Phenomenological Analysis of TPE Amplitudes from Elastic ep-scattering

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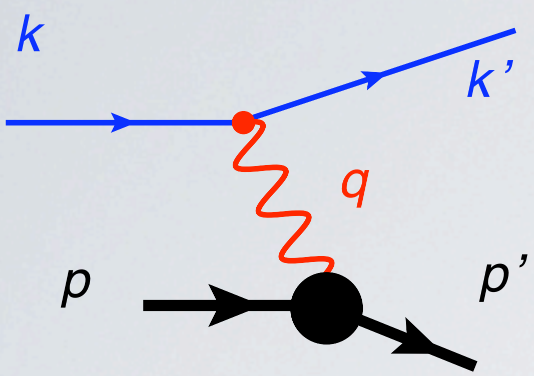
Outline

J. Guttman, NK, M. Meziane, M. Vanderhaeghen
arXiv:1012.0564, Eur.Phys.J. A (2011)

- Introduction
- Analysis of the TPE contributions in different observables and the fit of existing experimental data
- Estimate the TPE amplitudes and predictions for the ratio $R_{+/-}$
- Conclusions

Elastic *e-p* scattering: one-photon exchange



$$d\sigma^{ep} = d\sigma^{\mathrm{ns}}\left[\tau G_M^2 + \varepsilon G_E^2\right]$$

$$\tau = \frac{Q^2}{4M^2}$$

no structure cross section $\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ns}} = 4\alpha\cos^2\theta/2\frac{E'^3}{Q^4 E_{beam}}$

in Lab frame

photon polarization parameter $\varepsilon = \left[1 + 2(1+\tau)\tan^2\theta/2\right]^{-1}$

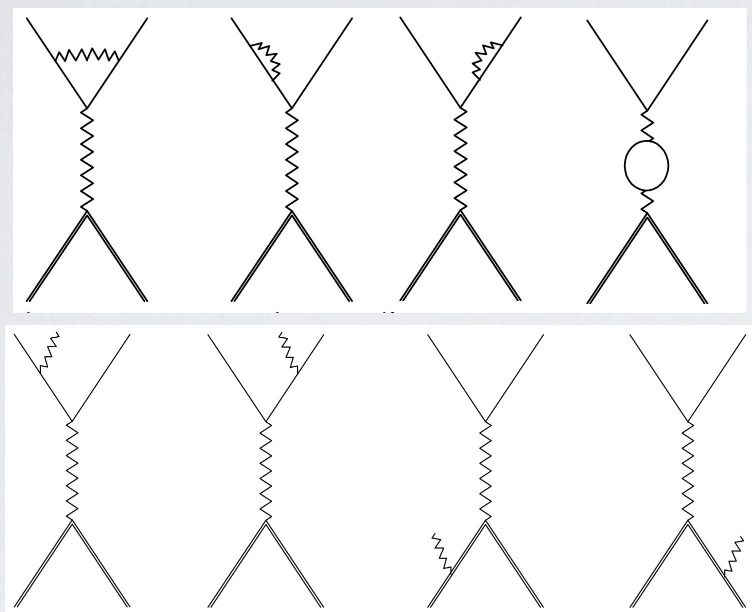
$$0 < \varepsilon < 1$$

reduced cross section $\sigma_{\mathrm{r}} = G_M^2\left(1 + \varepsilon R^2/\tau\right)$ $R \equiv \frac{G_E}{G_M}$

Elastic e-p scattering: radiative corrections

$$d\sigma = d\sigma_0(1+\delta)$$

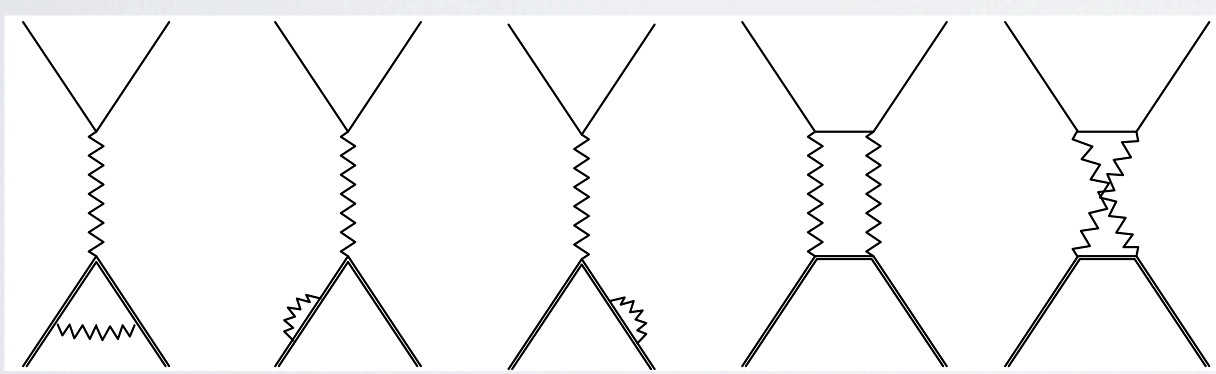
$$\sim \frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \times \ln \frac{4EE'}{E^\gamma_{min}}$$



electron and photon radiative corrections known and well understood

Soft bremsstrahlung involves only soft photons

effect of the nucleon structure is negligible



Vertex and box diagrams involve hard and soft photons

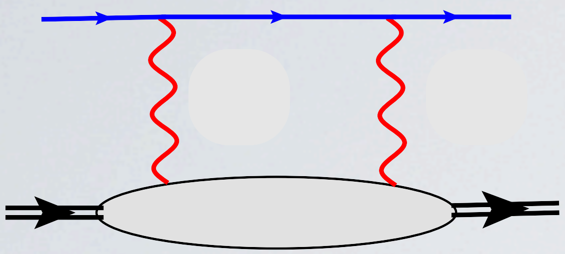
soft photon contribution is included in radiative correction (corresponding IR divergence is cancelled with electron proton bremsstrahlung interference)

computed in soft photon approximation (proton structure neglected)

Mo, Tsai Rev.Mod.Phys 1969

nontrivial ε-dependence arises only from the two photon exchange diagrams!

TPE contribution in the reduced cross section



3 TPE amplitudes $Y_M(\varepsilon, Q),\ Y_E(\varepsilon, Q),\ Y_3(\varepsilon, Q)$

What do we know about these amplitudes?

large Q behavior

QCD predictions

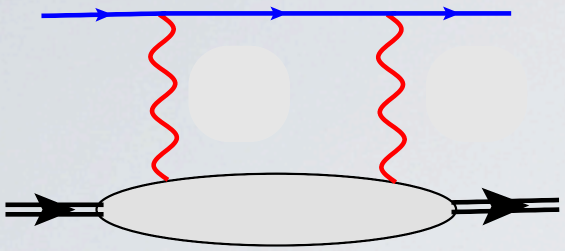
$$G_M \sim Q^{-4} \qquad Y_{M,3} \sim const$$

$$R = G_E/G_M \sim const \qquad Y_E \sim Q^{-2}$$

but it works only at very large Q: $Q\Lambda_{\rm QCD} \gg m_N^2$

intermediate Q^2=2.5-16 GeV^2 $Q\Lambda_{\rm QCD} \simeq 0.5 - 1.2 \mathrm{GeV}^2$

TPE contribution in the reduced cross section



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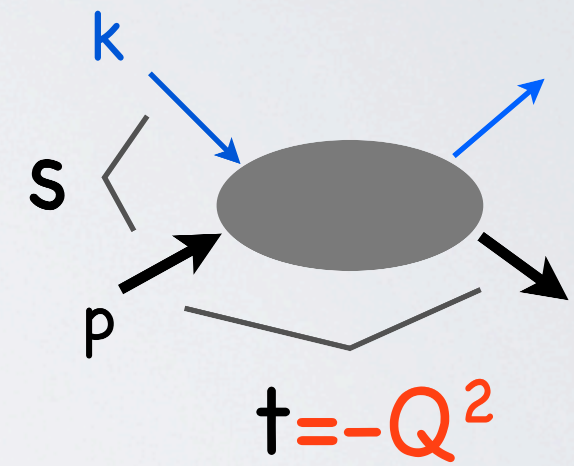
What do we know about these amplitudes?

End-point $\varepsilon \to 0, 1$ behavior



$\varepsilon \to 1$ Q fixed $s \to \infty$ Regge limit

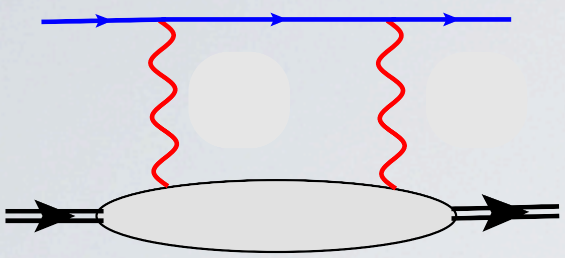
plausible assumption $Y_{M,3,E}(\varepsilon) \to 0$

agrees with all models, in pQCD $Y_{M,3}(\varepsilon) \sim (1-\varepsilon)^{1/2}$

$\varepsilon \to 0$ $Y_{M,3,E}(\varepsilon) \neq \infty$ are not singular

$Q^2 = s - 2m^2 - m^4/s$

TPE contribution in the reduced cross section



3 TPE amplitudes $Y_M(\varepsilon, Q), Y_E(\varepsilon, Q), Y_3(\varepsilon, Q)$

reduced cross section

1-γ exch interference 1-γ and 2-γ

$$\frac{\sigma_r}{G_M^2} = 1 + \varepsilon R^2/\tau + 2\mathcal{R}e\{\, Y_M + \varepsilon\, Y_3 + \varepsilon R/\tau(Y_M + Y_3)\,\}$$

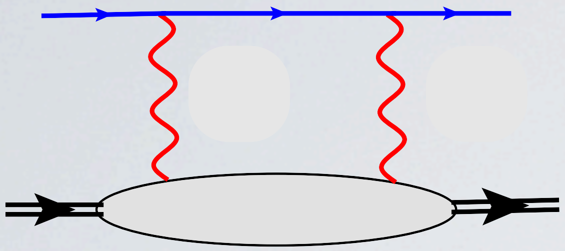
$$R = \frac{G_E}{G_M} \simeq 0.25$$

Q=2.5GeV^2 → R can be considered as a small parameter

simplified expression:

$$\frac{\sigma_r}{G_M^2} \simeq 1 + \varepsilon R^2/\tau + 2\mathcal{R}e\{\, Y_M + \varepsilon\, Y_3\}$$

TPE contribution in the reduced cross section



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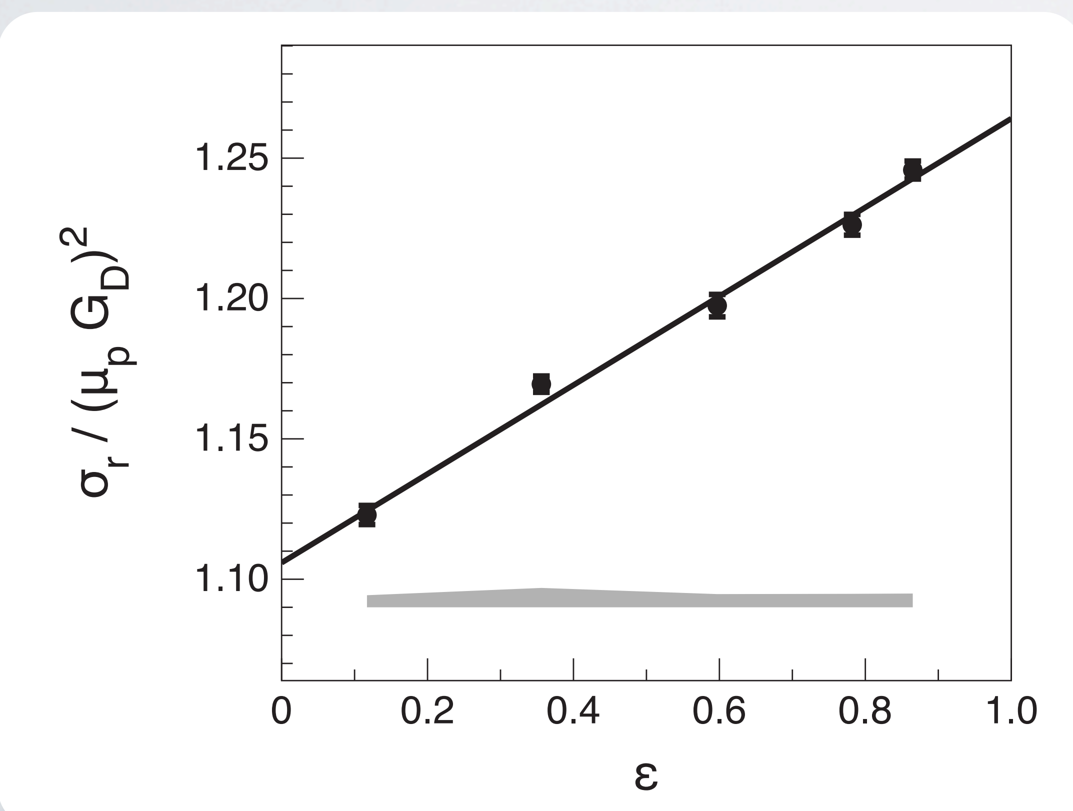


