Two–Photon Corrections from Dispersion Relations

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CONTENTS

Introduction

WHY DISPERSION RELATIONS ?

- Model-independent approach $\rightarrow$ important non-perturbative tool to analyze data
- Dispersion relations are based on fundamental principles: unitarity & analyticity
- Connect data from small to large momentum transfer
  as well as time- and space-like data
- Allow for a **simultaneous analysis** of all four em form factors
- Spectral functions encode perturbative and non-perturbative physics
  e.g. vector meson couplings, multi-meson continua, pion cloud, ...
- Spectral functions also encode information on the strangeness vector current
  $\rightarrow$ sea-quark dynamics, strange matrix elements
- Allow to extract nucleon electric and magnetic radii
- Can be matched to chiral perturbation theory
- Can be used to extract two–photon corrections $\surd$

Theoretical framework

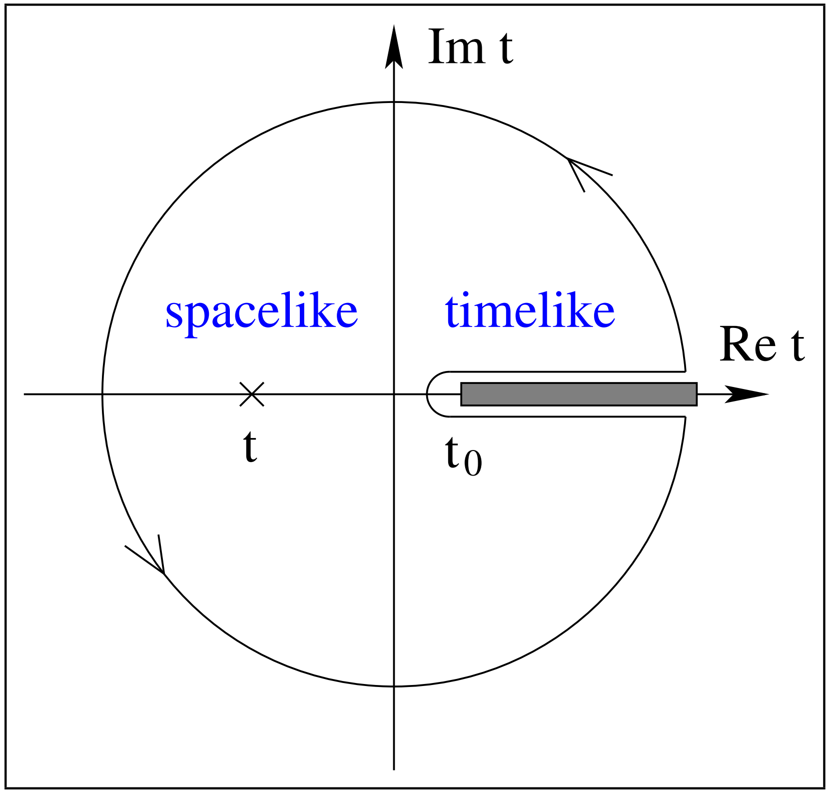
DISPERSION RELATIONS

Federbush, Goldberger, Treiman, Drell, Zachariasen, Frazer, Fulco, Höhler, . . .

- The form factors have cuts in the interval $[t_n, \infty[\ \ (n = 0, 1, 2, \ldots)$ and also poles

$\Rightarrow$ Dispersion relations for $F_i(t)$ $(i = 1, 2)$:

$$F_i(t) = \frac{1}{\pi}\int_{t_0}^{\infty} dt' \, \frac{\text{Im}\, F_i(t')}{t' - t}$$



- no subtractions
  [only proven in perturbation theory]
- suppression of higher mass states
- central objects: spectral functions

$$\boxed{\text{Im}\, F_i(t)}$$

  – cuts $\triangleq$ multi-meson continua
  – poles $\triangleq$ vector mesons

SPECTRAL FUNCTIONS – GENERALITIES

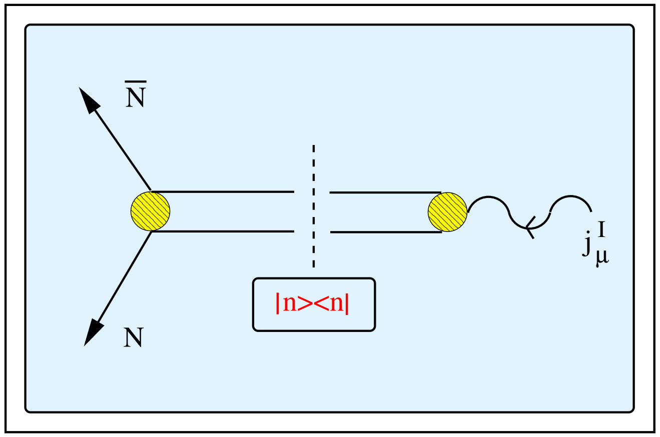
• Spectral decomposition:

$$\mathbf{Im}\,\langle \bar{N}(p')N(p)|J_\mu^I|0\rangle \sim \sum_n \langle \bar{N}(p')N(p)|n\rangle\langle n|J_\mu^I|0\rangle \Rightarrow \mathbf{Im}\,F$$

⋆ on-shell intermediate states

⋆ generates imaginary part

⋆ accessible physical states



• *Isoscalar* intermediate states: $3\pi, 5\pi, \ldots, K\bar{K}, K\bar{K}\pi, \pi\rho, \ldots +$ poles $\longrightarrow t_0 = 9M_\pi^2$

• *Isovector* intermediate states: $2\pi, 4\pi, \ldots +$ poles $\longrightarrow t_0 = 4M_\pi^2$

• Note that some poles are *generated* from the appropriate continua

ISOVECTOR SPECTRAL FUNCTIONS

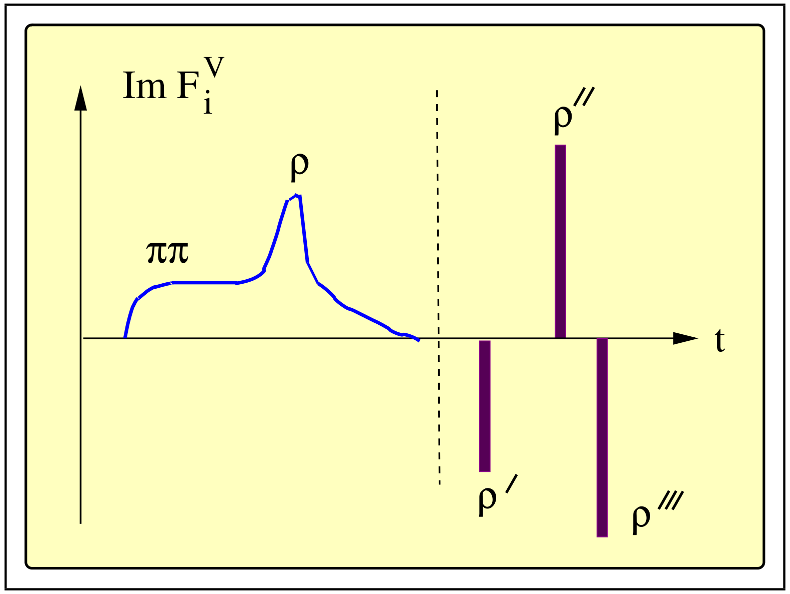
Frazer, Fulco, Höhler, Pietarinen, . . .

