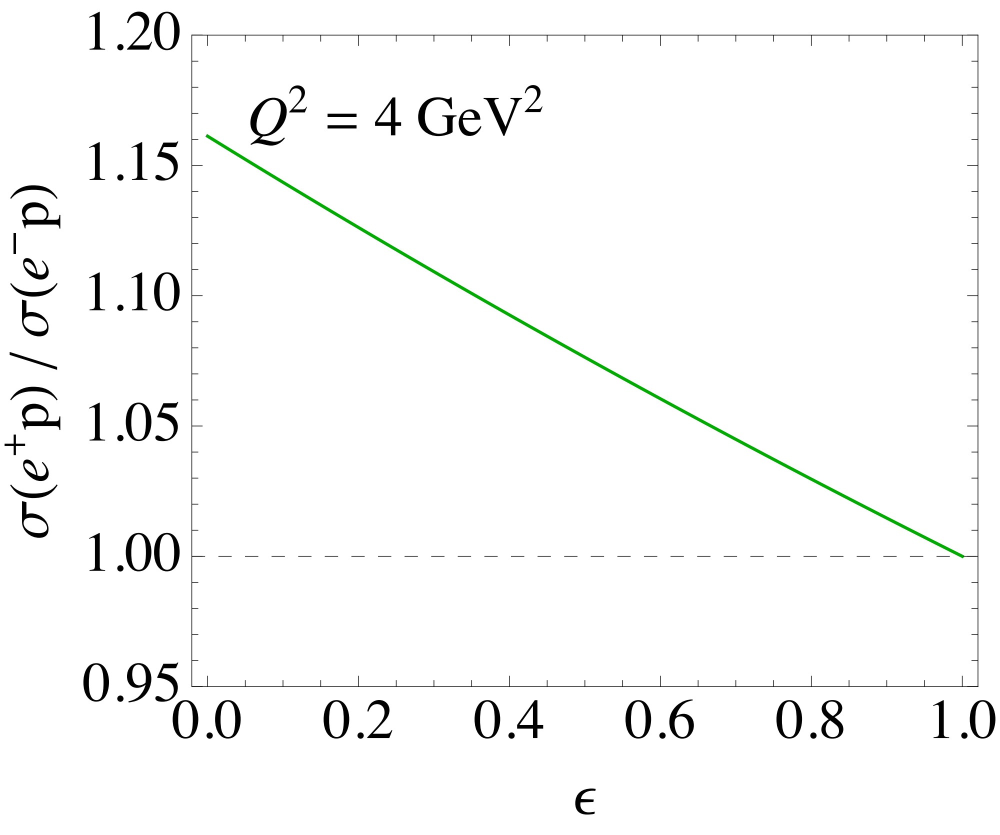
- Useful note: the Rosenbluth problem required us to fix a cross section.
- Hypothesis: due to 2-photon effects which had a suitable ε dependence. (Backed up by calculation, but take as hypothesis for moment.)
- If so, the main terms in the correction are interference terms and are C-odd.
- Hence what is subtracted from $\sigma(e^-p)$ is added to $\sigma(e^+p)$. Allows:

almost model independent predictions for $\sigma(e^+p)/\sigma(e^-p)$

- 1γ line uses polarization G_E/G_M
- Used form linear in ε. Suggested by data, but not required.
- Equality at $\varepsilon \approx 1$ also not required.
- Effect is large.
- Give "impossible" (in LO) $\sigma(e^+p)$.