Data $0.117 < \varepsilon < 0.865$

$Q^2 = 2.64\mathrm{GeV}^2$

JLab/Hall A (2005)

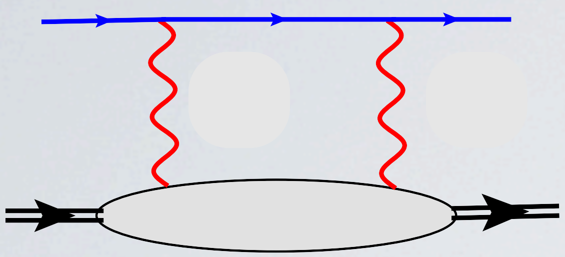
linear fit: a + εb $a = 1.106 \pm 0.006$

$b = 0.160 \pm 0.009$

linear extrapolation $\varepsilon \to 1$ $Y_{M,3,E}(\varepsilon) \to 0$

$$\frac{G_M^2(1 + R^2/\tau)}{(\mu_p G_D)^2} = a + b$$

TPE contribution in the reduced cross section



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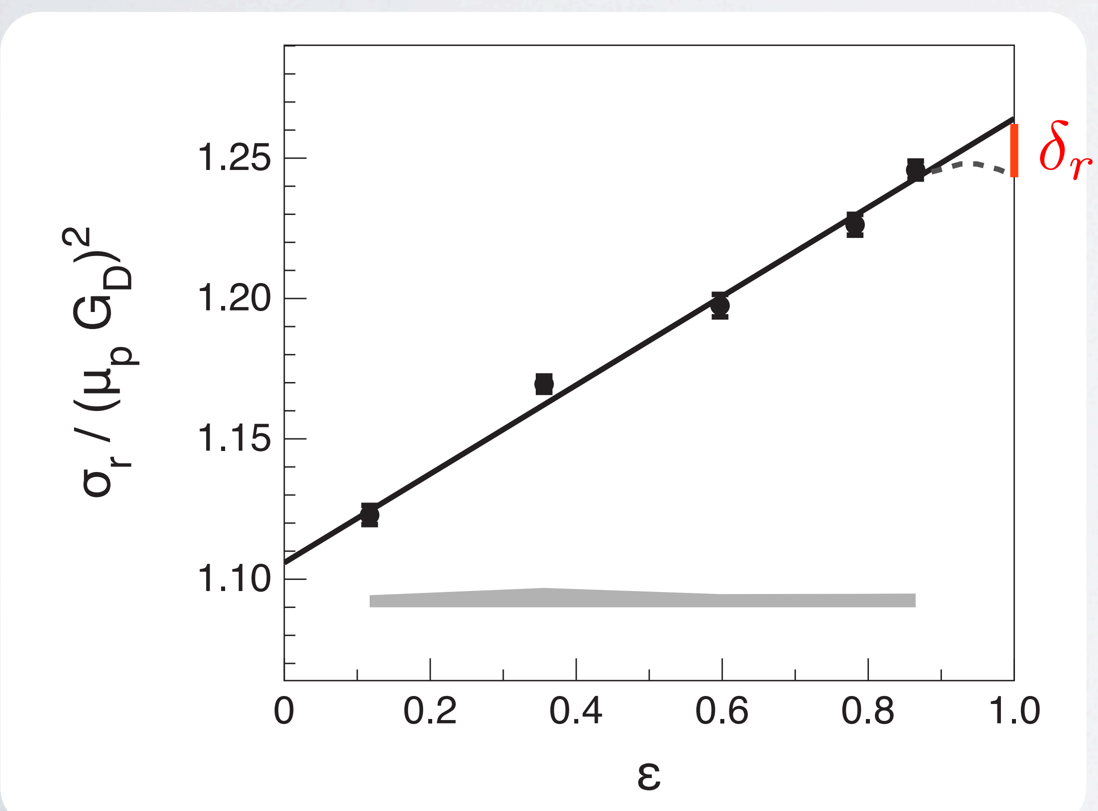

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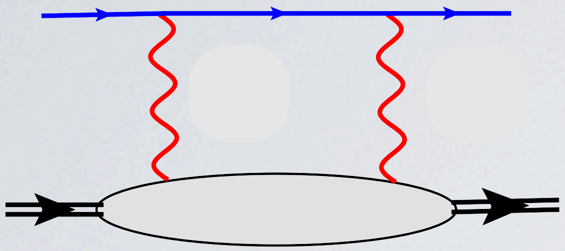

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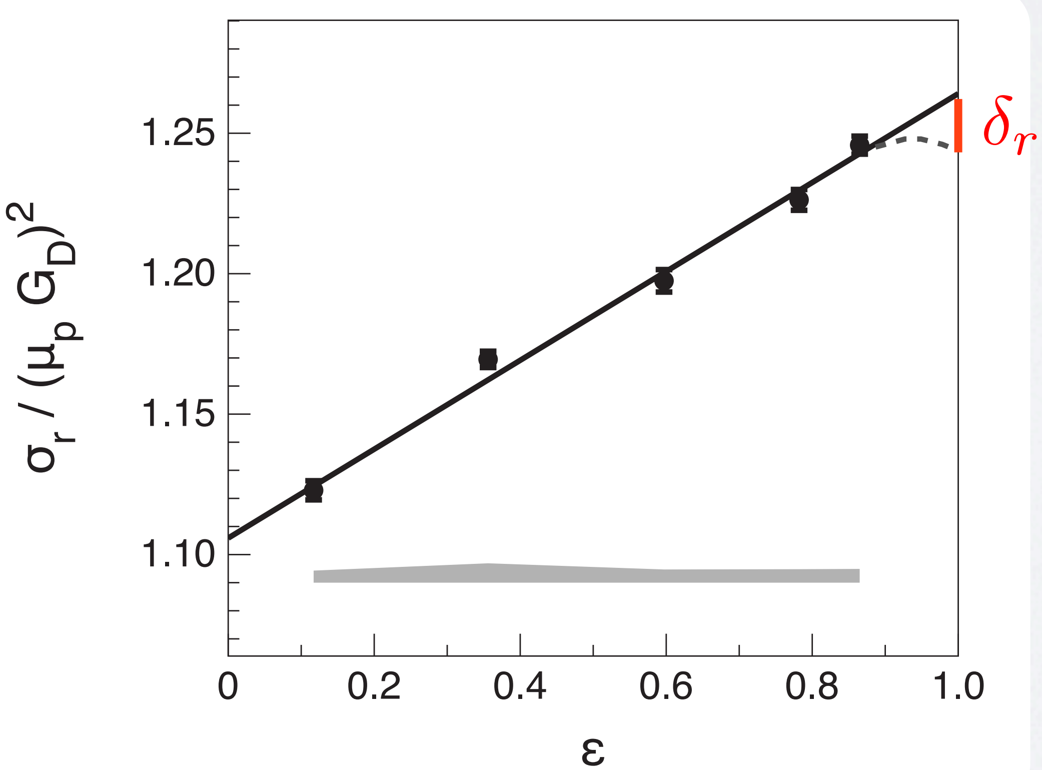

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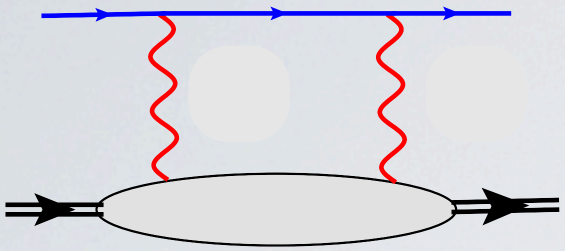
linear extrapolation $\varepsilon \to 1$ $Y_{M,3,E}(\varepsilon) \to 0$

$$\frac{G_M^2(1 + R^2/\tau)}{(\mu_p G_D)^2} = a + b \pm \delta_r$$

TPE contribution in the reduced cross section



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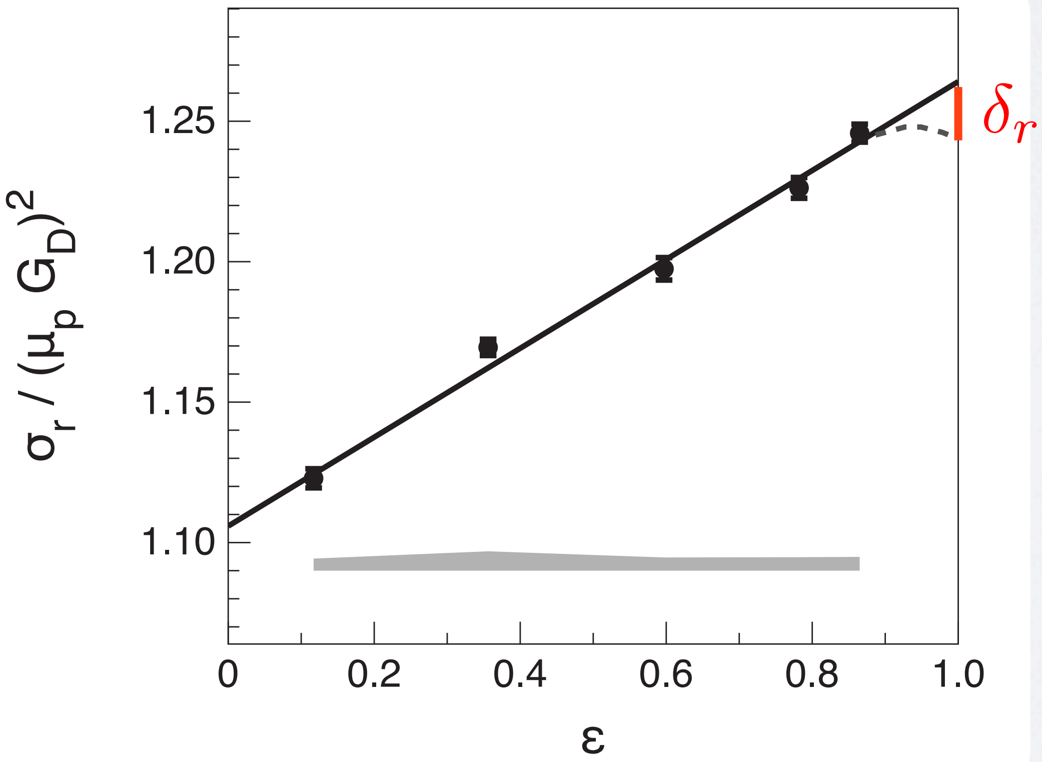
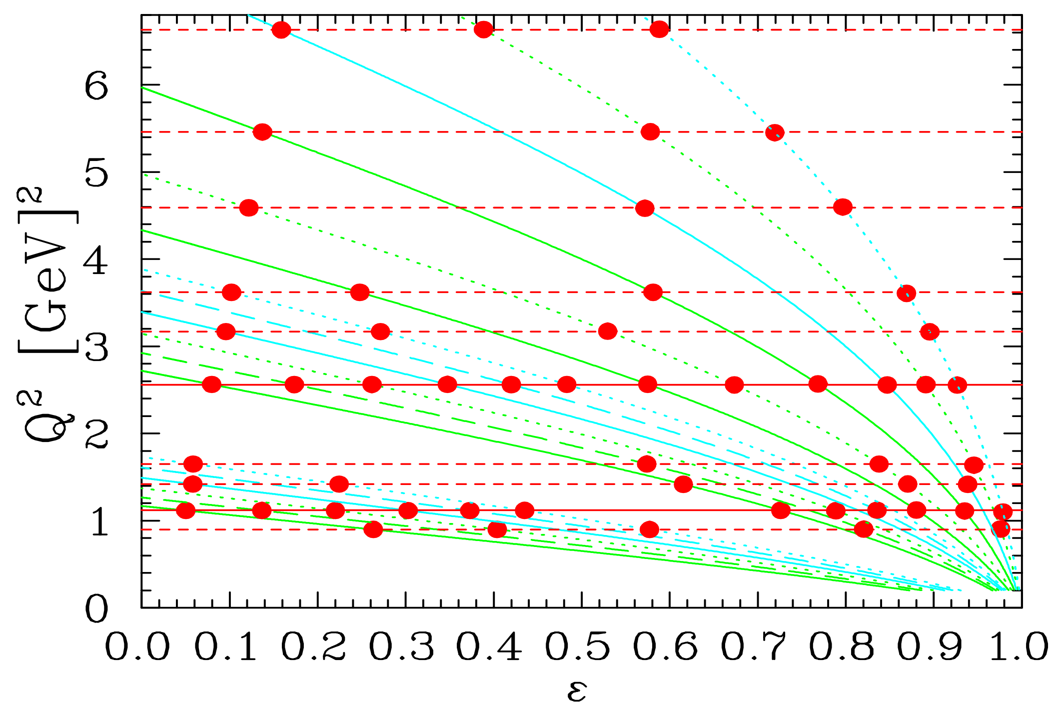