- exact 2π continuum is known from threshold $t_0 = 4M_\pi^2$ to $t \simeq 40\, M_\pi^2$

$$\mathrm{Im}\, F_i^V(t) = \frac{q_t^3}{\sqrt{t}} |F_\pi(t)|^2 J_i(t)$$

  ⋆ $F_\pi(t)$ = pion vector form factor

  ⋆ $J_i \sim$ P-wave pion-nucleon partial waves in the t-channel



- Spectral functions inherit singularity on the second Riemann sheet in $\pi N \to \pi N$

$$t_c = 4M_\pi^2 - M_\pi^4/m^2 \simeq 3.98\, M_\pi^2$$

$\to$ strong shoulder $\to$ isovector radii

- This singularity can also be analyzed in CHPT   Bernard, Kaiser, M, Nucl. Phys. A **611** (1996) 429

- For a recent determination of the 2π continuum, see BHM, PLB **633** (2006) 507

- Higher mass states represented by poles (not necessarily physical masses)

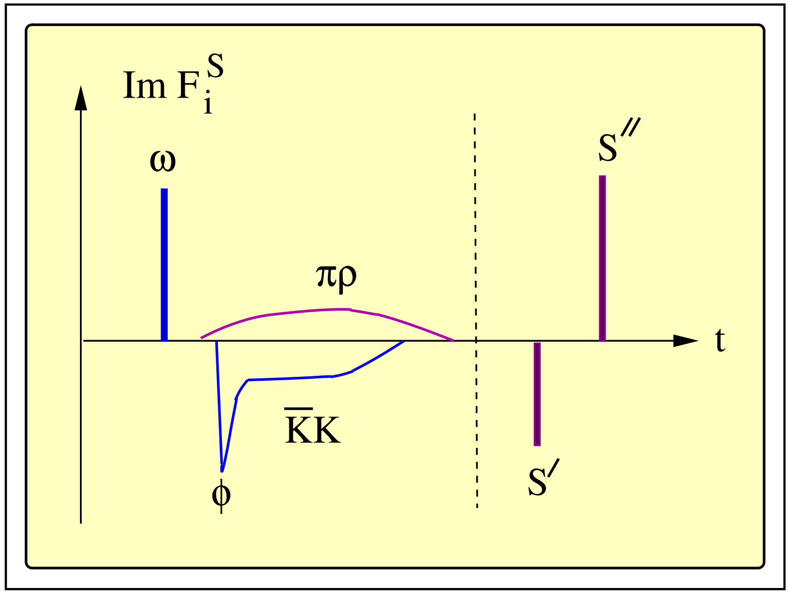
ISOSCALAR SPECTRAL FUNCTIONS

- $K\bar{K}$ continuum can be extracted from analytically cont. KN scattering amplitudes

  $\longrightarrow$ analytic continuation must be stabilized

  $\longrightarrow$ generates most of the ϕ contribution

Hammer, Ramsey-Musolf, Phys. Rev. C **60** (1999) 045204, 045205



- Further strength in the ϕ-region generated by correlated $\pi\rho$ exchange

  $\longrightarrow$ strong cancellations ($K\bar{K}, K^*K, \pi\rho$)

  $\longrightarrow$ takes away sizeable strength from the ϕ

M., Mull, Speth, van Orden, Phys. Lett. B **408** (1997) 381

- Spectral functions exhibit anomalous threshold (analyzed in 2-loop CHPT)

$$t_c = M_\pi^2 \left(\sqrt{4 - M_\pi^2/m^2} + \sqrt{1 - M_\pi^2/m^2}\right)^2 \simeq 8.9\, M_\pi^2$$

$\longrightarrow$ effectively masked

Bernard, Kaiser, M, Nucl. Phys. A **611** (1996) 429

- Higher mass states represented by poles (with a finite width)

CONSTRAINTS ON THE SPECTRAL FUNCTIONS

- Normalizations: electric charges, magnetic moments
- Superconvergence relations ≅ leading pQCD behaviour

$$\boxed{F_1(t) \sim 1/t^2 \, , F_2(t) \sim 1/t^3 \;\; (\text{helicity} - \text{flip})}$$

Brodsky et al.

$$\Rightarrow \boxed{\int_{t_0}^{\infty} \text{Im}\, F_1(t)\, dt = 0 \, , \quad \int_{t_0}^{\infty} \text{Im}\, F_2(t)\, dt = \int_{t_0}^{\infty} \text{Im}\, F_2(t)\, t\, dt = 0}$$

- Two ways of implementing the aymptotic QCD behaviour
  - SC relations alone, add broad resonance to generate imag part for $t \geq 4m^2$
  - Explicit pQCD term in addition to SC relations (smoother interpolation)

$$F_i^{(I,\text{pQCD})} = \frac{a_i^I}{1 - c_i^2\, t + b_i^2\, (-t)^{i+1}} \quad i = 1, 2\, , \quad I = S, V$$

Results

Belushkin, Hammer, M., Phys. Rev. **C 75** (2007) 035202 [hep-ph/0608337]

GENERAL COMMENTS ON THE FITS

- large MC sampling for initial values, successive improvement by pole reduction, new MCs, ...
- theoretical uncertainty (error bands) from $\chi^2_{\rm min} + 1.04$ [1-σ devs.]

→ first time: dispersive analysis w/ error bars !

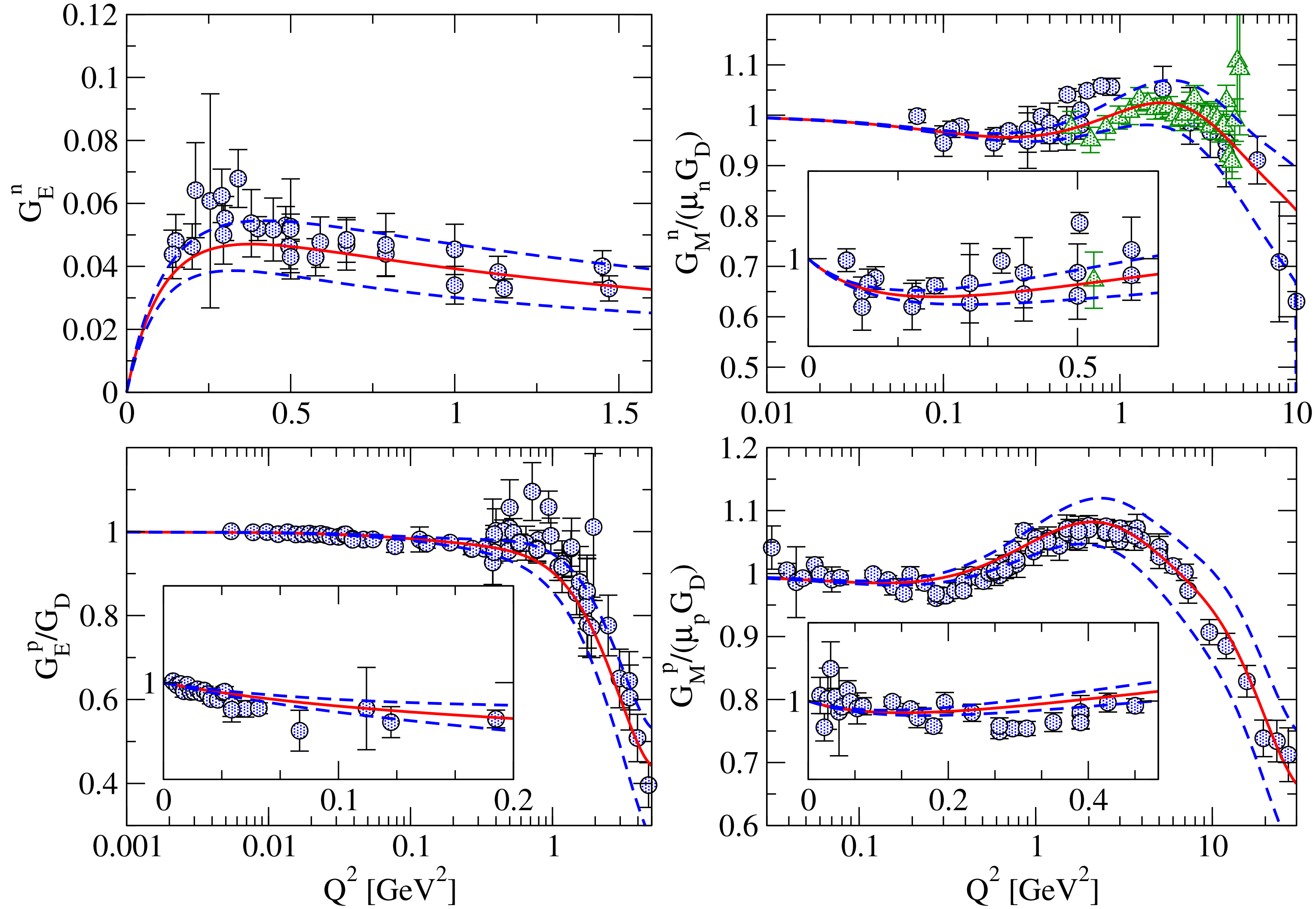
	this work	recent determinations
r_E^p [fm]	0.844 (0.840...0.852)	0.875(8), 0.84184(57) [1,2]
r_M^p [fm]	0.854 (0.849...0.859)	0.855(35) [3] (0.777(17) [2])
$(r_E^n)^2$ [fm^2]	−0.117 (−0.11...−0.128)	−0.115(4) [4]
r_M^n [fm]	0.862 (0.854...0.871)	0.873(11) [5]