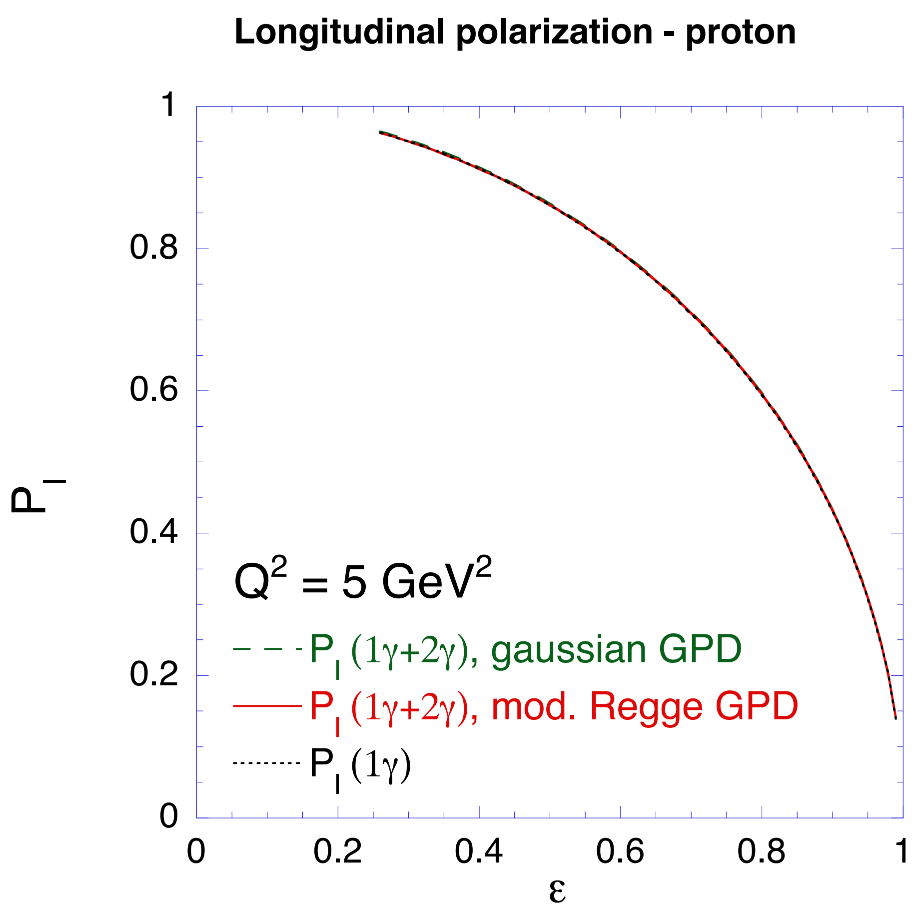


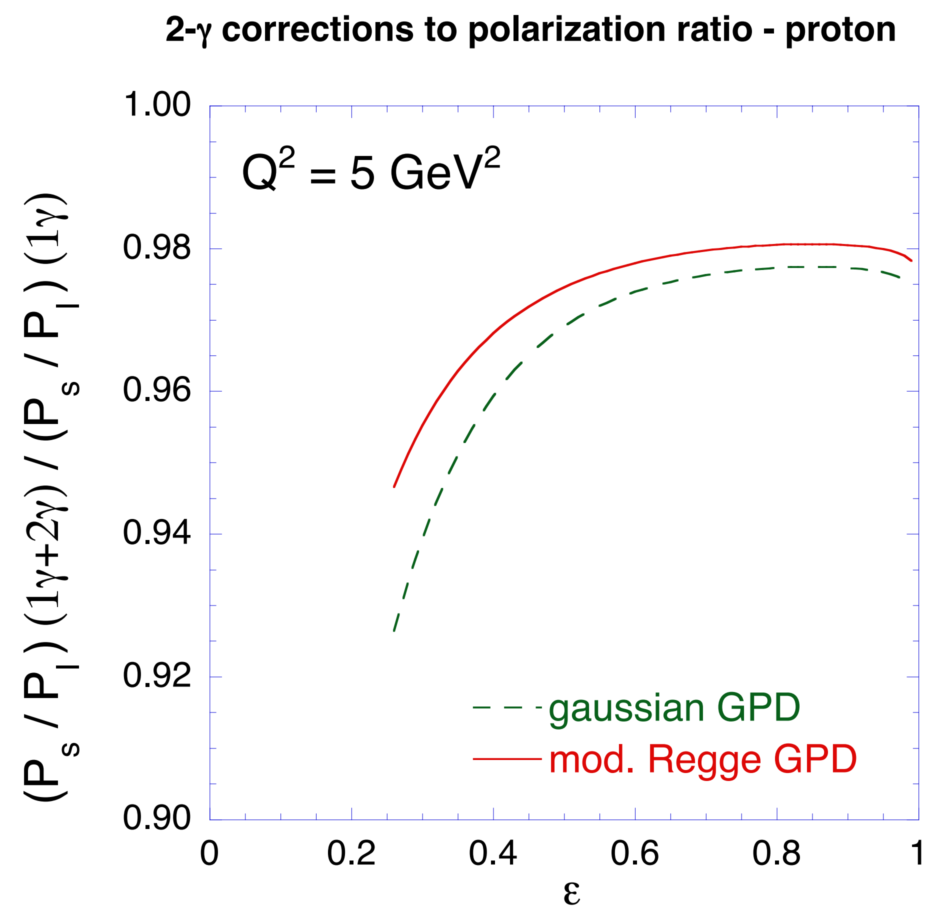
almost model independent predictions for $\sigma(e^+p)/\sigma(e^-p)$



- Slope is fixed, if this is to explain the polarization vs. Rosenbluth puzzle.
- Equality at $\varepsilon \approx 1$ not fixed by this requirement.
- Thanks to Nikolai Kivel

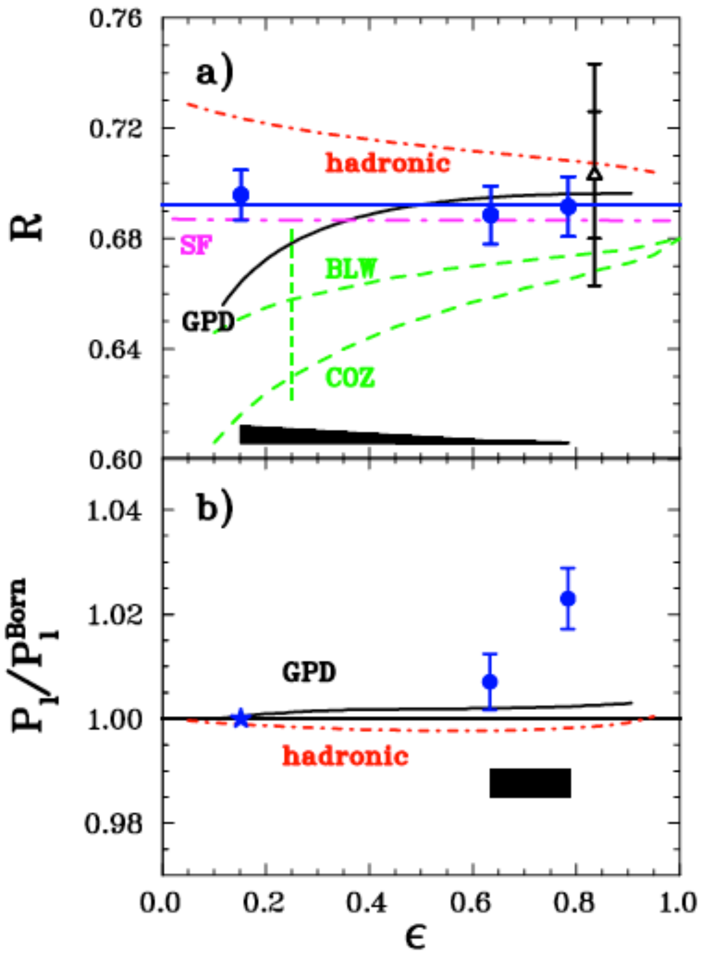
Another sample result: polarization from GPD calc.





Another sample result: polarization from GPD calc.

- New in late 2010 were polarization data as a function of ε for two polarizations, P_t and P_l
- GEp2γ experiment, $Q^2 = 2.5\ \mathrm{GeV}^2$



$$R \equiv -\mu_p \sqrt{\frac{(1+\epsilon)\tau}{2\epsilon}}\,\frac{P_t}{P_\ell} = \frac{\mu_p G_E}{G_M}(1+\ \mathrm{corr.})$$

- Take this data, plus Rosenbluth data, and reverse formulas to find actual δG_M, δG_E, and δF_3. See how well GPD (or other) calculation did! Done by Guttmann, Kivel, Meziane, & Vanderhaeghen (1012.0564).
- Next talk!