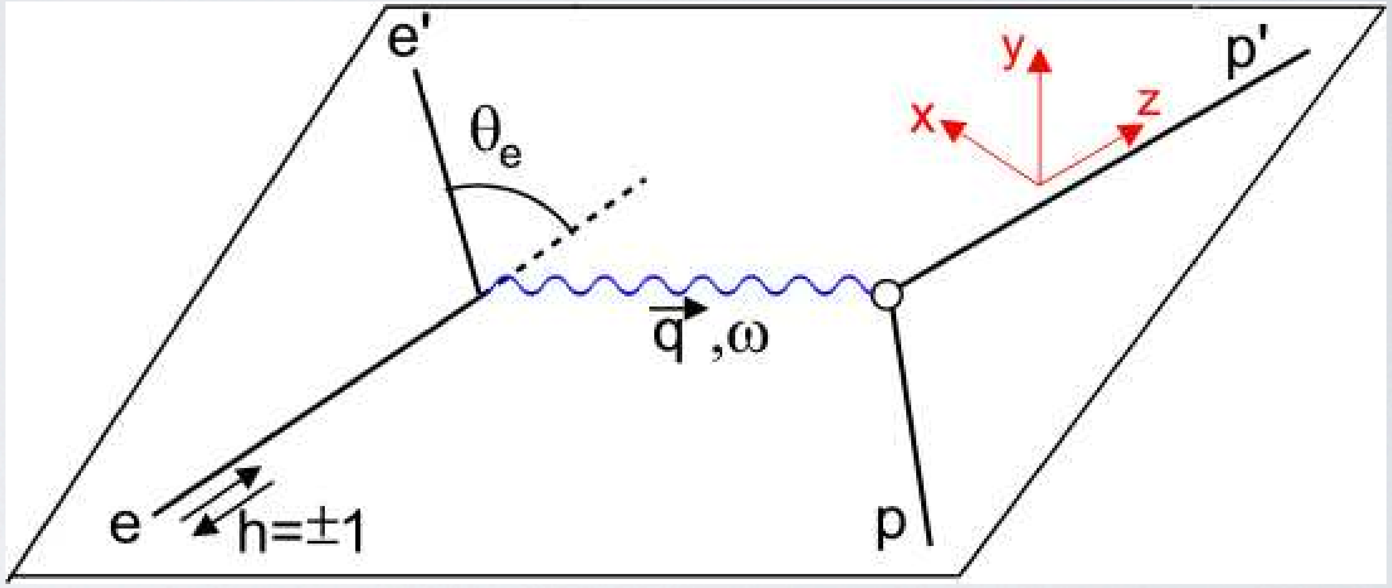
E05-017, proposal, expected



Elastic *ep*-scattering: recoil polarization technique

Akhiezer, Rekalo (1974)

$$\vec{e} + p \to e + \vec{p}$$



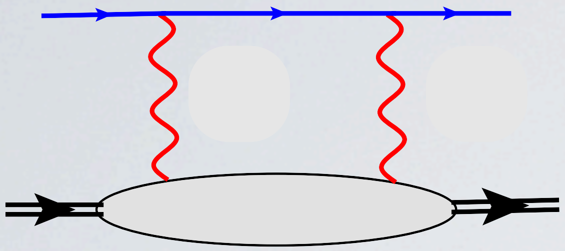
$$d\sigma_{pol} = d\sigma_{unpol}(1 + h\,S_x\,P_t + h\,S_z P_l)$$

$$P_t = -\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\,\frac{G_E G_M}{\tau\,\sigma_r}$$

$$P_l = \sqrt{1-\varepsilon^2}\,\frac{G_M^2}{\tau\,\sigma_r}$$

$$\rightarrow \quad -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\,\frac{P_t}{P_l} = R$$

TPE contribution in the polarizations



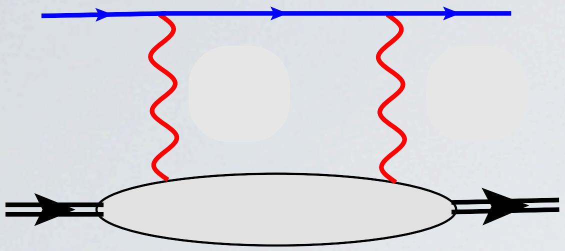
3 TPE amplitudes $Y_M(\varepsilon, Q),\ Y_E(\varepsilon, Q),\ Y_3(\varepsilon, Q)$

$$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} = R + \mathcal{R}e(Y_E + Y_3) - R\,\mathcal{R}e\left(Y_M + \frac{2\varepsilon}{1+\varepsilon}Y_3\right)$$

interference 1-γ and 2-γ

$$\frac{P_l}{P_l^\gamma} = 1 - \frac{2}{(1+\varepsilon R^2/\tau)}\mathcal{R}e\left(\frac{\varepsilon^2\,Y_3}{1+\varepsilon} + \varepsilon R/\tau(Y_3 + Y_E) - R^2/\tau\left(\frac{\varepsilon^2\,Y_3}{1+\varepsilon} + \varepsilon Y_M\right)\right)$$

TPE contribution in the polarizations



3 TPE amplitudes $Y_M(\varepsilon, Q),\ Y_E(\varepsilon,\ Q),\ Y_3(\varepsilon, Q)$

simplified expressions:

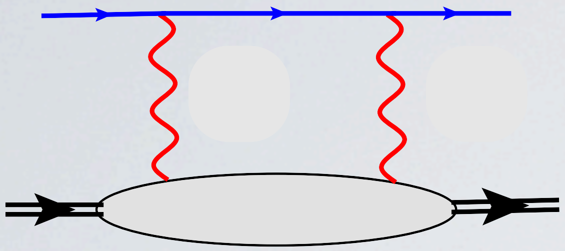
$$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} \simeq R + \mathcal{R}e(Y_E + Y_3)$$

interference 1-γ and 2-γ

$$\frac{P_l}{P_l^{\gamma}} \simeq 1 - 2\mathcal{R}e\left(\frac{\varepsilon^2\, Y_3}{1+\varepsilon} + \varepsilon R/\tau(Y_3 + Y_E)\right)$$

$$\varepsilon \lesssim R/\tau$$

TPE contribution in the polarizations



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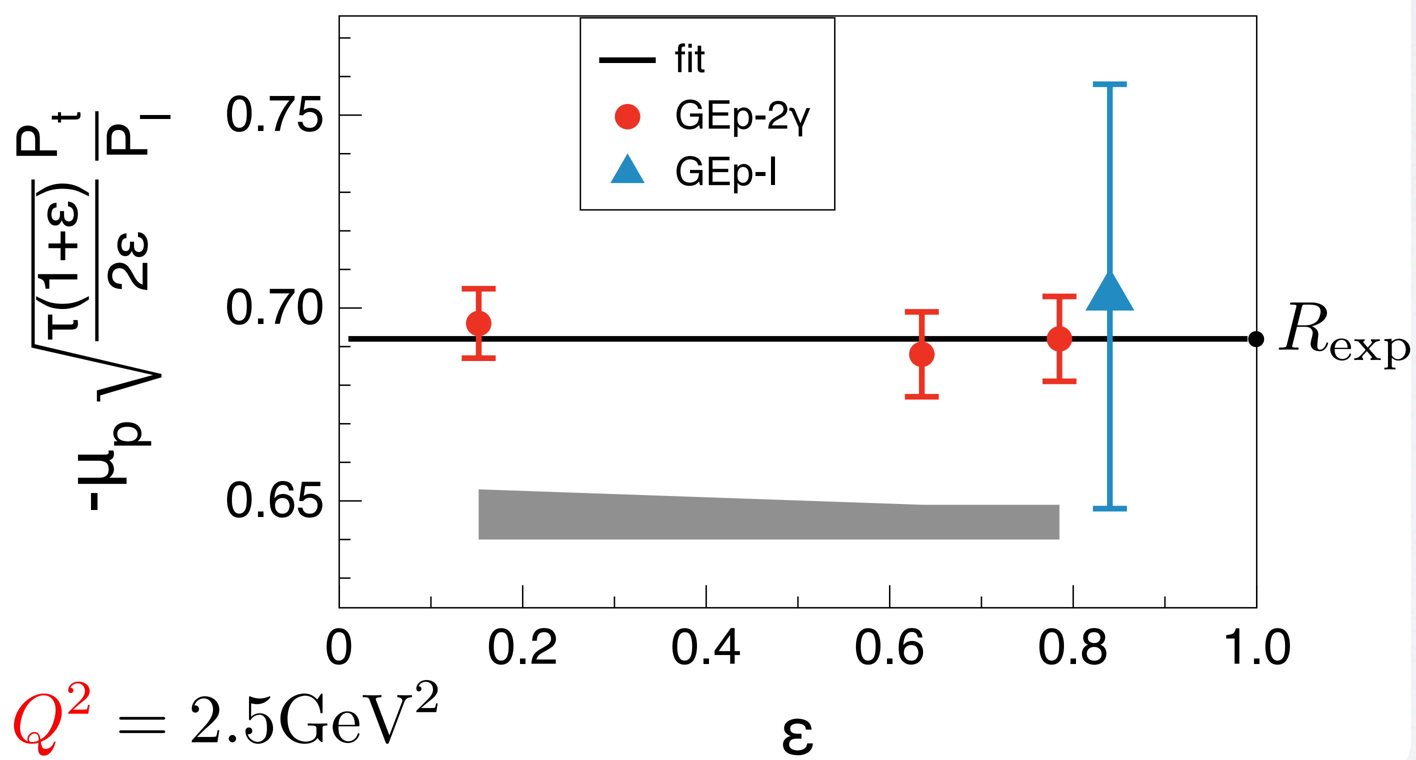
$$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} \simeq R + \mathcal{R}e(Y_E + Y_3)$$

- $\mathcal{R}e(Y_M + Y_3) \simeq 0$

$$0 < \varepsilon < 1$$

$$\Rightarrow R = R_{\mathrm{exp}}$$



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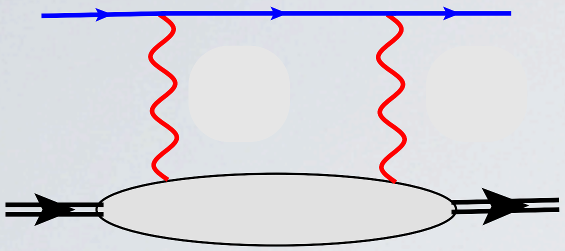

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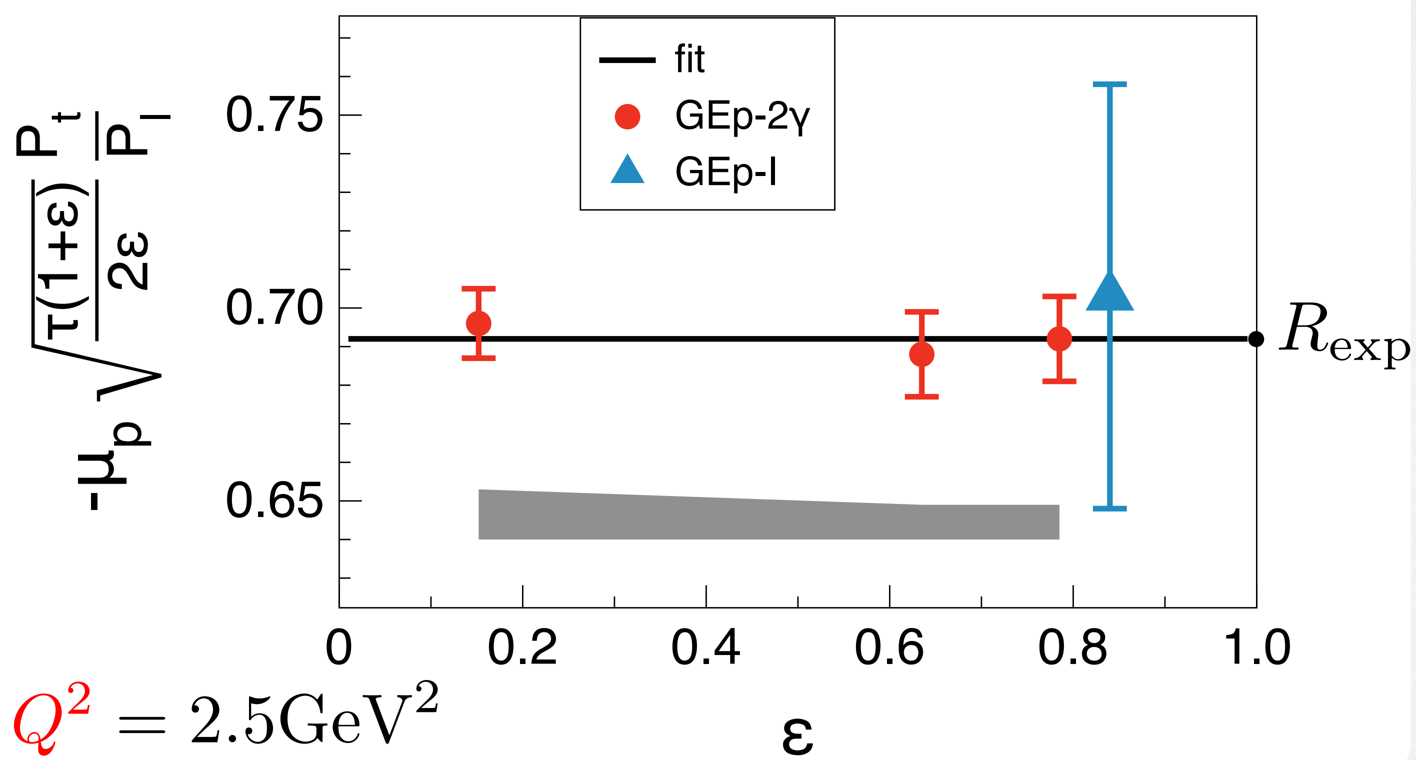