[1] Bernauer et al., Phys. Rev. Lett. **105** (2010) 242001
[2] Pohl et al., Nature **466** (2010) 231
[3] Sick, Phys. Lett. B **576** (2003) 62
[4] Kopecky et al., Phys. Rev. C **56** (1997) 2229
[5] Kubon et al., Phys. Lett. B **524** (2002) 26

⋆ Magnetic radii in good agreement with recent determinations (except Mainz)

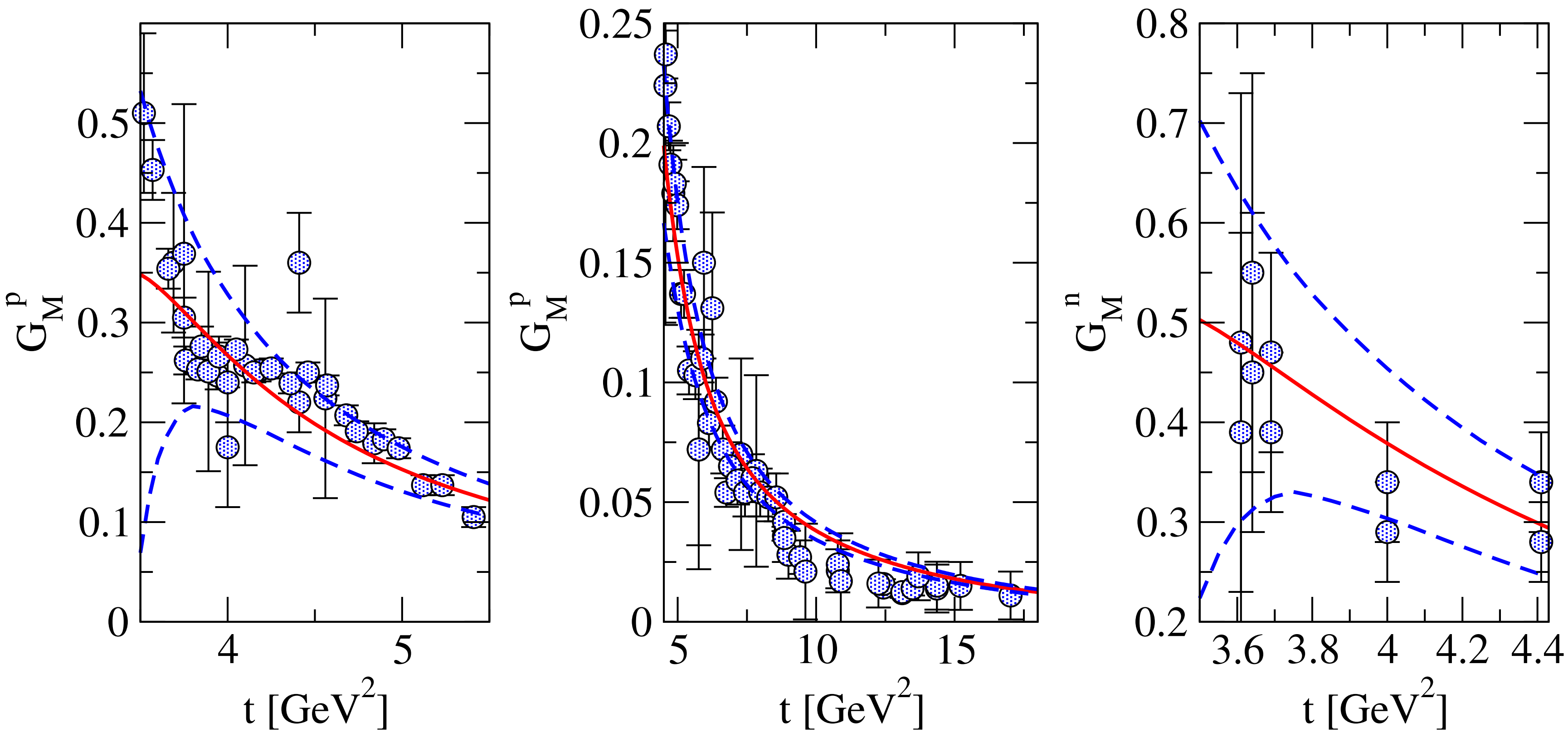
⋆ Proton electric radius comes out $\lesssim$ 0.855 fm (consistent with PSI)

SPACE-LIKE FORM FACTORS



→ excellent representation of the data

TIME-LIKE FORM FACTORS



- Only proton data participate in the fits
- All data within one sigma – first time consistent fit w/ space-like ffs

$\Rightarrow$ Need more data on time-like G_M^n

Two-photon corrections

Belushkin, Hammer, M., Phys. Lett. **B 658** (2008) 138 [arXiv:0705.3385 [hep-ph]]

INTRO: TWO-PHOTON CORRECTIONS

- Discrepancy between Rosenbluth and polarization transfer (PT) data

  $\Rightarrow$ two-photon exchange effects

- Direct (model-dependent) calculations

  $\Rightarrow$ right direction, effect too small

[1] Blunden et al., Phys. Rev. Lett. **91** (2003) 142304
[2] Blunden et al., Phys. Rev. C **72** (2005) 034612
[3] Kondryatuk et al., Phys. Rev. Lett. **95** (2005) 172503
[4] Chen et al., Phys. Rev. Lett. **93** (2004) 122301
[5] Afanasev et al., Phys. Rev. D **72** (2005) 013008

- Model-independent extraction from the data?