Final remarks (1/2)

- Clear evidence that 2-photon processes exist
  - Original Rosenbluth vs. polarization conflict
  - Observation of SSA in e^{-}-n scattering
  - no apparent evidence from polarization vs. ε
  - Other experiments expected
    - curvature in Rosenbluth plot
    - $e^{+}p$ vs. $e^{-}p$ comparison (VEPP, Olympus@DESY, CLAS)
- Reverse: measuring nucleon structure
  - Different observables sensitive to different quantities, as Re(δG_M), Re(F_3 or $G_A^{2\gamma}$), and Imaginary parts of extended FF

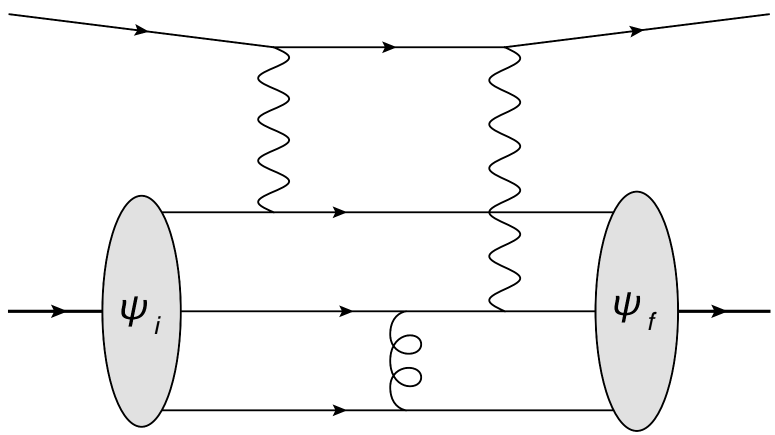
Final remarks (2/2)

- Theory still not complete
  - Partonic calculation explains about half discrepancy at $Q^2 = 5.6$ GeV^2
  - Hadronic calculation perhaps a bit better in this regard
  - Questions of applicability at experimental Q^2

Extra slides

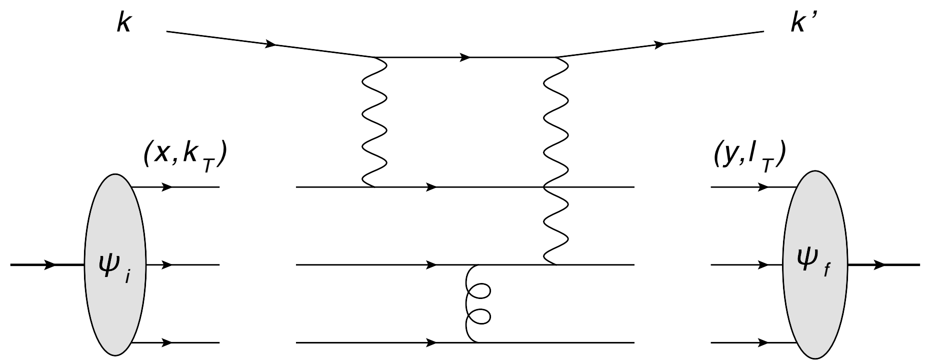
2-photon calculations

- 3rd calculation
- Kivel and Vanderhaeghen (2009): 2-photon contributions to *e-p* elastic scattering from perturbative QCD
- Sample diagram (24 total):



- Lowest order diagrams to convert three parallel moving quarks into three quarks moving parallel in different direction

2γX in pQCD



- Result is convolution of process specific hard scattering amplitude and general wave function for quarks in proton
- High enough momentum transfer, neglect transverse momentum of quarks, defining distribution amplitude,

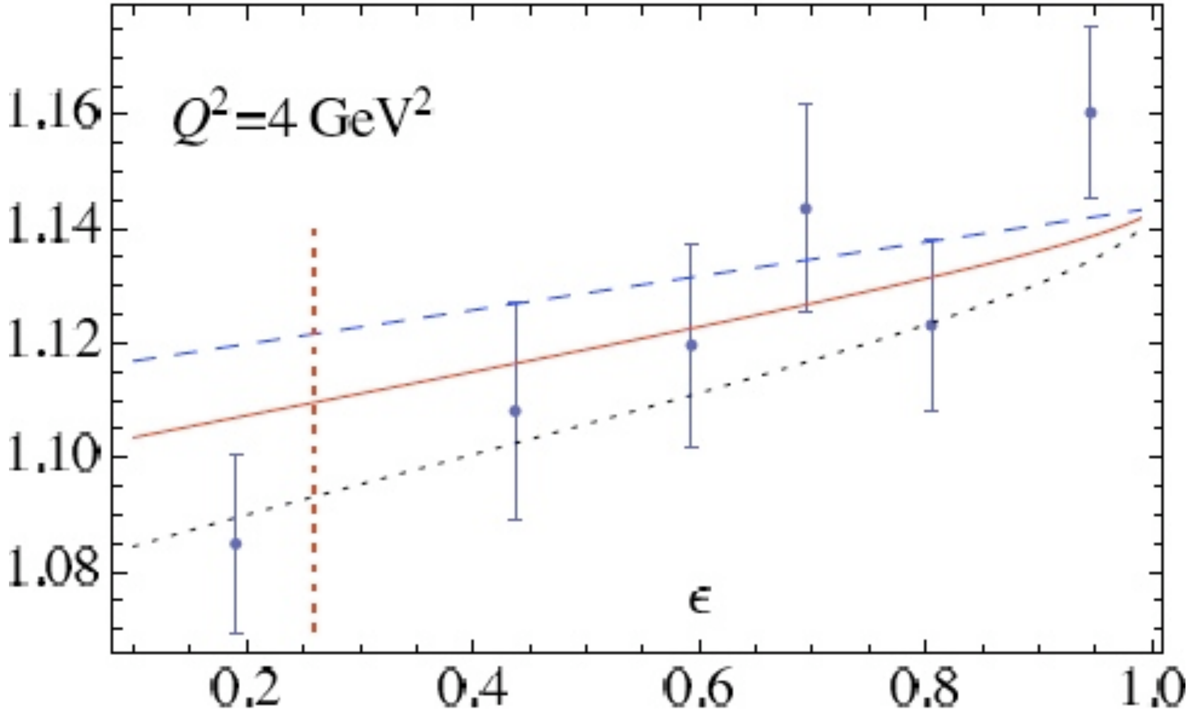
$$\phi(x) = \int [d^2k_\perp]\, \psi(x, k_\perp)$$

- Whence two-photon contribution to FF is (generically),

$$\delta\tilde{F}_i = \frac{1}{Q^4} \int [dx][dy] \phi^*(y)\, T_H(x, y, k, k')\, \phi(x)$$

2γX in pQCD

- The $1/Q^4$ factored out of the hard scattering amplitude
- Same falloff as one-photon exchange terms
- Leading twist. GPD contribution is higher twist. Hence pQCD dominates GPD at high enough momentum transfer.
- Sample result,



- blue dash -- 1 photon
- solid red -- BLW
- dotted black -- COZ

Other 2-γ exchange observables

- Different observables measure different 2γ contributions

Recall three generalized form factors

$$\begin{aligned} \tilde{G}_M &= G_M(Q^2) + \delta\tilde{G}_M(\varepsilon, Q^2) \\ \tilde{G}_E &= G_E(Q^2) + \delta\tilde{G}_E(\varepsilon, Q^2) \\ \tilde{F}_3 &= \quad 0 \qquad + \delta\tilde{F}_3(\varepsilon, Q^2) \\ & \quad\;\; \upuparrows \qquad\qquad \upuparrows \end{aligned}$$

ordinary FF TPE

Form factors G_M and G_E are defined from matrix elements of the electromagnetic current,

the "δ" quantities come from two-photon exchange.