$Q^2 = 2.5\mathrm{GeV}^2$

- $\mathcal{R}e(Y_M + Y_3) \simeq 0$

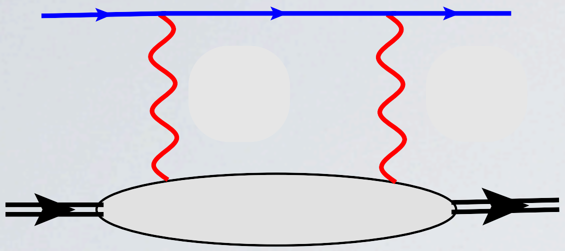
  $0 < \varepsilon < 1$

  $\Rightarrow R = R_{\rm exp}$

- $\mathcal{R}e(Y_M + Y_3) \simeq const$

  $\varepsilon_{min} < \varepsilon < \varepsilon_{max}$

TPE contribution in the polarizations



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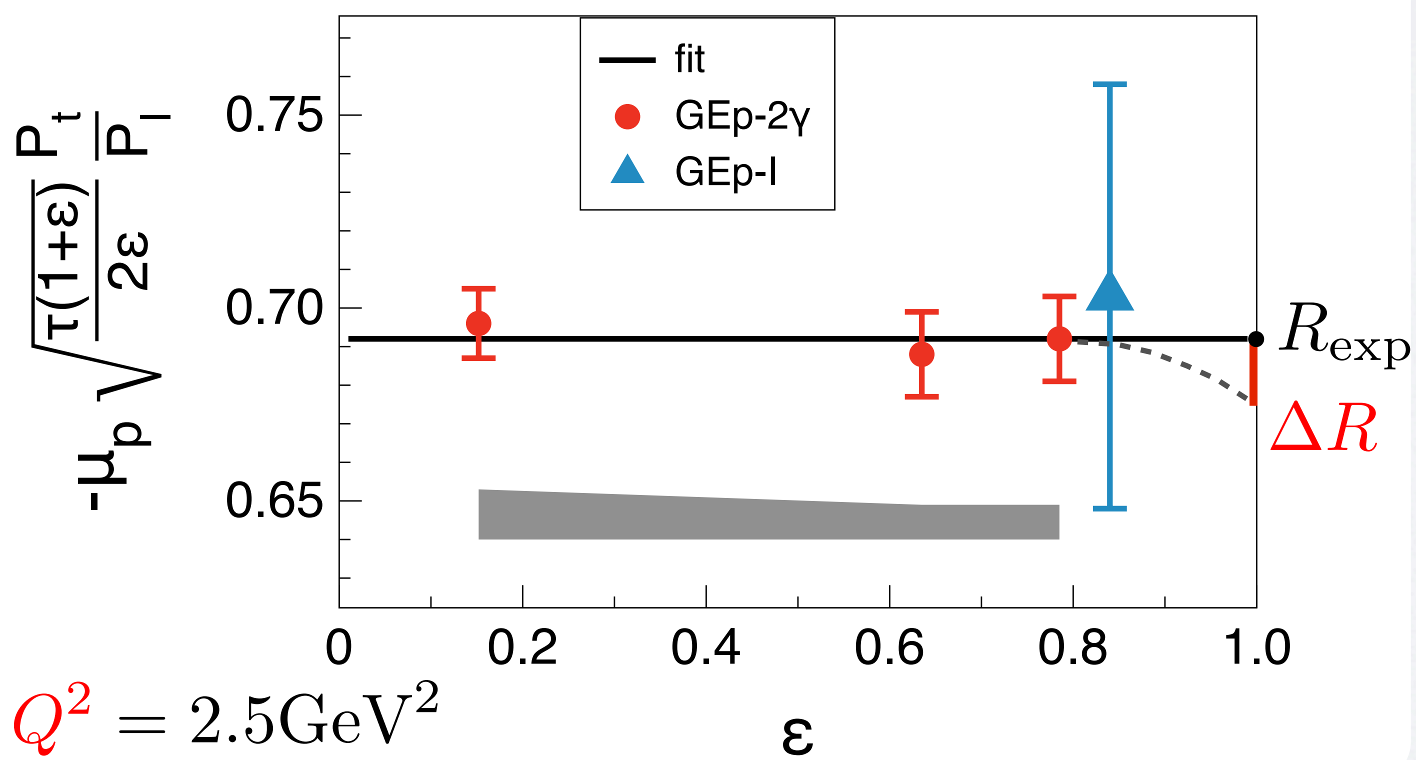


- $\mathcal{R}e(Y_M + Y_3) \simeq 0$

  $0 < \varepsilon < 1$

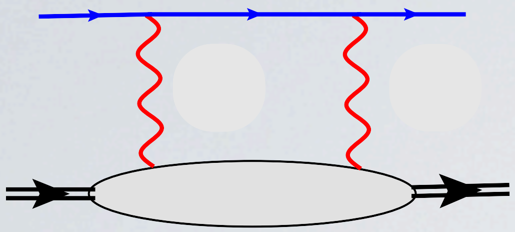
  $\Rightarrow R = R_{\exp}$

- $\mathcal{R}e(Y_M + Y_3) \simeq const$

  $\varepsilon_{min} < \varepsilon < \varepsilon_{max}$

  $\Rightarrow R = R_{\exp} - \Delta_R$

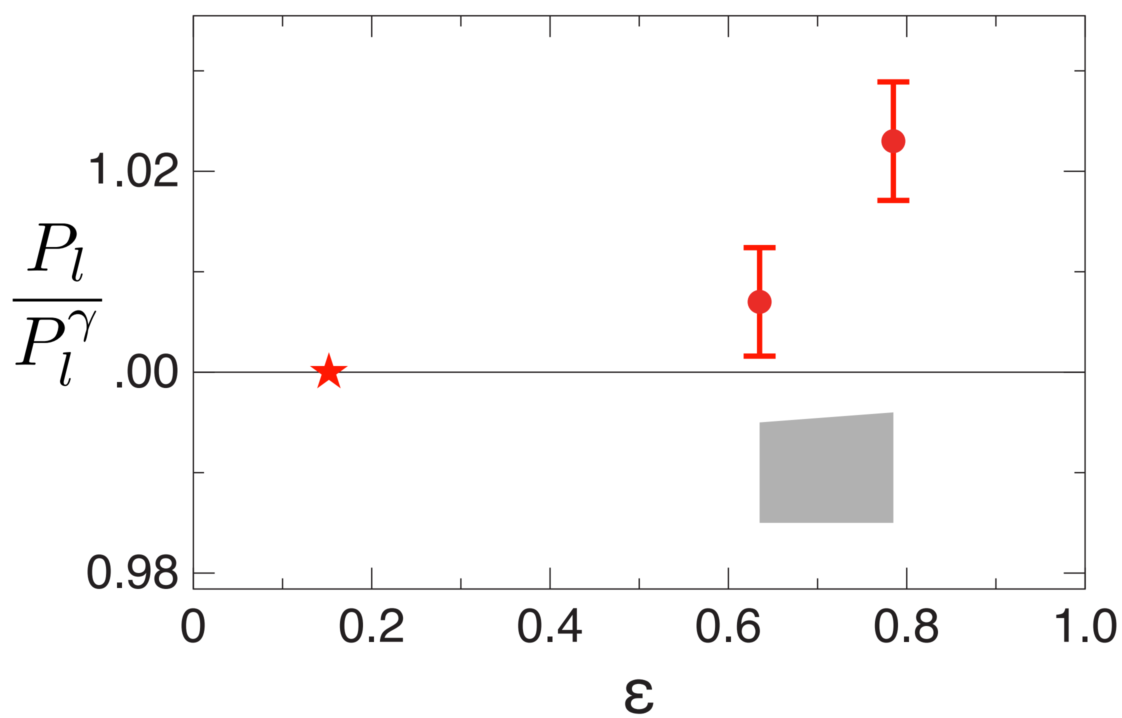
TPE contribution in the polarizations



3 TPE amplitudes $Y_M(\varepsilon, Q), Y_E(\varepsilon, Q), Y_3(\varepsilon, Q)$

$$\frac{P_l}{P_l^\gamma} \simeq 1 - 2\mathcal{R}e\left(\frac{\varepsilon^2 Y_3}{1+\varepsilon} + \varepsilon R/\tau(Y_3 + Y_E)\right) \simeq 1 - 2\mathcal{R}e\left(\frac{\varepsilon^2 Y_3}{1+\varepsilon}\right)$$

$Q^2 = 2.5\text{GeV}^2 \quad R/\tau \approx 0.33$



- $Y_3(\varepsilon) \sim \varepsilon^{2+\omega} \quad \omega \geq 0$

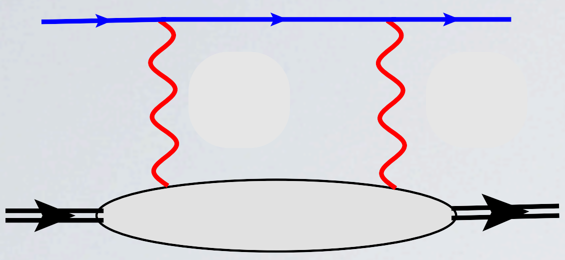
  $\varepsilon \to 0$

- indication about the nonlinear behavior of $Y_3(\varepsilon)$

  $\varepsilon \to 1 \quad Y_3(\varepsilon) \to 0$

  $0.8 < \varepsilon < 1 \quad \Delta Y_3 \simeq 2\%$

TPE contribution in the polarizations



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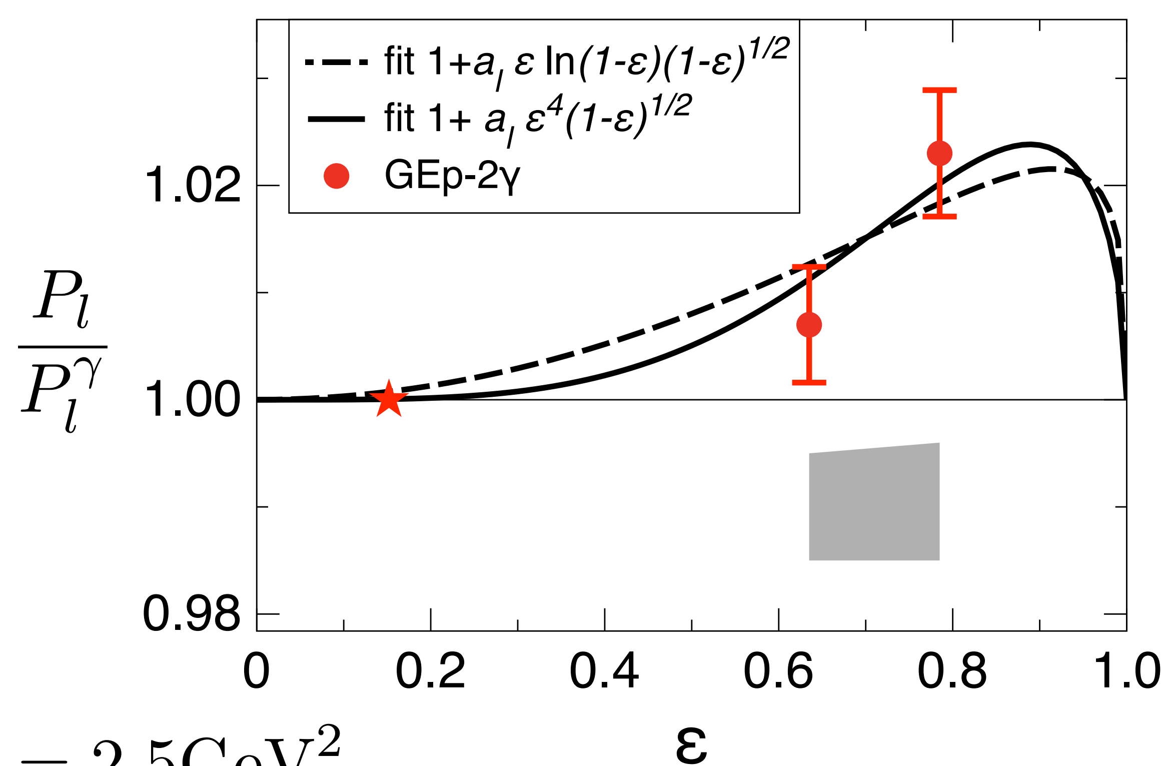
$$\frac{P_l}{P_l^\gamma} \simeq 1 - 2\mathcal{R}e\frac{\varepsilon^2\, Y_3}{1+\varepsilon}$$