- Assumption: no significant two-photon effects in PT data

$\Rightarrow$ Estimate hard 2γ corrections from comparison of our previous analysis (mainly PT data) and direct analysis of Rosenbluth cross section for the proton (including Coulomb corrections $\rightarrow$ soft 2γ corrections)

ANALYSIS of TWO-PHOTON CORRECTIONS

- Hybrid analysis: FF data for the neutron, cross sections for the proton
- Easiest to compare at cross section level

$\Rightarrow$ reconstruct “PT cross section” from FF data (A = SC, pQCD)

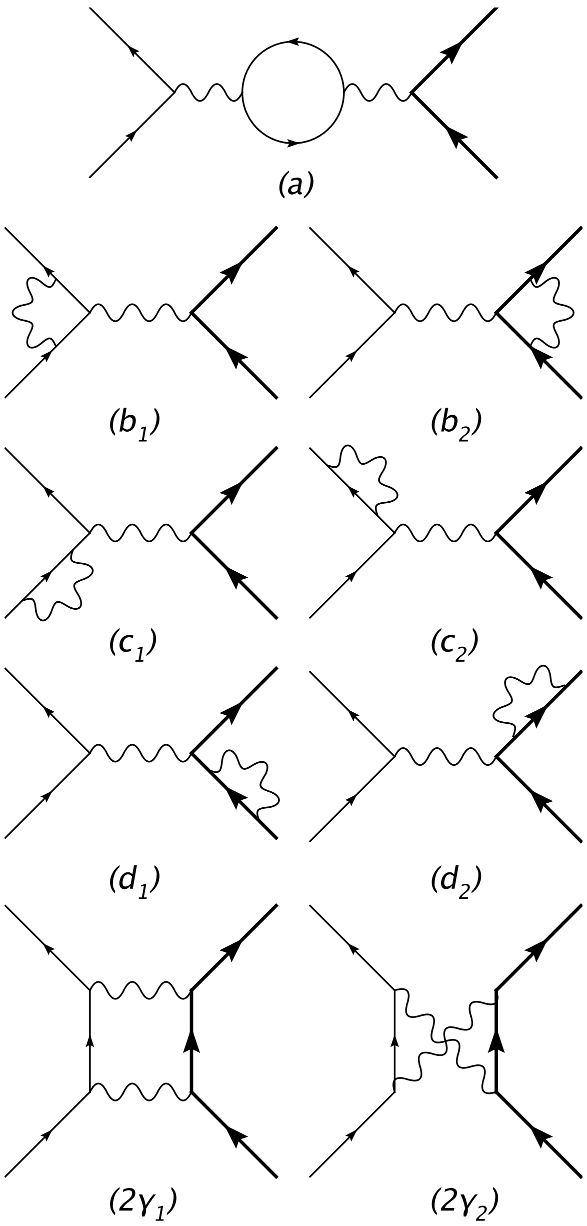
$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ros,A}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{A}} (1 + \delta^{2\gamma})$$

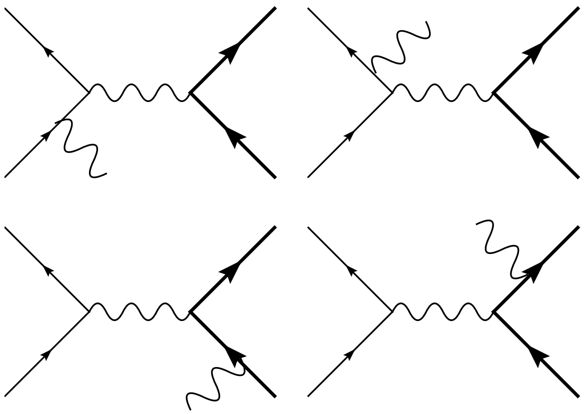
- Comparison with direct calculation (Blunden et al.)

$\rightarrow$ add in Coulomb correction: $\boxed{\Delta^{2\gamma} = \delta^{2\gamma} + \delta^{C}}$

[Coulomb corrections from Arrington and Sick, Phys. Rev. C **70** (2004) 028203]