Sometimes (esp. Guichon-Vdh, 2003) replace F_3 by

$$Y_{2\gamma} \equiv \mathrm{Re}\left(\frac{\nu\tilde{F}_3}{m_N^2 G_M}\right)$$

Other 2-γ exchange observables

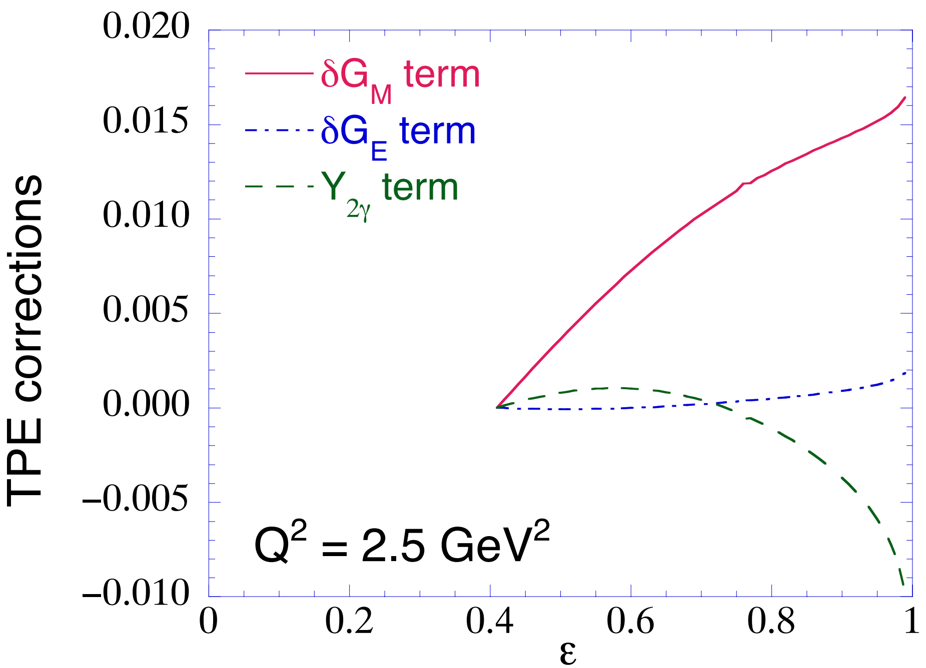
Cross section with two-photon corrections

Experimenters usually apply the Mo-Tsai corrections, so work with

$$R \equiv \frac{\sigma_R^{MTcorr}}{\mu_p^2 G_{\rm dipole}^2} = R^{(1\gamma)}(1+\pi\alpha) + \frac{2\tau G_M \Re\delta\tilde{G}_M^{hard} + 2\varepsilon G_E \Re\delta\tilde{G}_E^{hard} + 2\varepsilon G_M^2\left(\tau + \frac{G_E}{G_M}\right) Y_{2\gamma}}{\tau\mu_p^2 G_{\rm dipole}^2}$$

The extra terms change the slope of *R* vs. ε.

Check term-by-term contributions to ε dependence using the GPD model



(Started all corrections at same point ε, to make slope clear)

- δG_M dominates total for Rosenbluth
- δG_E term is small
- $Y_{2\gamma}$ by itself has the wrong sign

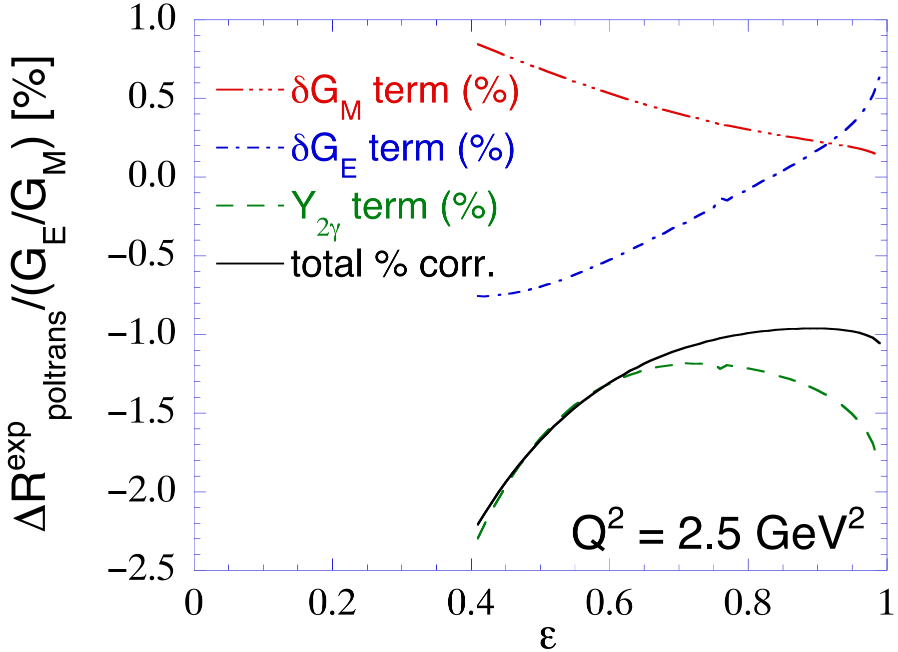
Other 2-γ exchange observables

Polarization transfer and two-photon corrections

One measures

$$R^{exp}_{poltrans} \equiv -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\,\frac{\mathcal{P}_s}{\mathcal{P}_l} = \frac{G_E}{G_M}\left\{1 - \frac{\Re\delta\tilde{G}_M^{hard}}{G_M} + \frac{\Re\delta\tilde{G}_E^{hard}}{G_E} + \left(\frac{G_M}{G_E} - \frac{2\varepsilon}{1+\varepsilon}\right) Y_{2\gamma}\right\}$$