- $Y_3(\varepsilon) \sim \varepsilon^{2+\omega} \quad \omega \geq 0$

  $\varepsilon \rightarrow 0$



$Q^2 = 2.5\mathrm{GeV}^2$

model 1 $\quad \frac{P_l}{P_l^\gamma} \simeq 1 + a_l\, \varepsilon^4 (1-\varepsilon)^{1/2}$

with $\quad a_l = 0.11 \pm 0.03$

model 2

$$\frac{P_l}{P_l^\gamma} \simeq 1 + a_l\, \varepsilon \ln(1-\varepsilon)(1-\varepsilon)^{1/2}$$

with $\quad a_l = -0.032 \pm 0.008$

Estimate the TPE amplitudes: small nonlinearities

$$Q^2 = 2.64 \mathrm{GeV}^2$$

1. $\frac{\sigma_r}{G_M^2} \simeq 1 + \varepsilon R^2/\tau + 2\mathcal{R}e\{Y_M + \varepsilon Y_3\}$ = a + εb $0 < \varepsilon < 1$

linear behavior

$$Q^2 = 2.5 \mathrm{GeV}^2$$

2. $-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} \simeq R + \mathcal{R}e(Y_E + Y_3)$ = R $0 < \varepsilon < 1$

$$\mathcal{R}e(Y_M + Y_3) \simeq 0$$

3. $\frac{P_l}{P_l^\gamma} \simeq 1 - 2\mathcal{R}e\frac{\varepsilon^2 Y_3}{1+\varepsilon}$

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unknown $G_M,\ R,\ Y_M(\varepsilon),\ Y_E(\varepsilon),\ Y_3(\varepsilon)$

- model assumptions about the extrapolation to $\varepsilon \to 1$

define G_M and R

Results: $Q^2 = 2.64\mathrm{GeV}^2$

Guttmann, NK,
Meziane, Vanderhaeghen, 2011

$$G_M^2 = (1.168 \pm 0.010)\mu_p^2 G_D^2$$

$$\mu_p R = 0.693 \pm 0.006$$

1. $$\frac{\sigma_r}{G_M^2} \simeq 1 + \varepsilon R^2/\tau + 2\mathcal{R}e\{ Y_M + \varepsilon\, Y_3\}$$

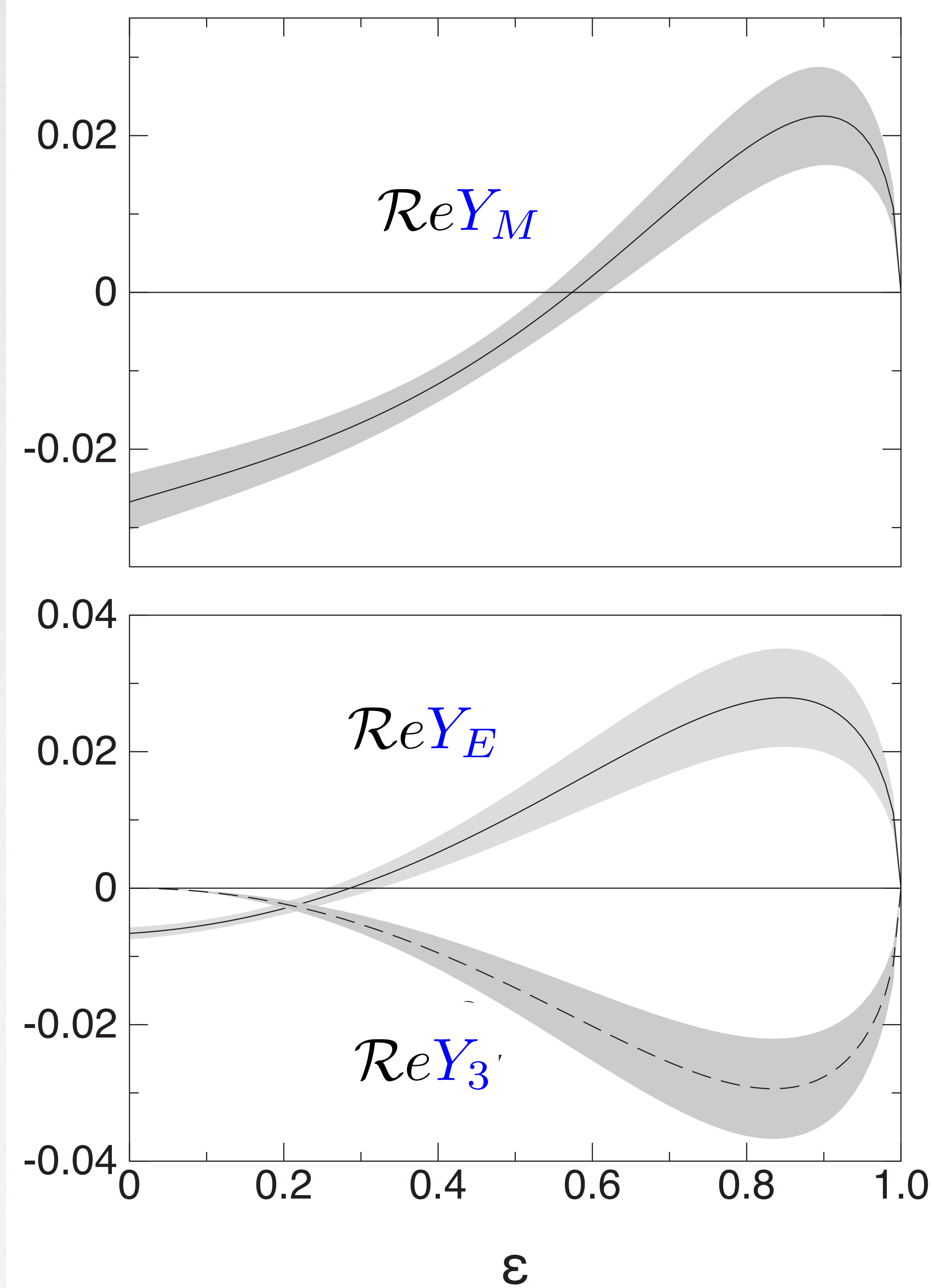
linear fit a + εb

2. $$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} \simeq R + \mathcal{R}e(Y_E + Y_3)$$

$$\mathcal{R}e(Y_M + Y_3) \simeq 0$$

3. $$\frac{P_l}{P_l^\gamma} \simeq 1 + a_l\, \varepsilon^4 (1-\varepsilon)^{1/2}$$

$$\mathcal{R}eY_3 \simeq -\frac{a_l}{2}\varepsilon^2(1-\varepsilon)^{1/2}(1+\varepsilon)$$



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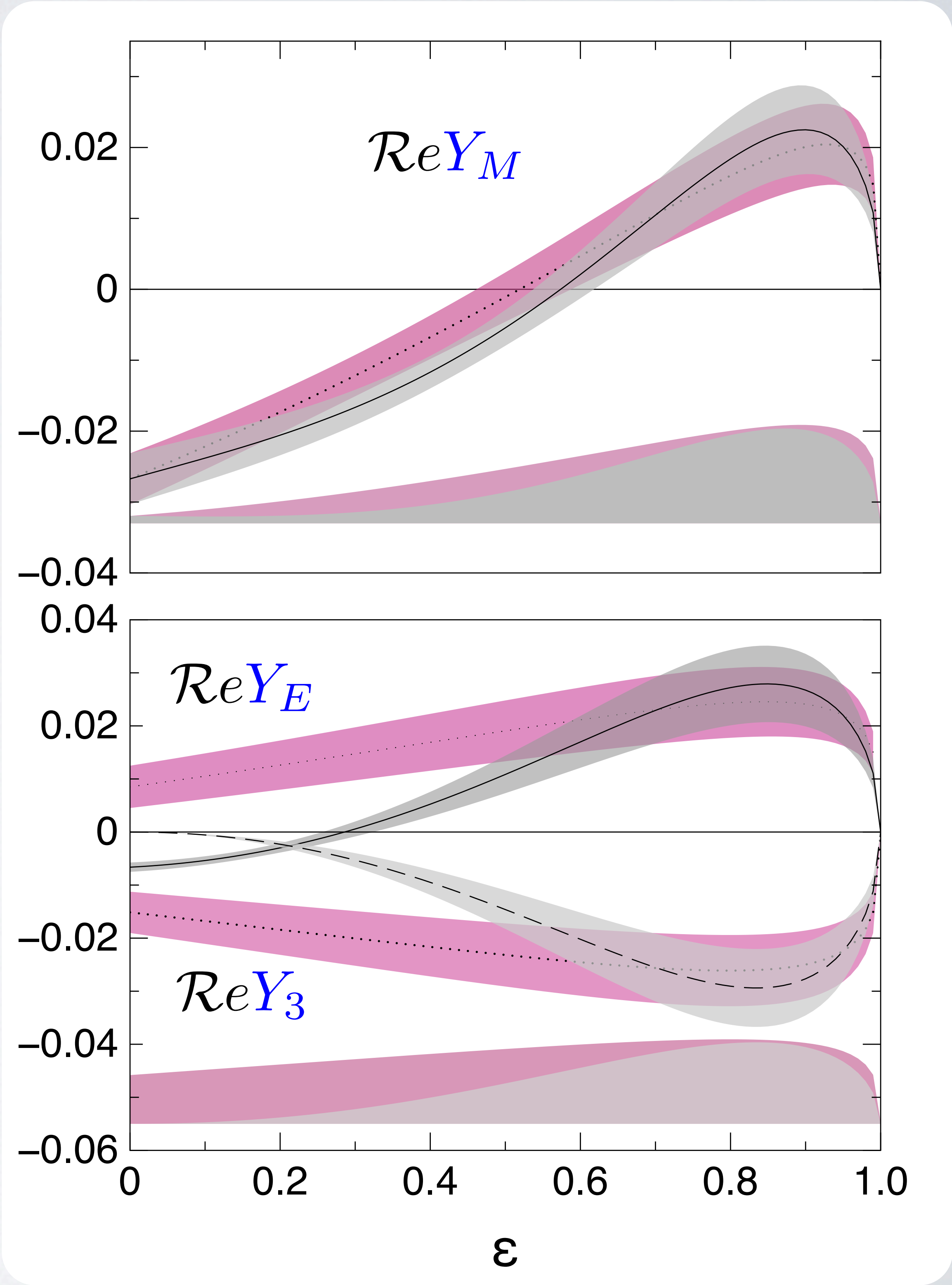
linear fit a + εb

2. $$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\,\frac{P_t}{P_l} \simeq R + \mathcal{R}e(Y_E + Y_3)$$

small TPE $\mathcal{R}e(Y_E + Y_3) \simeq 0$

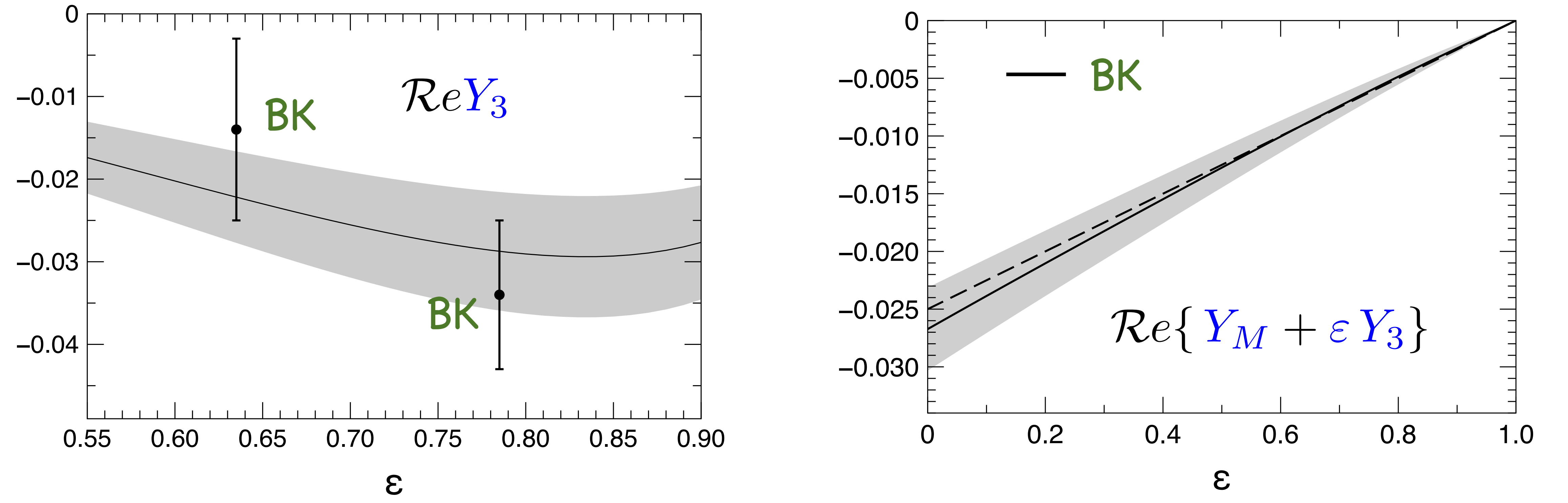
3. $$\frac{P_l}{P_l^\gamma} \simeq 1 + a_l\, \varepsilon \ln(1-\varepsilon)(1-\varepsilon)^{1/2}$$

$$\mathcal{R}eY_3 \simeq -\frac{a_l}{2}\,\frac{\ln(1-\varepsilon)}{\varepsilon}(1-\varepsilon)^{1/2}(1+\varepsilon)$$