TWO-PHOTON CORRECTIONS: DIAGRAMS



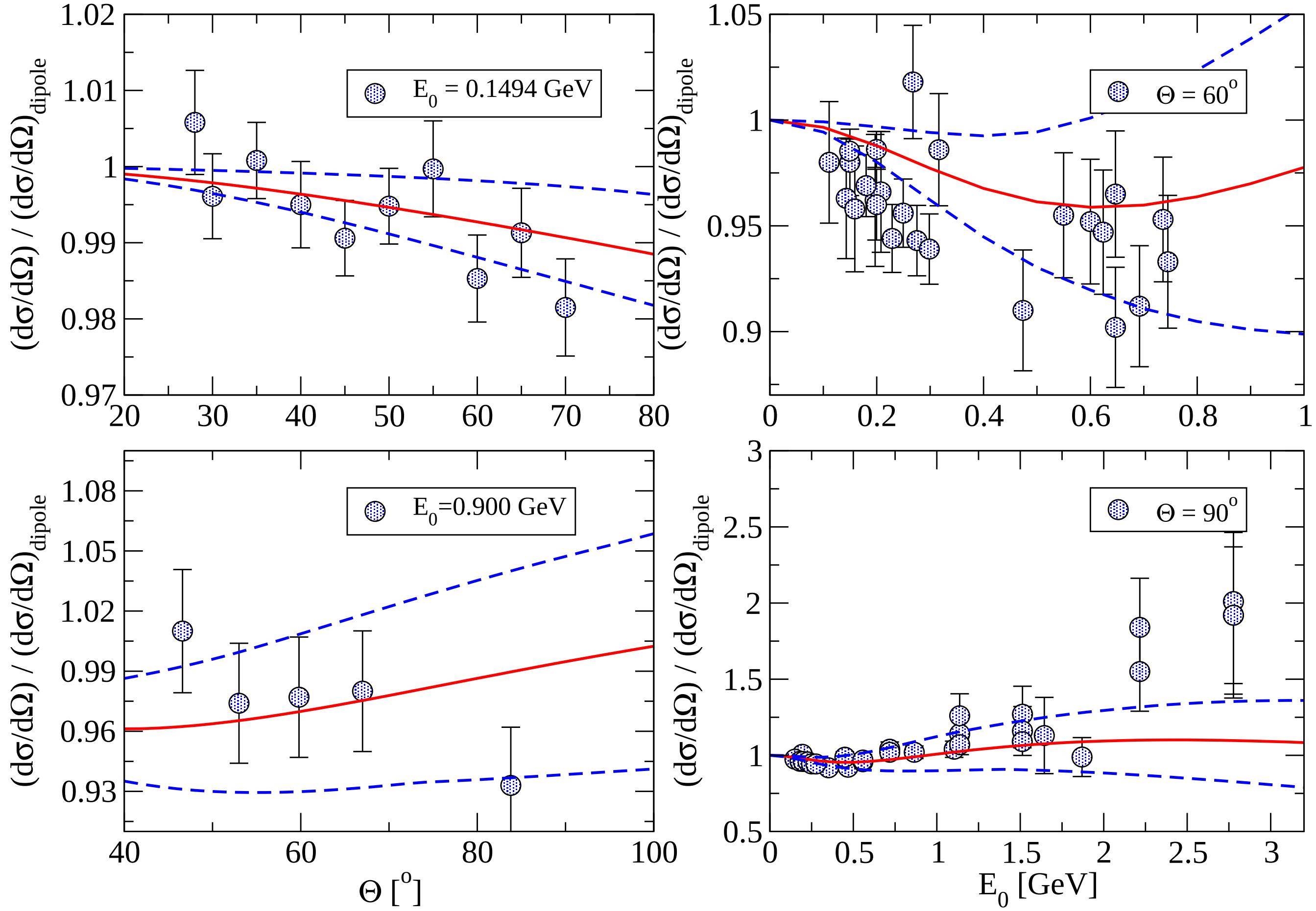


The music plays here →
soft & hard contributions

CROSS SECTION ANALYSIS

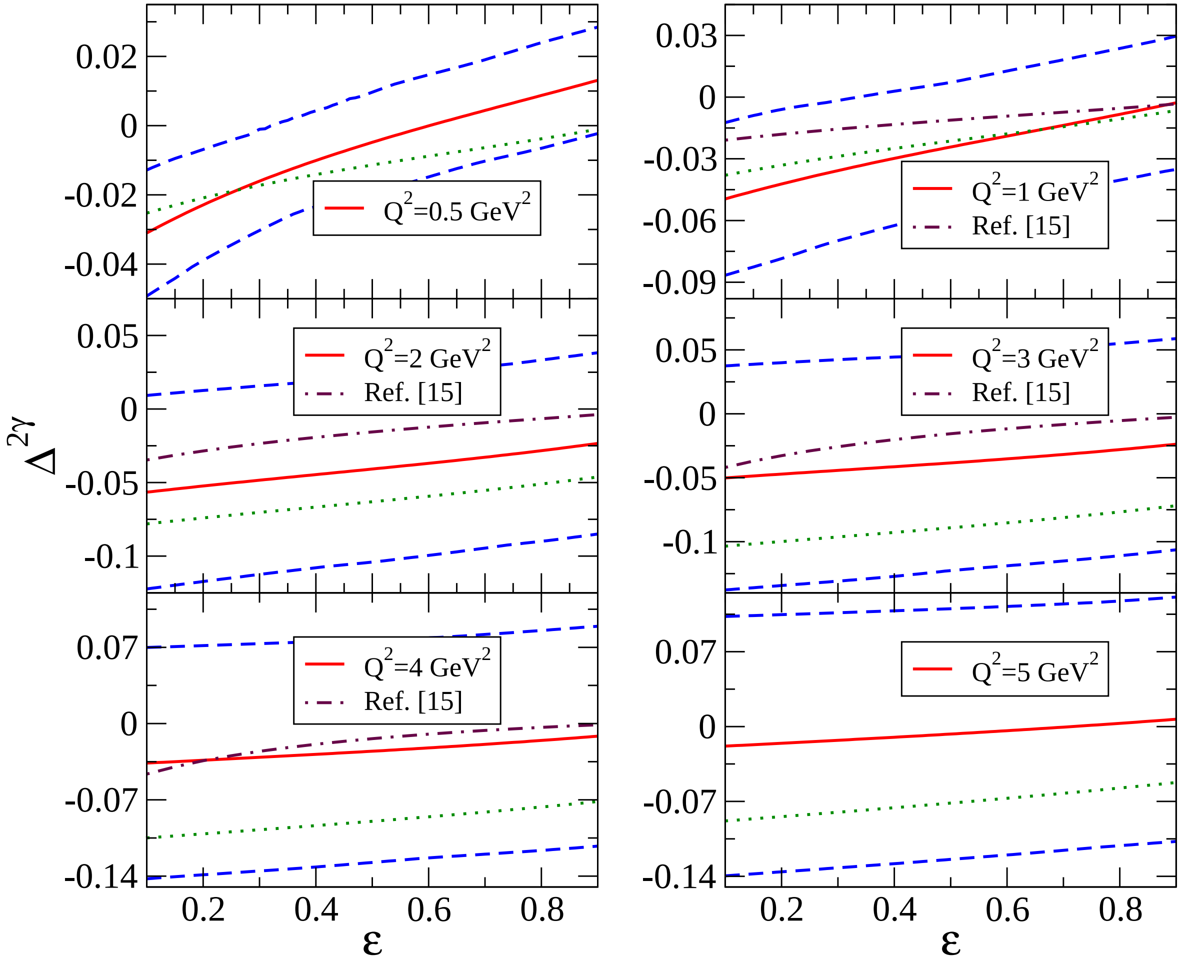
- Example: cross section analysis in SC approach

fixed scattering angle   fixed electron energy



TWO–PHOTON CORRECTIONS

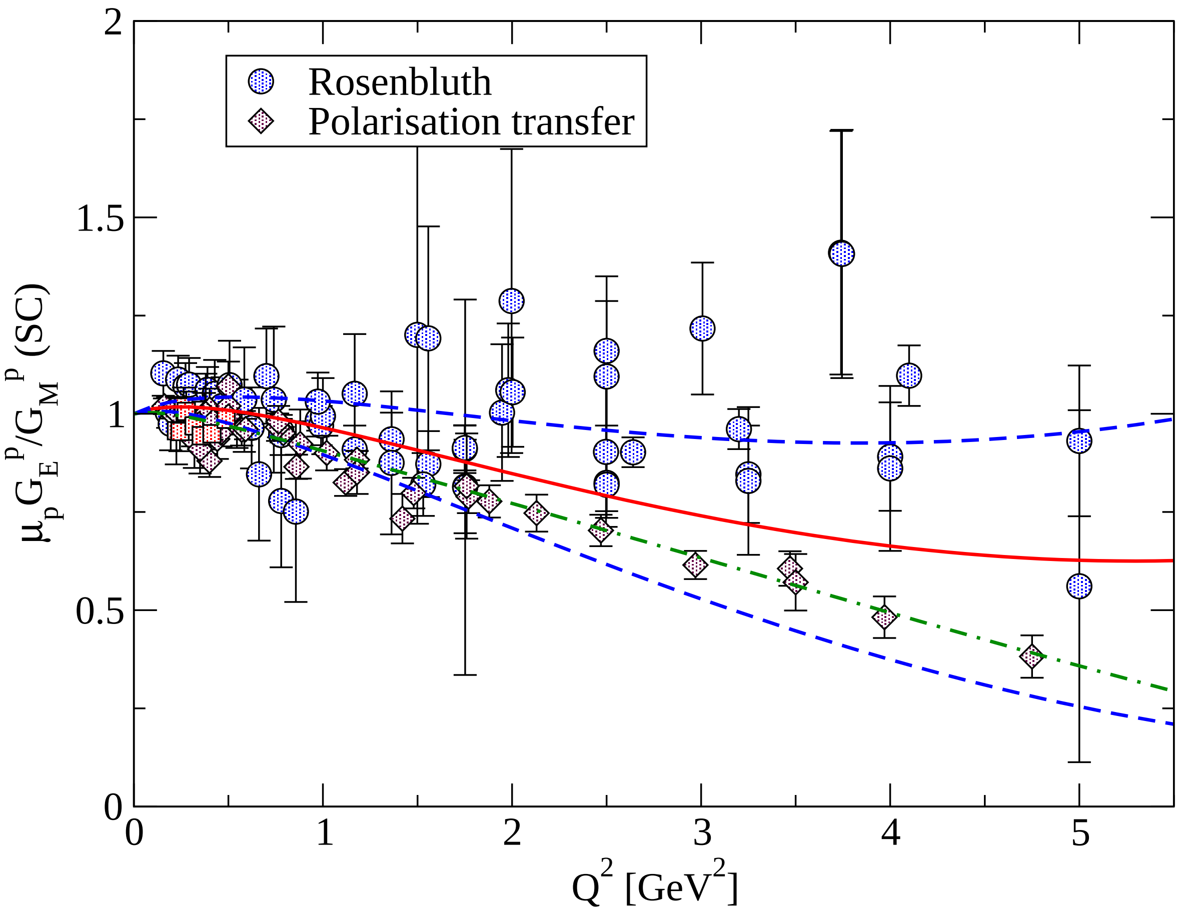
- $\Delta^{2\gamma} = \delta^{2\gamma} + \delta^{C}$
- ——— SC
- ...... pQCD
- -.-.-.- Blunden et al.