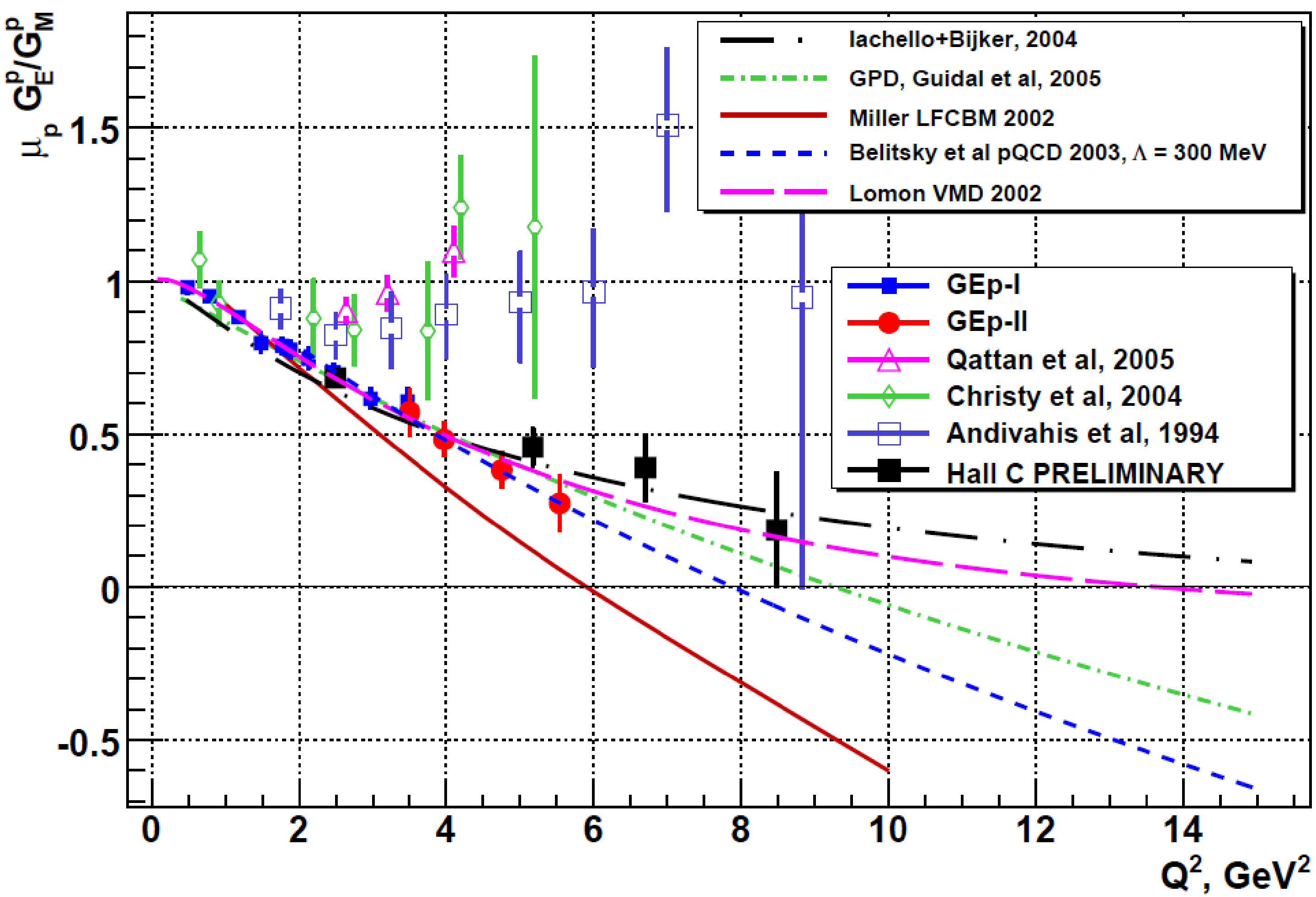
Using the GPD calculation, the corrections are



For polarization transfer, net corrections small, –1 to –2% at this Q^2, and come mainly from F_3 (or $Y_{2\gamma}$). BTW, $Y_{2\gamma}$ is ε dependent and about –(1/2)%

New data

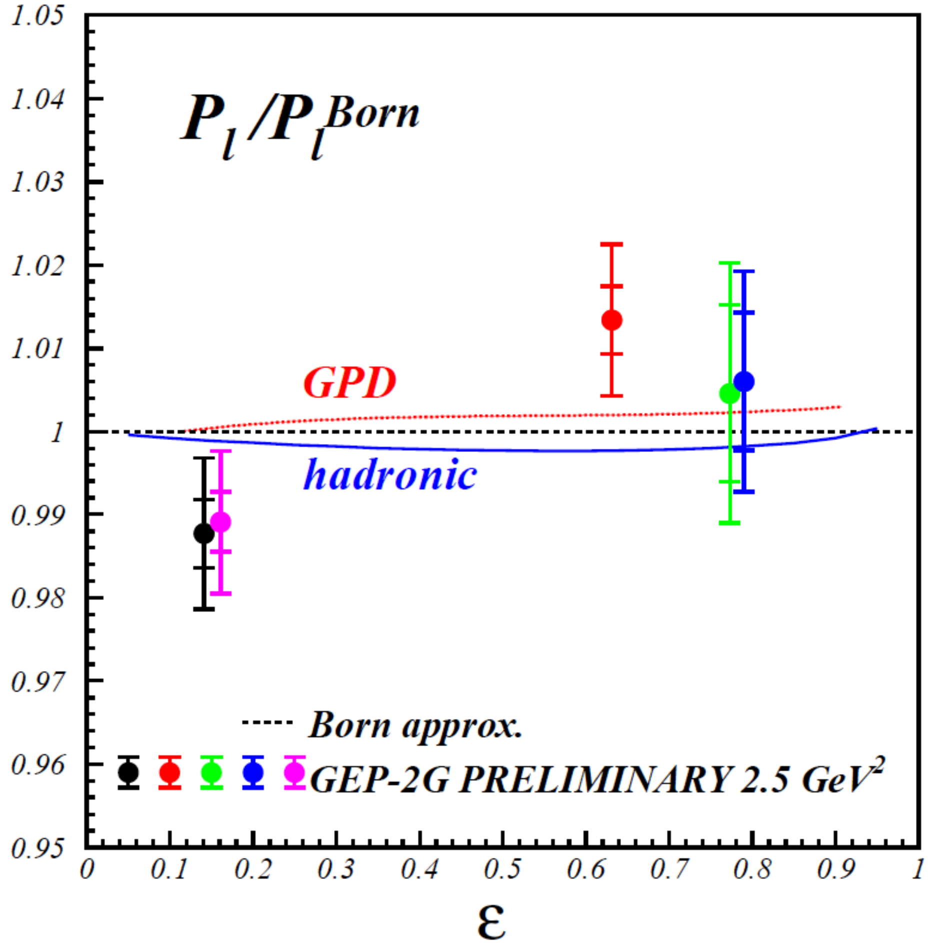
- New data was presented at this meeting and User's meeting
- From GEp-3,