Comparison with other results

Borisyuk, Kobushkin 2011



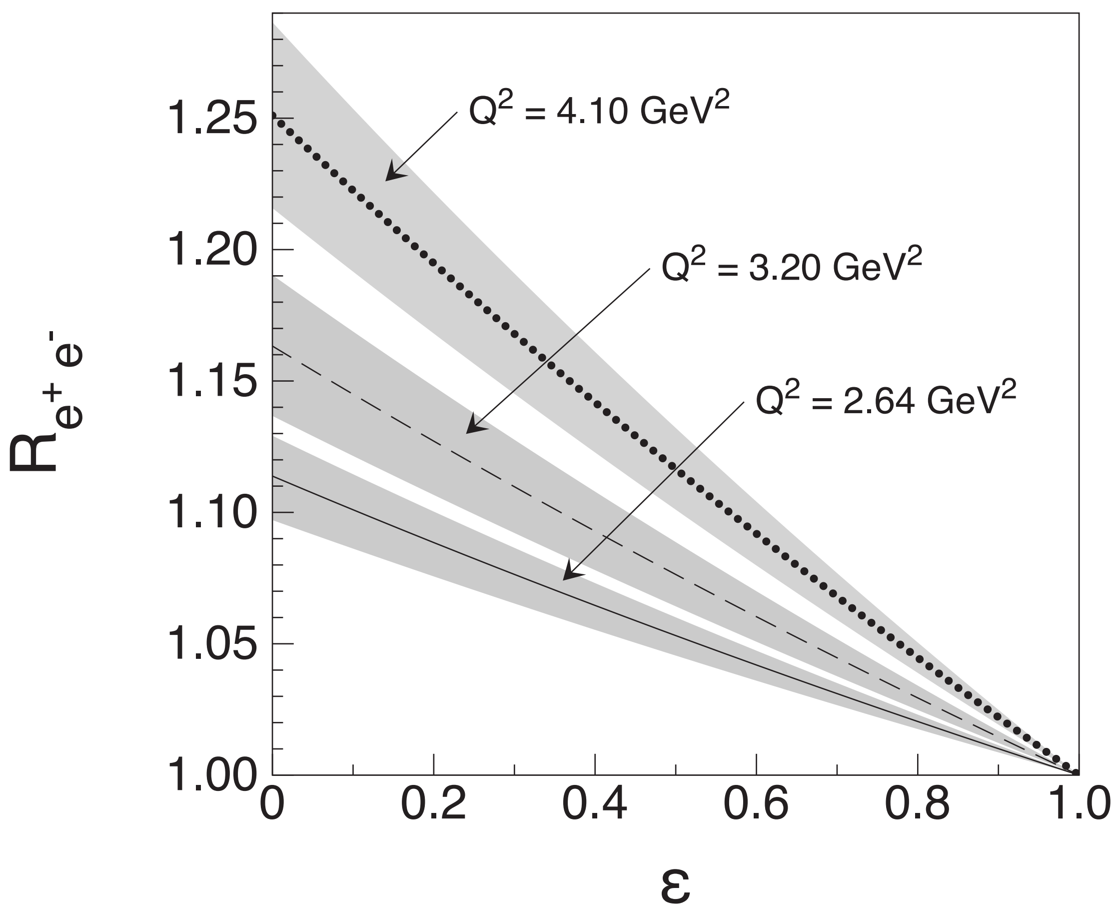
$$\mathcal{R}e\{ Y_M + \varepsilon Y_3 \} = (1-\varepsilon) \left\{ \frac{a+b}{1+R^2/\tau} \frac{R^2}{\tau} - b \right\}$$

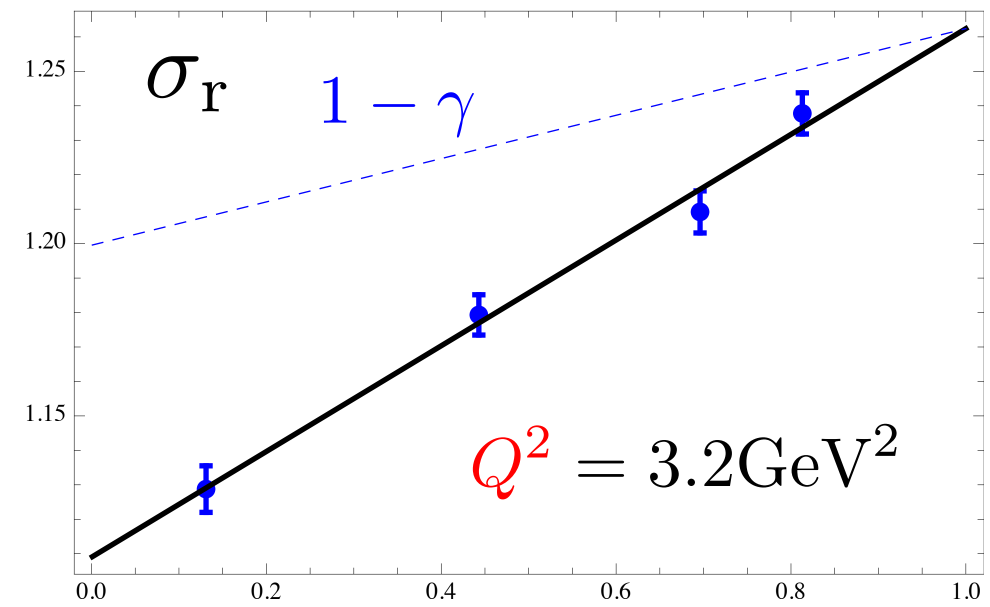
Ratio $R_{\pm}$

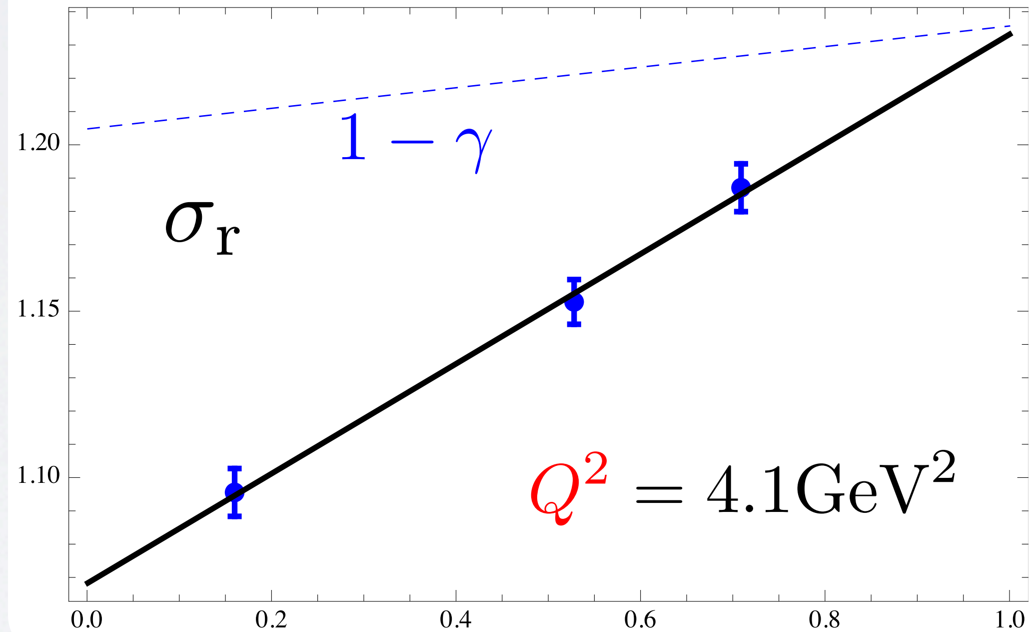
$$\frac{\sigma_r^{\mp}}{G_M^2} \simeq 1 + \varepsilon R^2/\tau \pm 2\mathcal{R}e\{Y_M + \varepsilon Y_3\}$$

$$\mathcal{R}e\{Y_M + \varepsilon Y_3\} = (1-\varepsilon)\left\{\frac{a+b}{1+R^2/\tau}\frac{R^2}{\tau} - b\right\}$$