⇒ good agreement w/ existing calcs where applicable

COMPARISON w/ FORM FACTOR RATIOS

- FF ratio from X sec analysis (SC) compared to PT data



- consistent within error bands
- form factor data not included in the analysis

SUMMARY & OUTLOOK

- Dispersion relations: powerful tool to analyse the nucleon em form factors
- Dispersive representation can also be used to compare XS w/ PT data
  - model-independent extraction of two-photon corrections

  $\rightarrow$ discrepancy between Rosenbluth and PT data resolved
- Still much to be done, e.g.
  - two-photon effects – fit also to n cross sections & PT data
  - analyse the new Mainz data (as their analysis is flawed)
  - to which precision can the proton charge radius really be deduced?

SPARES

BASIC DEFINITIONS

- Nucleon matrix elements of the em vector current J^I_μ

$$\langle N(p')|J^I_\mu|N(p)\rangle = \bar{u}(p')\left[F^I_1(t)\,\gamma_\mu + i\,\frac{F^I_2(t)}{2m}\,\sigma_{\mu\nu}q^\nu\right]u(p)$$

⋆ isospin $I = S, V$ (isoScalar, isoVector)

⋆ four-momentum transfer $t \equiv q^2 = (p'-p)^2 \equiv -Q^2$

⋆ F_1 = Dirac form factor, F_2 = Pauli form factor

⋆ Normalizations: $F^V_1(0) = F^S_1(0) = 1/2$, $F^{S,V}_2(0) = (\kappa_p \pm \kappa_n)/2$

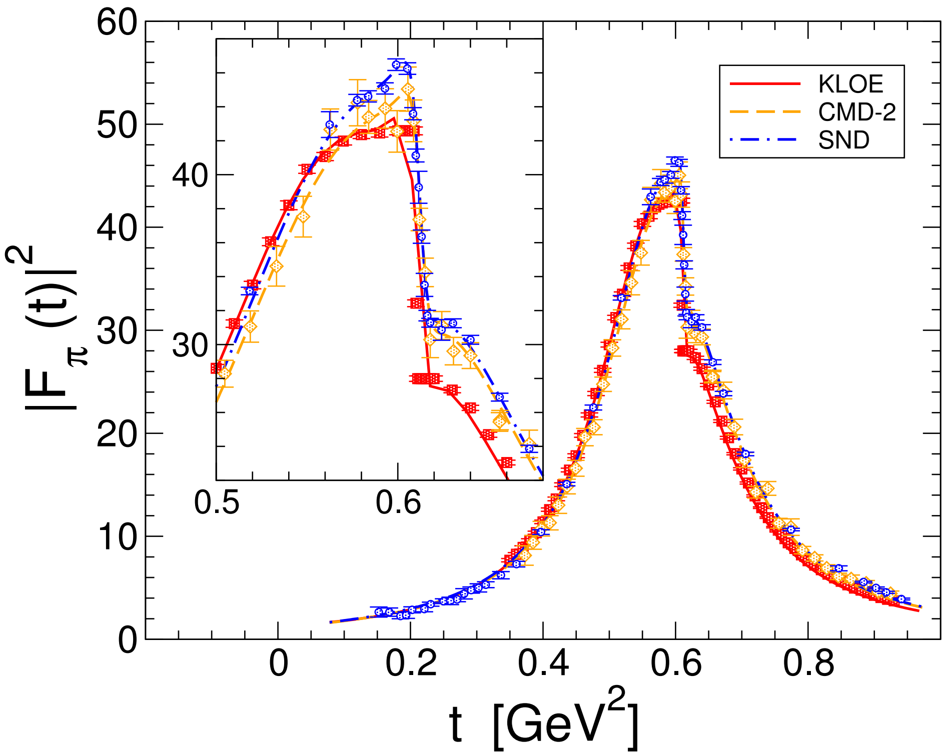
⋆ Sachs form factors: $G_E = F_1 + \frac{t}{4m^2}F_2\ ,\ \ G_M = F_1 + F_2$

⋆ Nucleon radii: $F(t) = F(0)\ \left[1 + t\langle r^2\rangle/6 + \ldots\right]$ [except for the neutron charge ff]

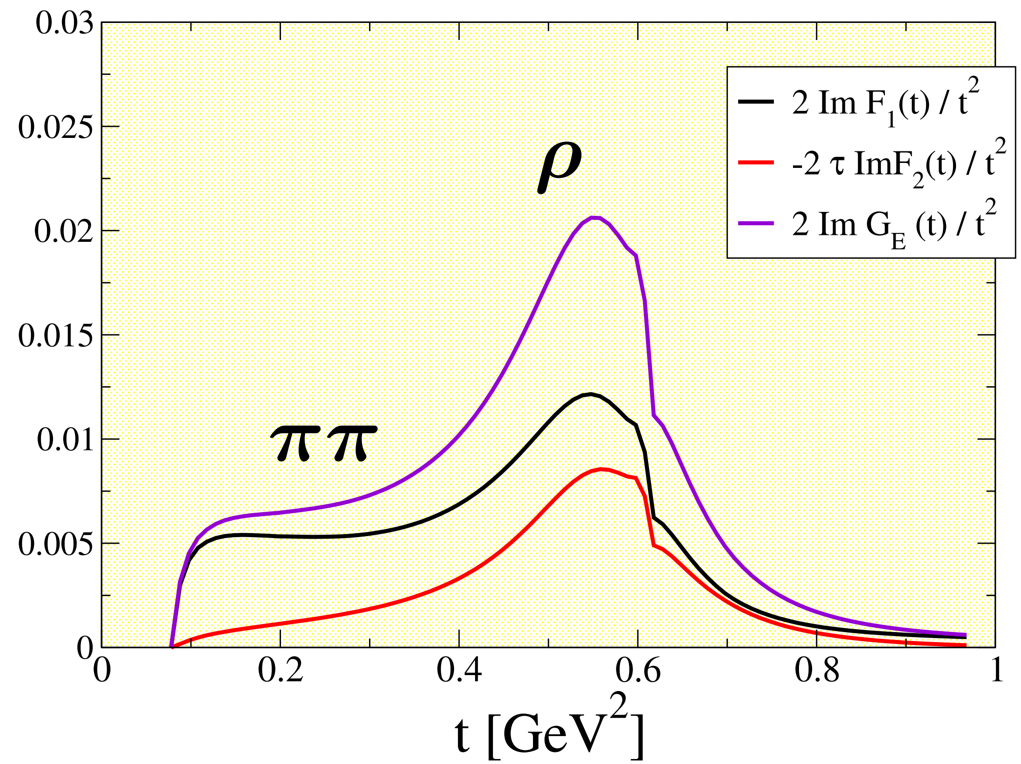
NEW DETERMINATION OF THE 2π CONTINUUM

Belushkin, Hammer, M., Phys. Lett. B **633** (2006) 507 [arXiv:hep-ph/0510382].