Lubomar Pentchev, User's meeting

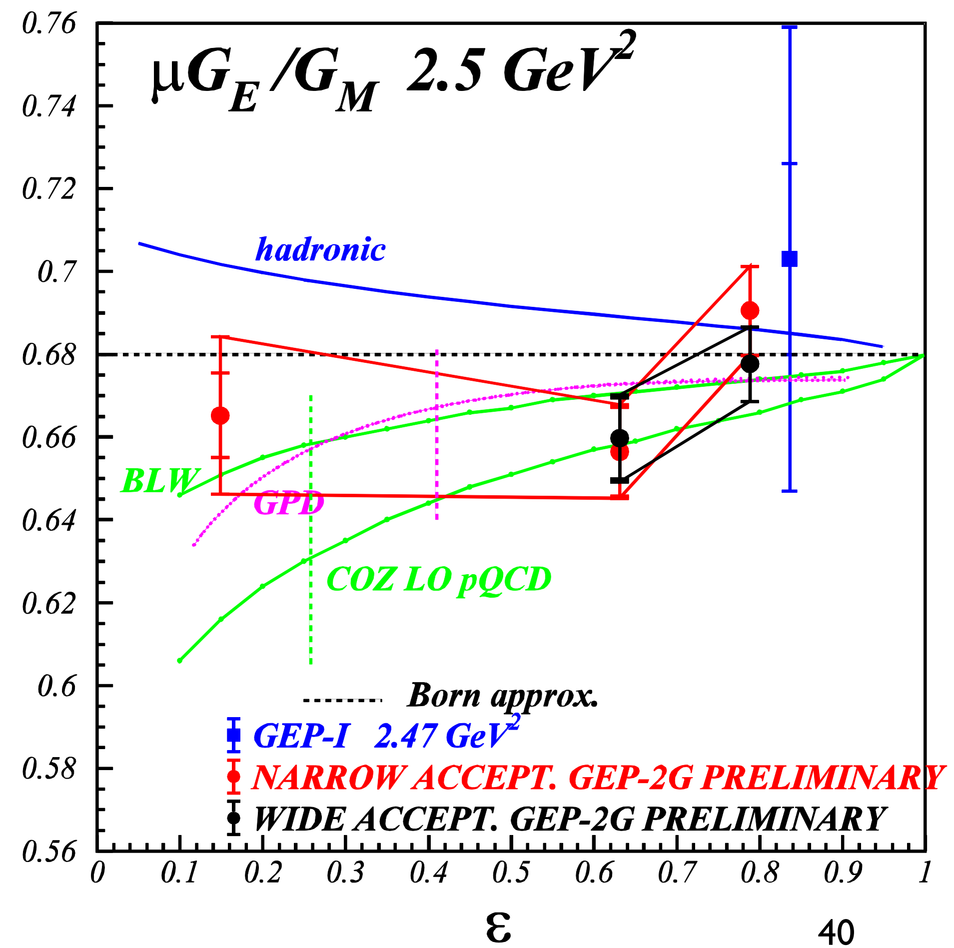
New data

- from GEp-2γ, longitudinal polarization
- predicted effect of 2γ is small for this observable



New data

- from GEp-2γ, ratio P_t/P_l polarizations at varying ε
- with kinematic factor removed, would be $\mu G_E/G_M$ and flat for one-photon exchange
- Famously opposite slope for hadronic and partonic calculations

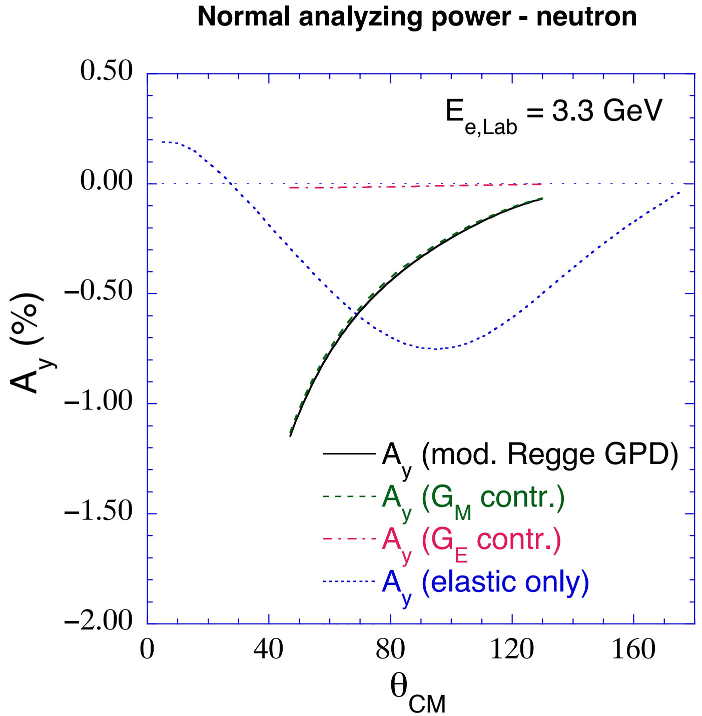


New data

- Single spin asymmetry (P_y or P_n) experiments
- zero if only one-photon exchange — any non-zero result means multiple photons
- Depends on imaginary part of new form factor functions

$$P_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}}\,\frac{1}{\sigma_R}\left\{-\,G_M\,\mathrm{Im}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E\,\mathrm{Im}\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right)\right\}$$

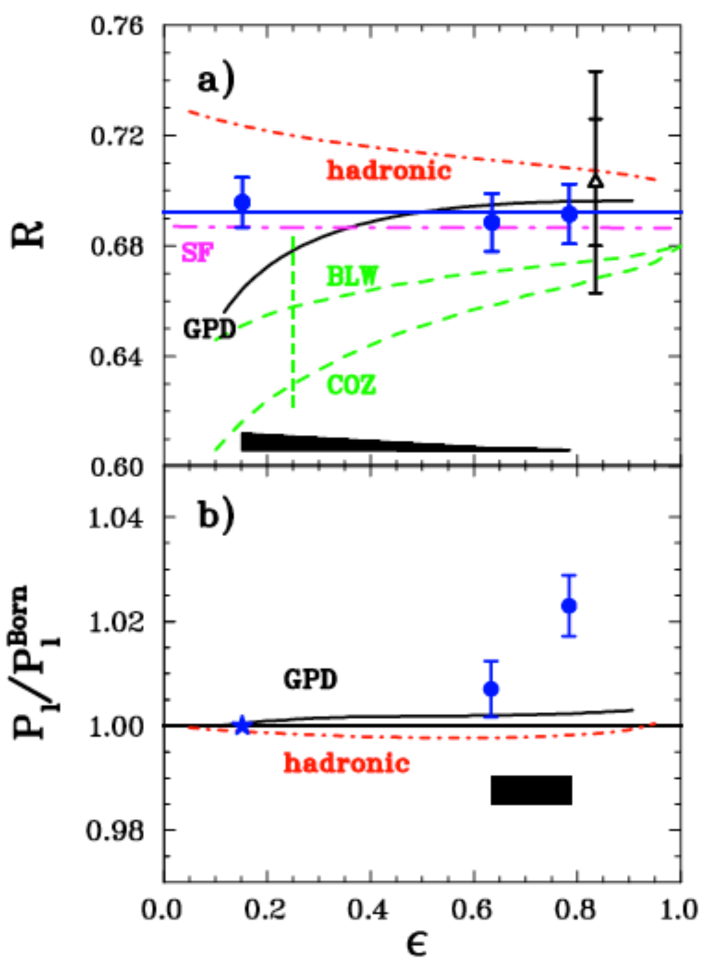
- There exist "on-line" results showing P_n 10σ from 0 for neutron (YaWei Zhang, this morning).
- Calculated result shown