$$R_{\pm} \simeq 1 - 4\mathcal{R}e\{Y_M + \varepsilon Y_3\}$$

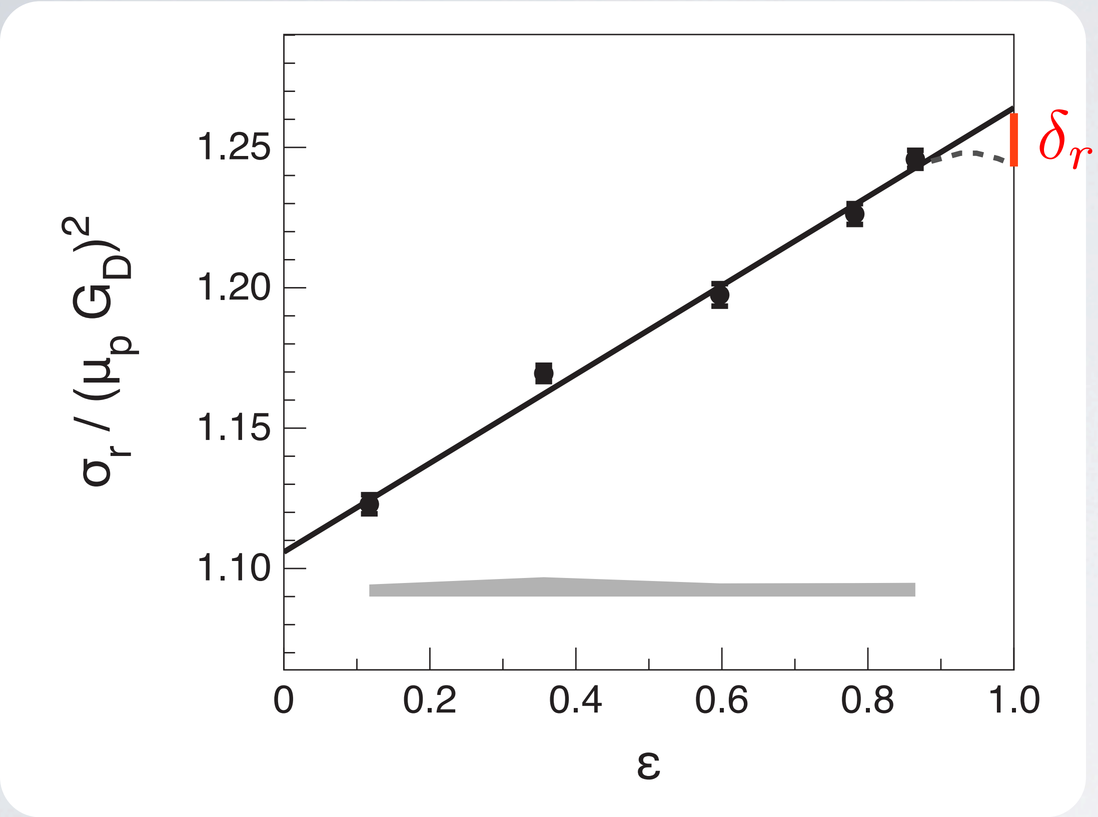


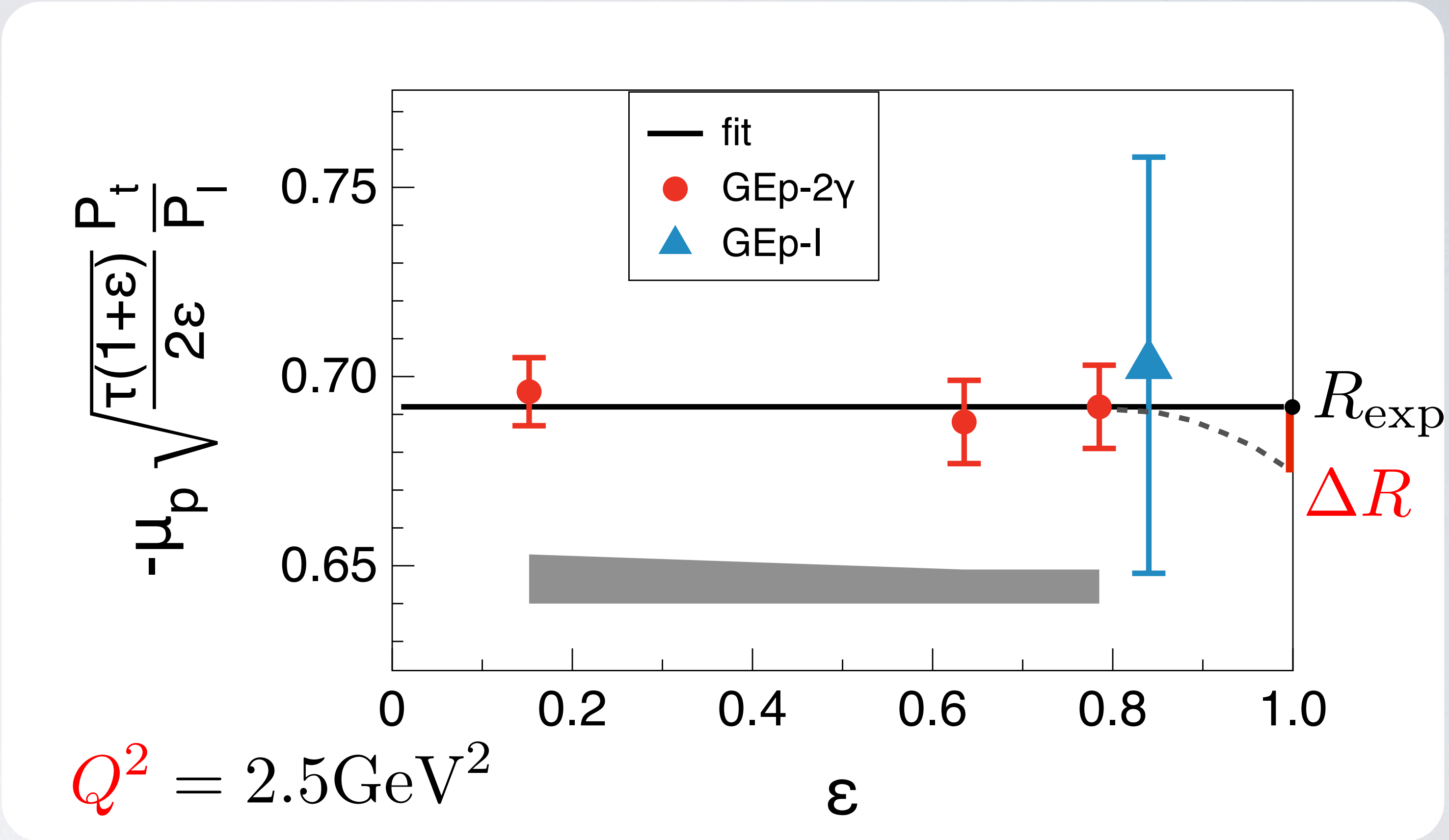




How large could be effects from the nonlinear behavior?