- Pion FF from KLOE/CMD-2/SND



- Nucleon isovector spectral functions



$\star$ pronounced $\rho - \omega$ mixing

KLOE Coll., Phys. Lett. B **606** (2005) 12
CMD-2 Coll., Phys. Lett. B **578** (2004) 285
SND Coll., J. Exp. Theor. Phys. **101** (2005) 1053

$\star$ pronounced ρ peak

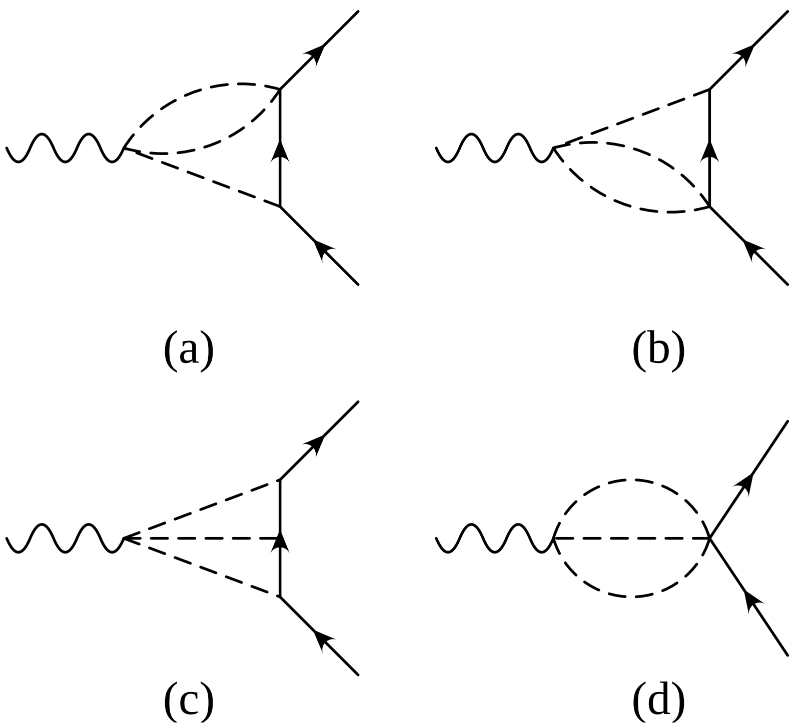
$\star$ strong shoulder on the left wing

$\Rightarrow$ isovector radii

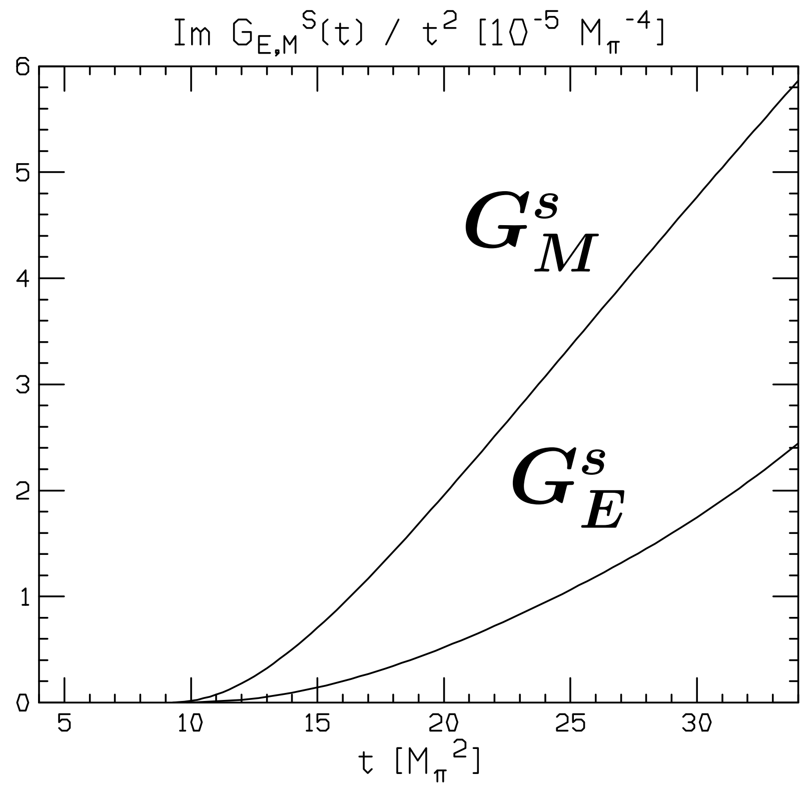
ISOSCALAR SPECTRAL FUNCTION

Bernard, Kaiser, M., Nucl. Phys. A **611** (1996) 429 [hep-ph/9607428]

- Two-loop CHPT calculation



- Electric/magnetic spectral fcts



⋆ **no** shoulder on the left wing

⋆ **clean** omega-pol dominance

SUMMARY: SPECTRAL & FIT FUNCTIONS

- Representation of the pole contributions: vector mesons
  [NB: can be extended for finite width]

$$\text{Im}\, F_i^V(t) = \sum_v \pi a_i^v\, \delta(t - M_v^2)\,, \quad a_i^v = \frac{M_v^2}{f_V} g_{vNN} \;\Rightarrow\; F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}$$

- *Isovector* spectral functions:

$$\text{Im}\, F_i^V(t) = \text{Im}\, F_i^{(2\pi)}(t) + \sum_{v=\rho',\rho'',\ldots} a_i^v \delta(t - M_v^2)\,, \quad (i = 1, 2)$$

- *Isoscalar* spectral functions:

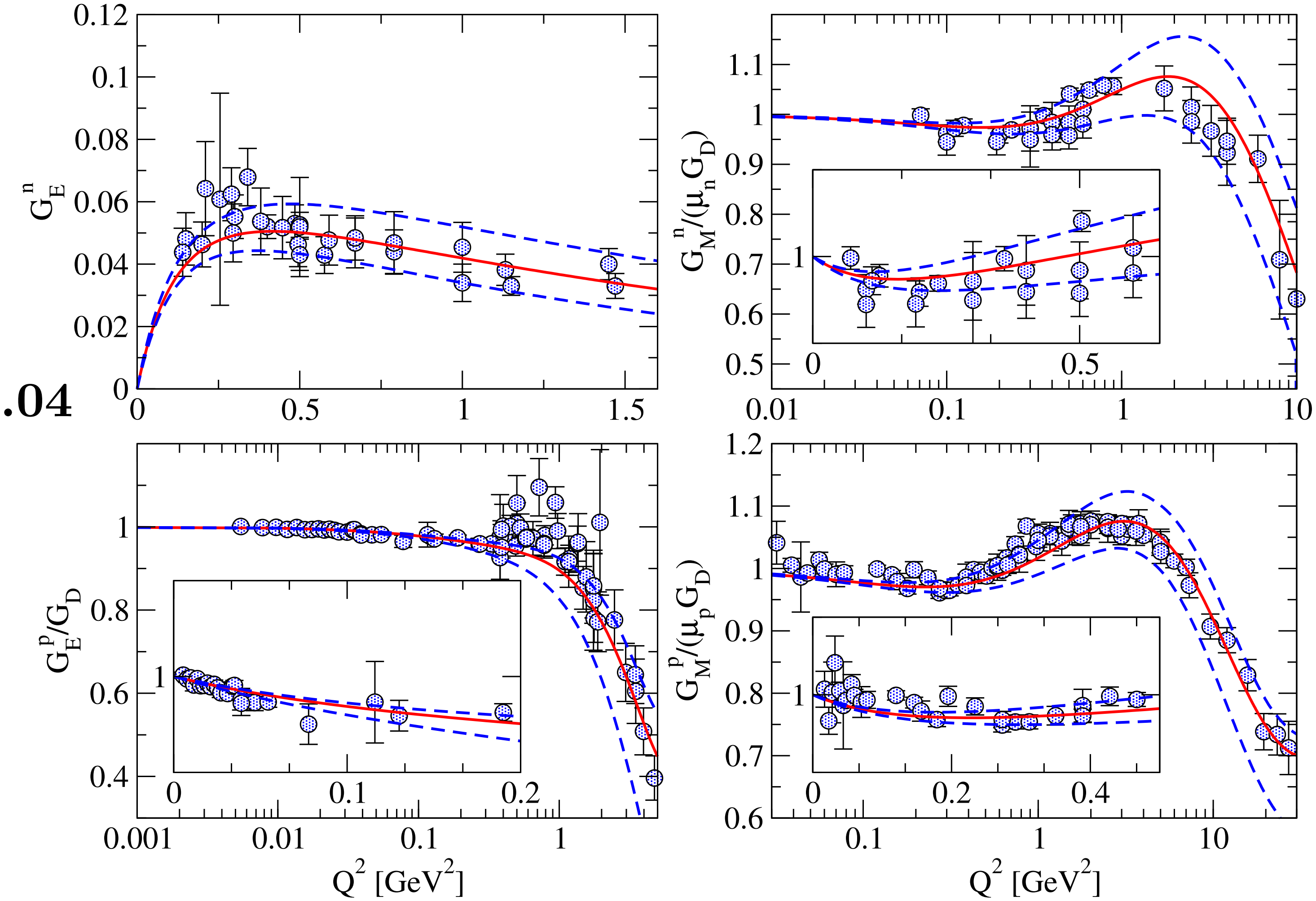
$$\text{Im} F_i^S(t) = \pi\, a_i^\omega\, \delta(t - M_\omega^2) + \text{Im} F_i^{(K\bar{K})}(t) + \text{Im} F_i^{(\pi\rho)}(t) + \sum_{v=S',S'',\ldots} a_i^v \delta(t - M_v^2)$$