Two-photon section

- New in late 2010 were polarization data as a function of ε for two polarizations, P_t and P_l

- GEp2γ experiment, $Q^2 = 2.5\ GeV^2$

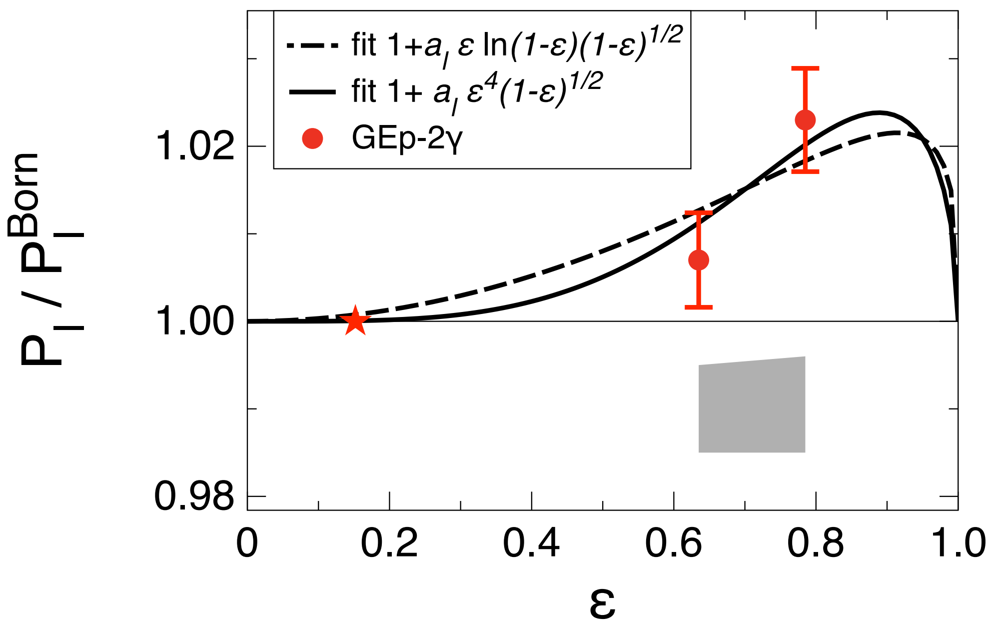


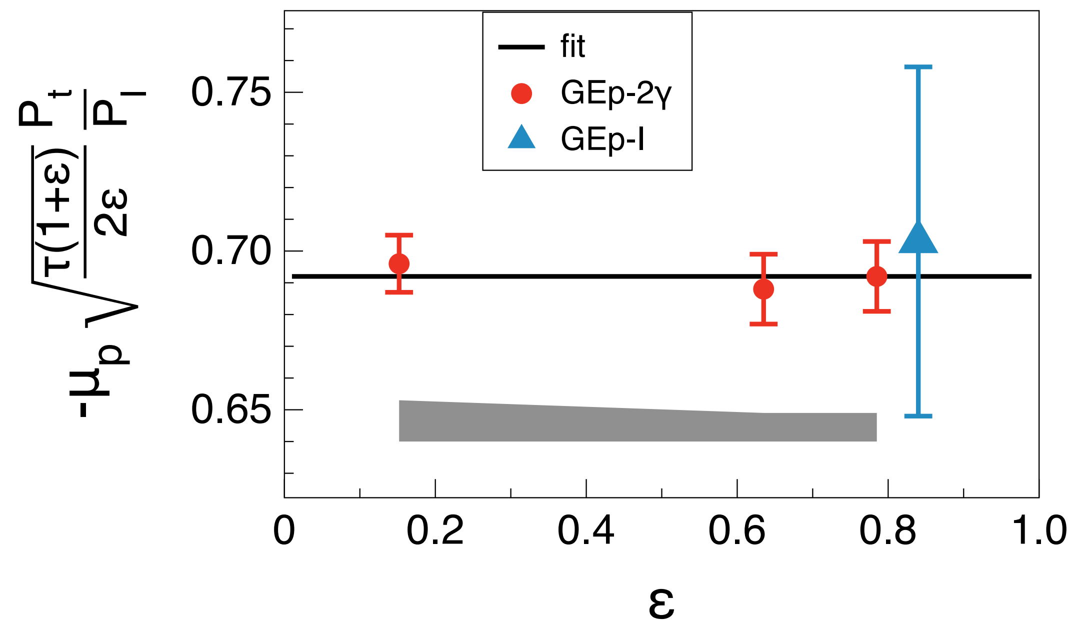
$$R \equiv -\mu_p \sqrt{\frac{(1+\epsilon)\tau}{2\epsilon}}\, \frac{P_t}{P_\ell} = \frac{\mu_p G_E}{G_M}\,(1+\ \text{corr.})$$

- Take this data, plus Rosenbluth data, and reverse formulas to find actual δG_M, δG_E, and δF_3. See how well GPD (or other) calculation did! Done by Guttmann, Kivel, Meziane, & Vanderhaeghen (1012.0564).