Ratio $R_\pm$

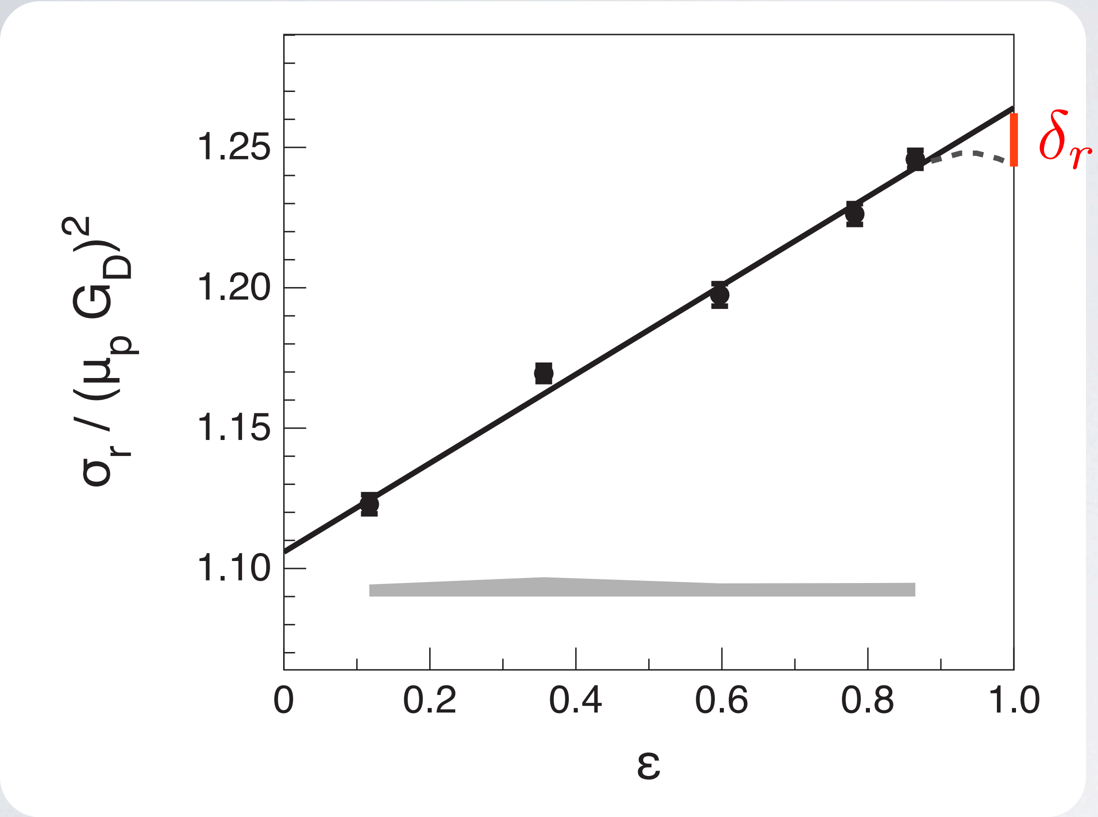


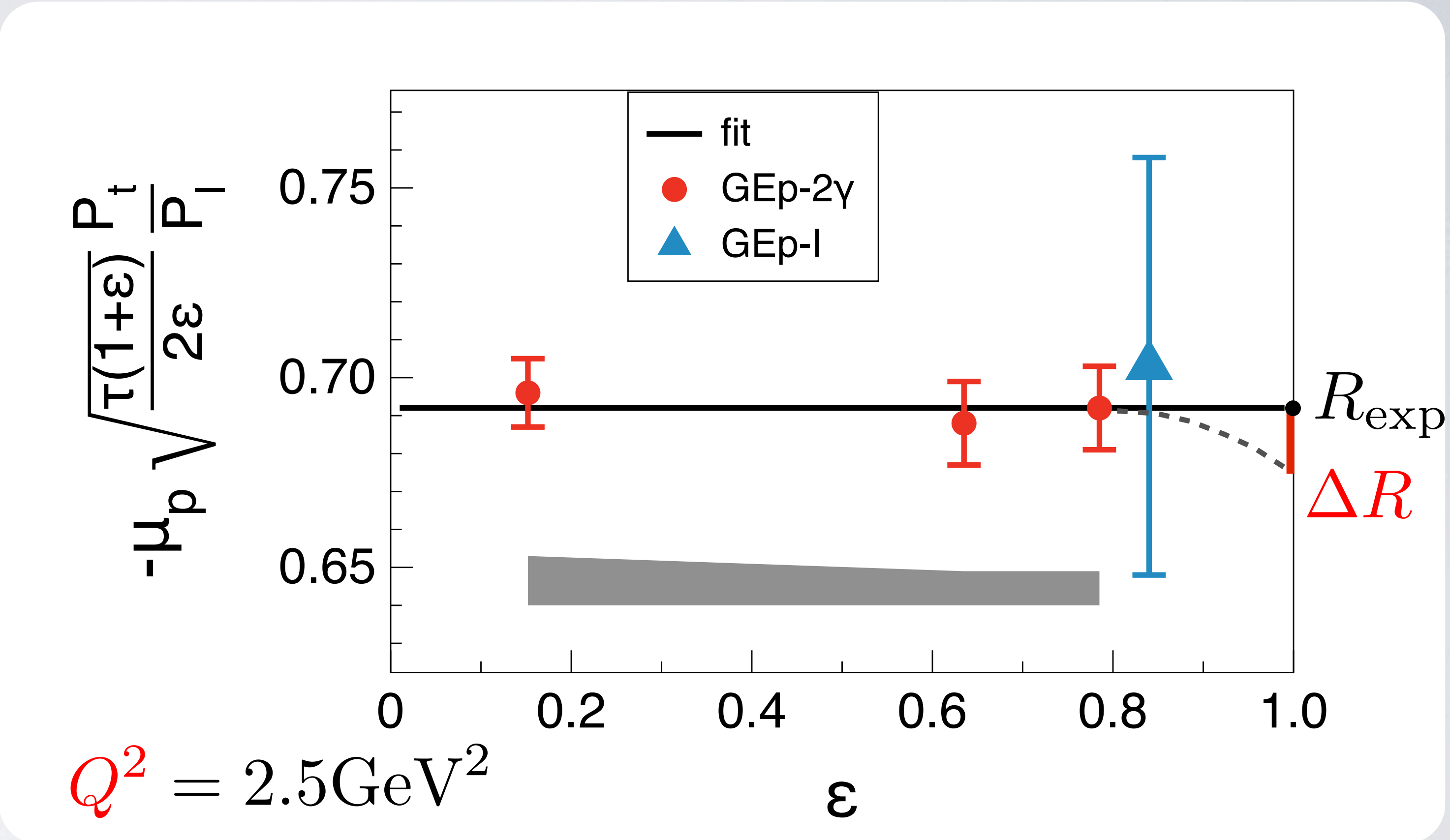


$$\frac{G_M^2(1 + R^2/\tau)}{(\mu_p G_D)^2} = a + b$$

$$\Rightarrow R = R_{\rm exp} - \Delta_R$$

Ratio $R_\pm$

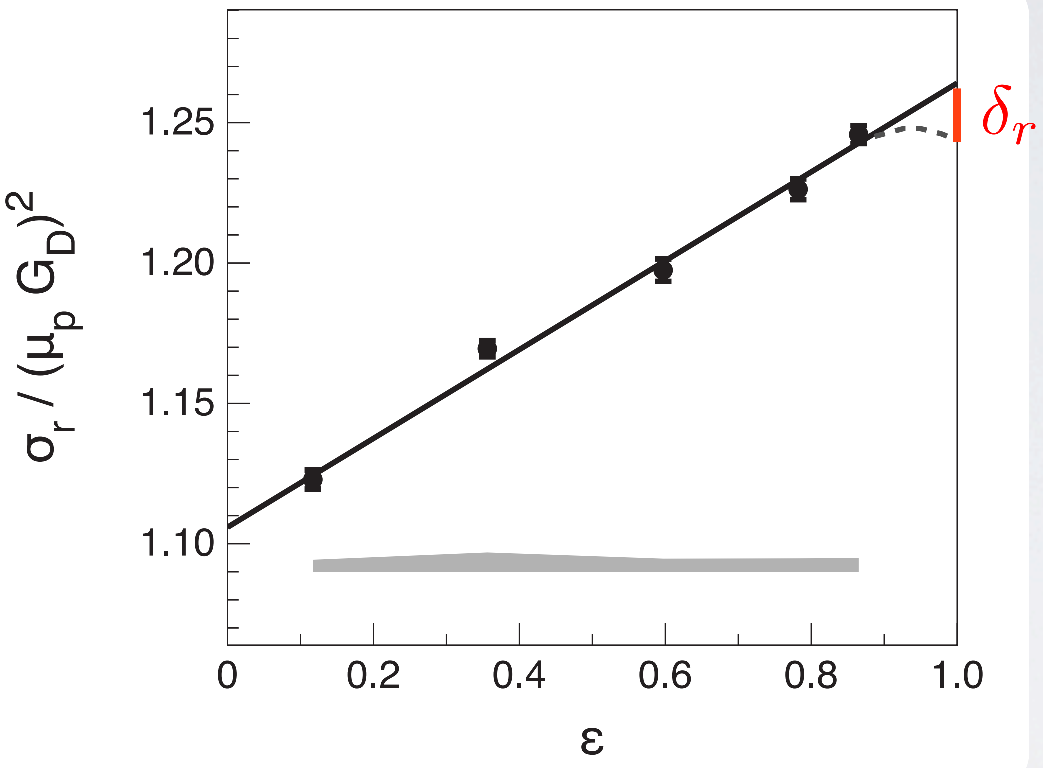




$$\frac{G_M^2(1+R^2/\tau)}{(\mu_p G_D)^2} = a + b \pm \delta_r$$

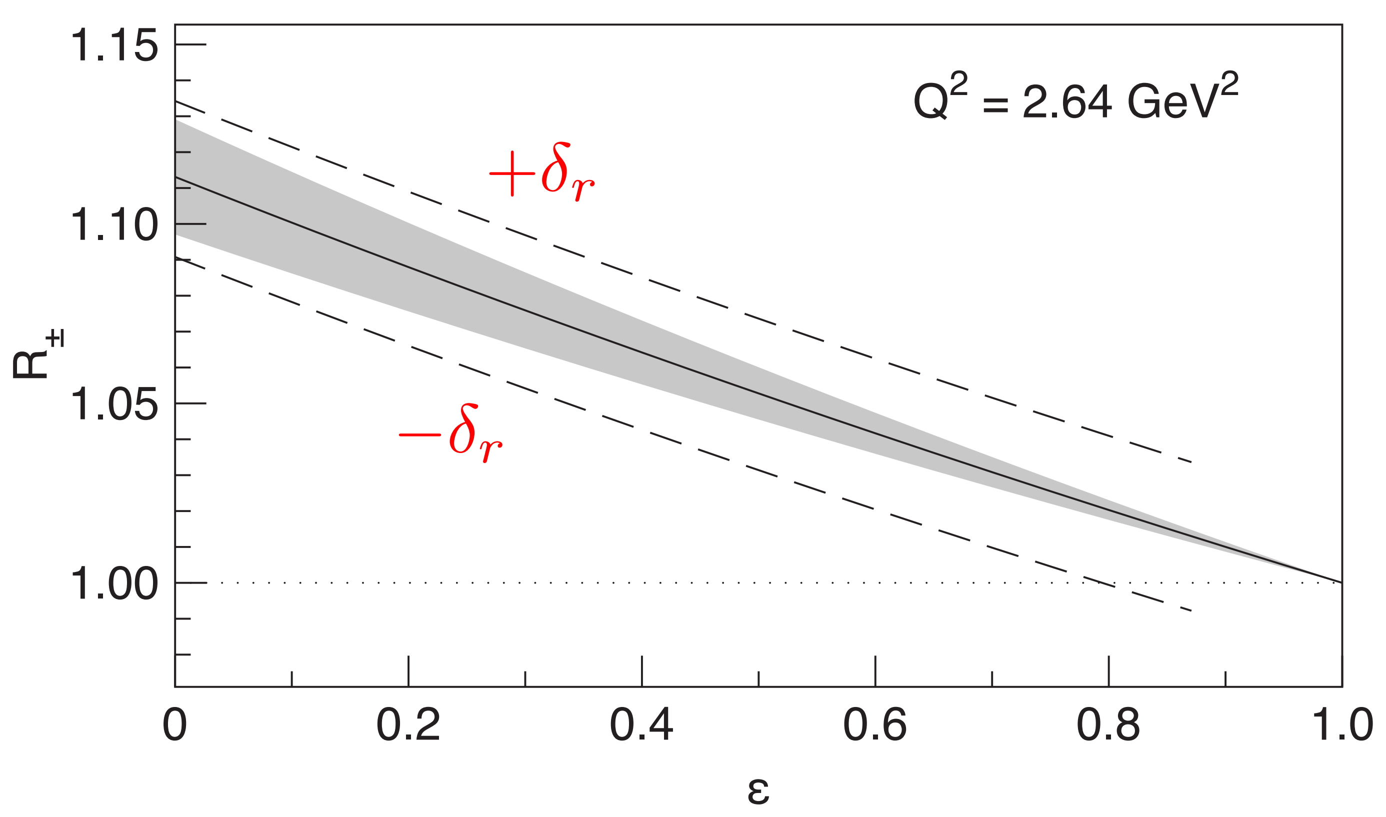
$$\Rightarrow R = R_{\rm exp} - \Delta_R$$

Ratio $R_{\pm}$



$$\frac{G_M^2}{(\mu_p G_D)^2} = \frac{a + b \pm \delta_r}{(1 + R^2/\tau)}$$

$$\delta_r = 0.013 (1\%)$$

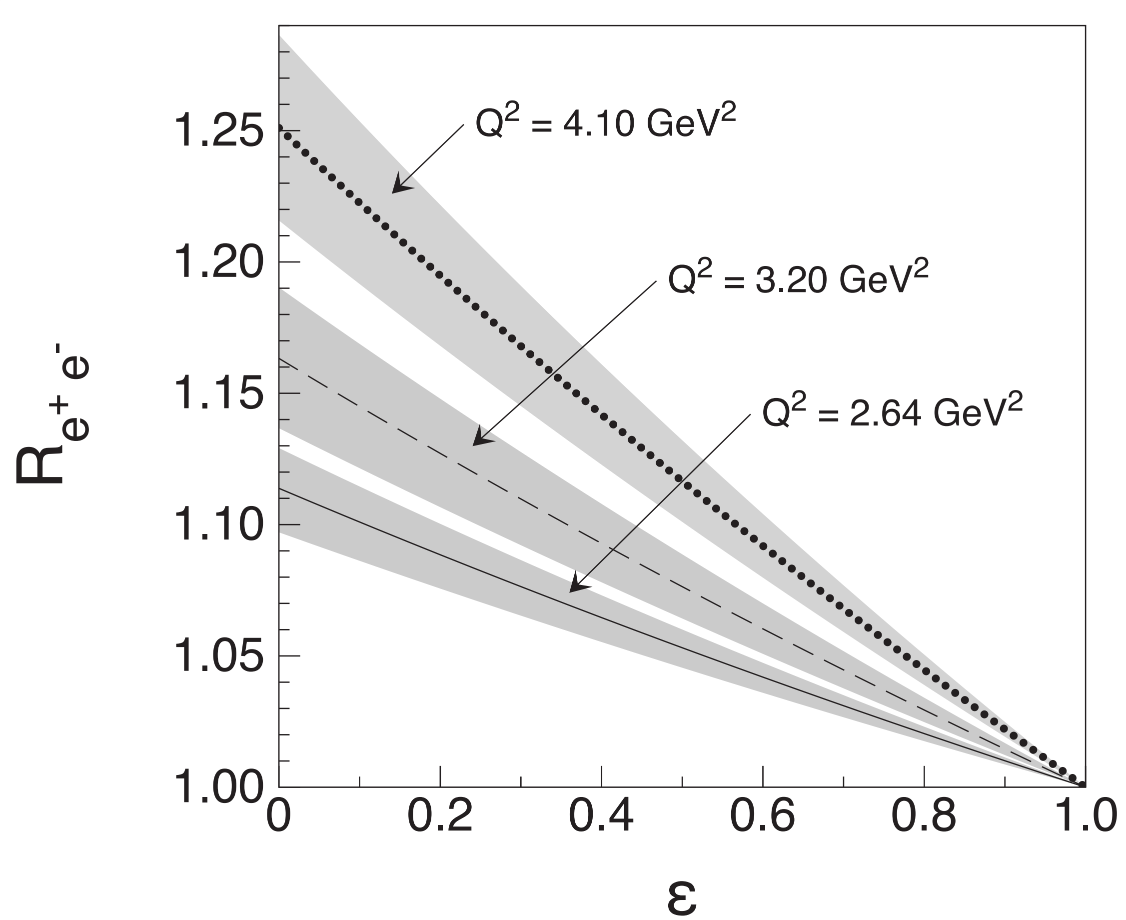


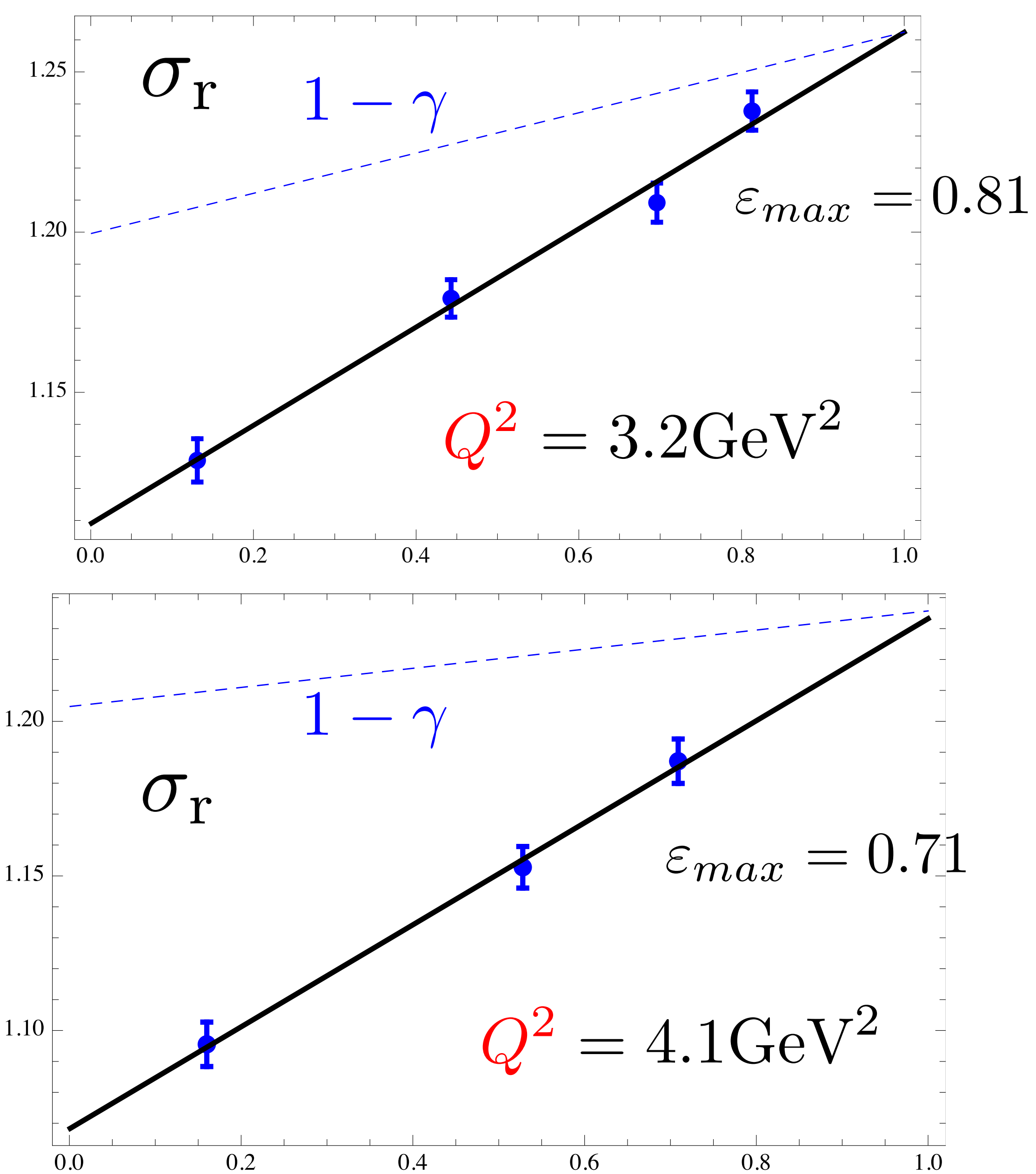
Ratio $R_\pm$

$$\frac{\sigma_r^{\mp}}{G_M^2} \simeq 1 + \varepsilon R^2/\tau \pm 2\mathcal{R}e\{Y_M + \varepsilon Y_3\}$$

$$\mathcal{R}e\{Y_M + \varepsilon Y_3\} = (1-\varepsilon)\left\{\frac{a+b}{1+R^2/\tau}\frac{R^2}{\tau} - b\right\}$$

$$R_\pm \simeq 1 - 4\mathcal{R}e\{Y_M + \varepsilon Y_3\}$$





How large could be effects from the nonlinear behavior?

Conclusions

- *Combining the data* $\sigma_{\rm r},\ P_l,\ P_t,\ R_\pm$

  and assumption that TPE amplitudes vanish at $\varepsilon = 1$

  one can extract information about TPE amplitudes

- *Practical problems: large error bars and ambiguity*

  in the extrapolation to $\varepsilon \to 1$

- *Using JLab data* $\sigma_{\rm r}(Q^2 = 2.64 {\rm GeV}^2, \varepsilon)$ *and* $P_{l,t}(\varepsilon)$ $Q^2 = 2.5 {\rm GeV}^2$

  and assuming small nonlinearity at $\varepsilon \to 1$

$\varepsilon \simeq 0.6 - 0.8$ $\quad \mathcal{R}eY_3 \simeq -(1.5 - 3.6 \pm 0.6_{st} \pm 1_{sys})\%$

$\mathcal{R}eY_E \simeq -\mathcal{R}eY_3 > 0 \quad \mathcal{R}eY_M > 0 \quad \mathcal{R}eY_M \simeq (0 - 2.2)\%$

$\varepsilon = 0 - 0.6$ $\quad \mathcal{R}eY_M < 0 \quad \mathcal{R}eY_M \simeq (-2.7 - 0 \pm 0.8_{st})\%$