Two-photon section

- Plots with their parameterizations compared to polarization data,

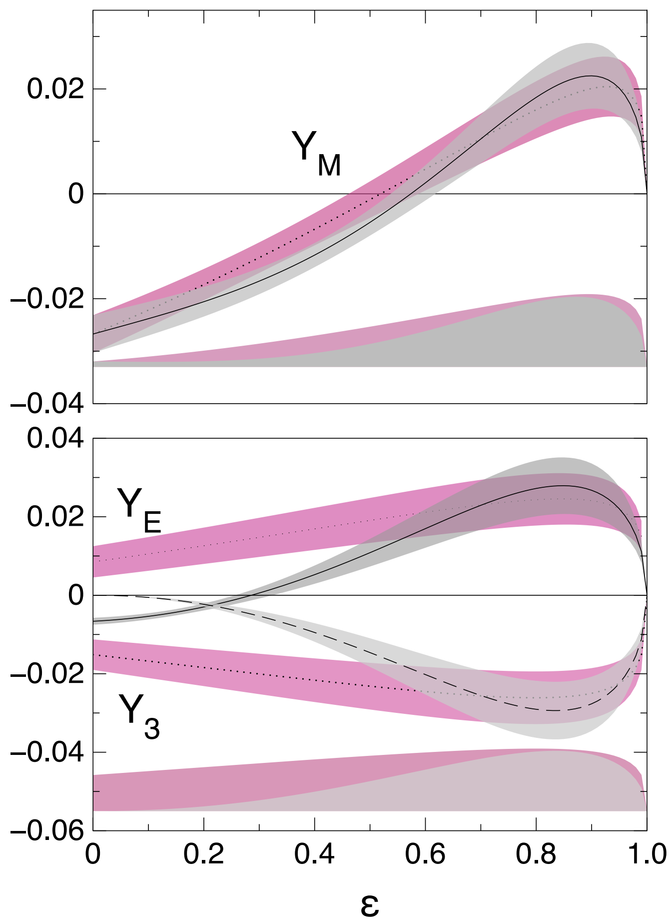




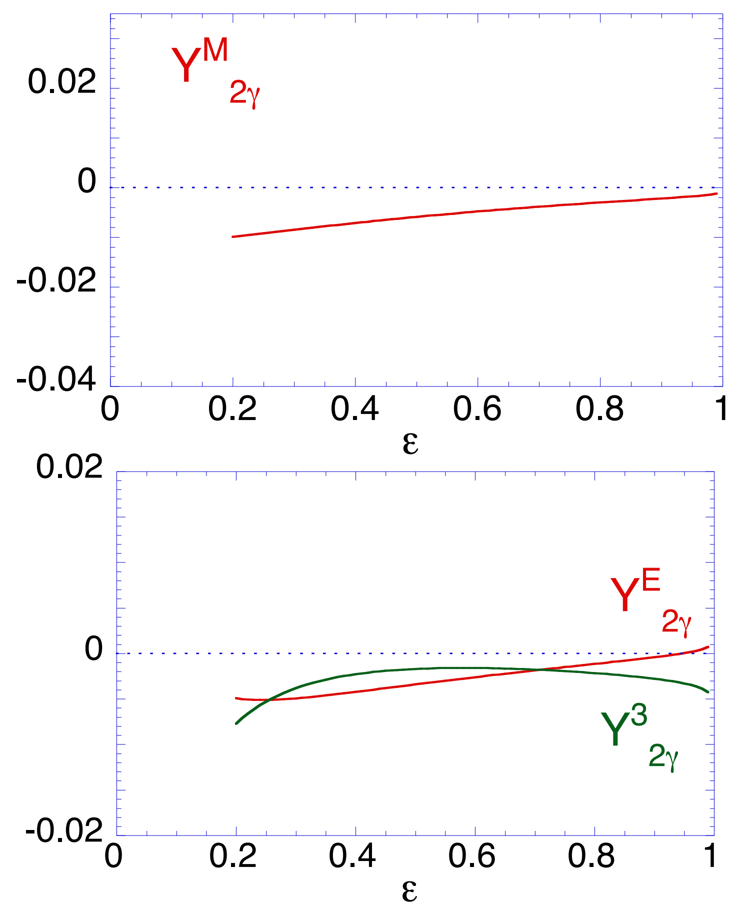
Two-photon section

- Results given in terms of ratios (plots for Q^2 = 2.64 GeV^2)

$$Y_{2\gamma}^{M} = \mathrm{Re}\,\frac{\delta \tilde{G}_M}{G_M} \qquad Y_{2\gamma}^{E} = \mathrm{Re}\,\frac{\delta \tilde{G}_E}{G_M} \qquad Y_{2\gamma}^{3} = \frac{s-u}{4M^2}\,\mathrm{Re}\,\frac{\delta \tilde{F}_3}{G_M}$$



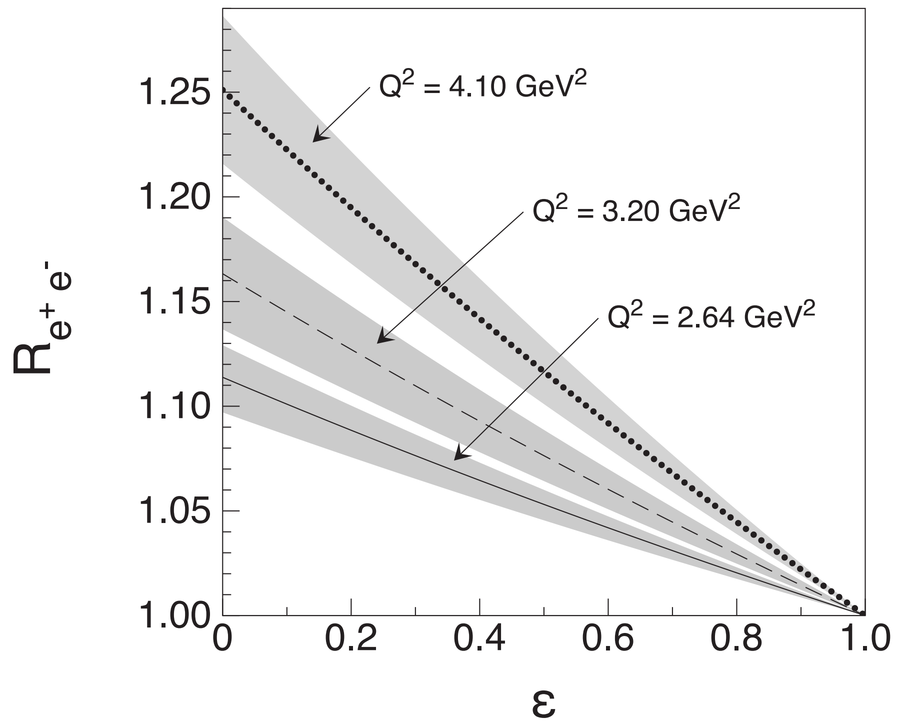
extraction from data, 2010



calculation, style of 2004

Two-photon section

- New prediction for $\sigma(e^+p)/\sigma(e^-p)$ considerably larger in magnitude.



- ∃ preliminary data point from VEPP-3, at $\varepsilon = 0.5$, $Q^2 = 1.43$ GeV^2, with $R(e^+e^-) = (2.6 \pm 1.0)$ %.