- Parameters: 2 for the ω, 3 (4) for each other V-mesons minus # of constraints

- Ill-posed problem $\rightarrow$ extra constraint: minimal # of poles to describe the data

SPACE-LIKE FORM FACTORS

- present best fit
  incl. time-like data
- ω, ϕ + 2 eff. IS poles
- 4 (?)effective IV poles
- weighted χ^2/dof = 1.8
  error bands: $\chi^2_{\mathbf{min}} + \mathbf{1.04}$



Improved description

⋆ JLab data described

⋆ higher mass poles
not at physical values

MMD 96, HMD 96, HM 04

$$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2}$$

TIME-LIKE FORM FACTORS

Haidenbauer, Hammer, M., Sibirtsev, Phys. Lett. B **643** (2006) 29

- fitting also time-like data more complicated

- experimental extraction ambiguous

– E/M separation

– $\bar{N}N$ final-state interactions?

similar to $J/\psi \to \gamma\bar{p}p, \omega\bar{p}p$ from BES

Sibirtsev et al., Phys. Rev. D **71** (2005) 054010
Haidenbauer et al., arXiv:0804.1469 [hep-ph]

similar to $B^+ \to \bar{p}pK^+$ from BaBar

Haidenbauer et al., Phys. Rev. D **74** (2006) 017501

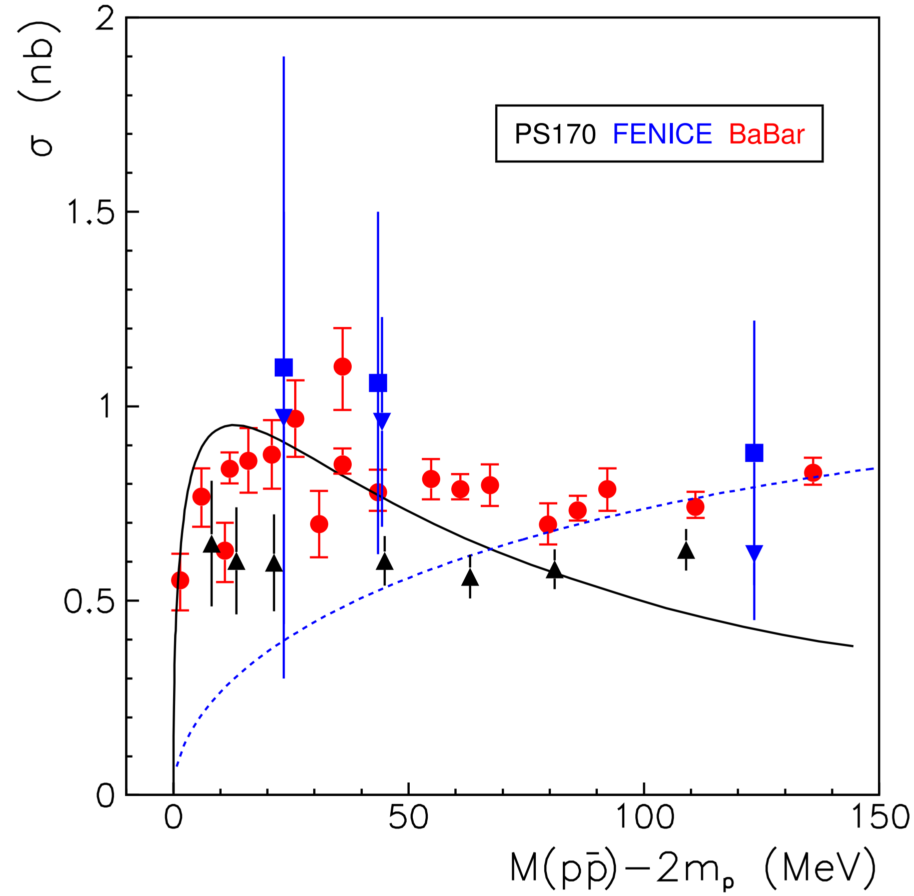
– subthreshold resonance ? (or FSI ?)

Antonelli et al., Nucl. Phys. B **517** (1998) 3

- many new proton data (radiative return)

BES, CLEO, BaBaR

$$\sigma(e^+e^- \leftrightarrow p\bar{p}, e^+e^- \to n\bar{n})$$



ON THE PION CLOUD OF THE NUCLEON

Hammer, M., Drechsel, Phys. Lett. B **586** (2004) 291

- FW find a very long-ranged contribution of the pion could, $r \simeq 2$ fm

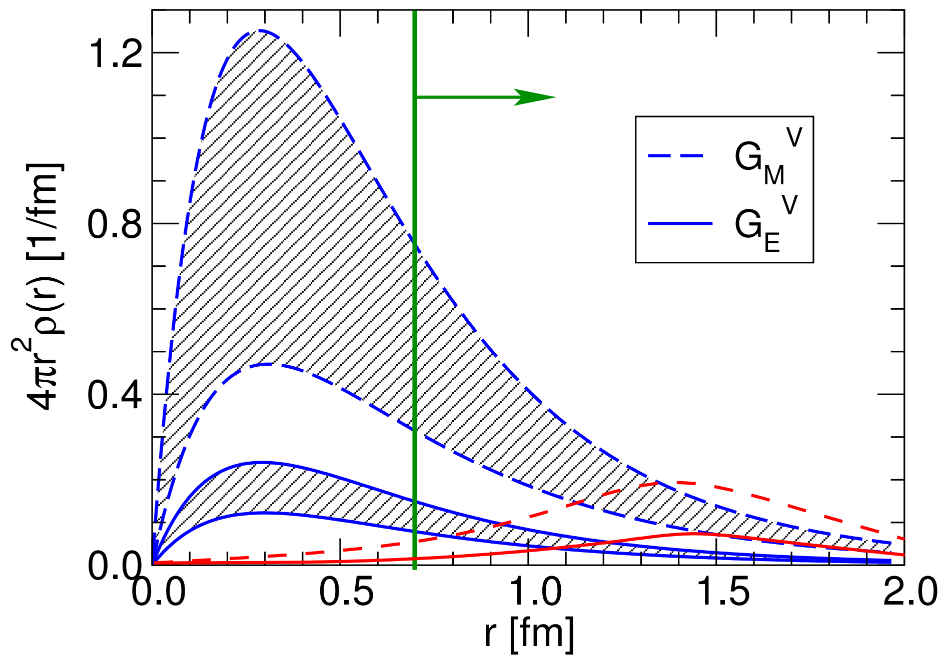
  Friedrich, Walcher, EPJ A **17** (2003) 607

- longest range component can be extracted from the isovector spectral function

  $\longrightarrow$ separation of the ρ-contribution

  $\longrightarrow$ three methods applied to do this

  $\longrightarrow$ theoretical band

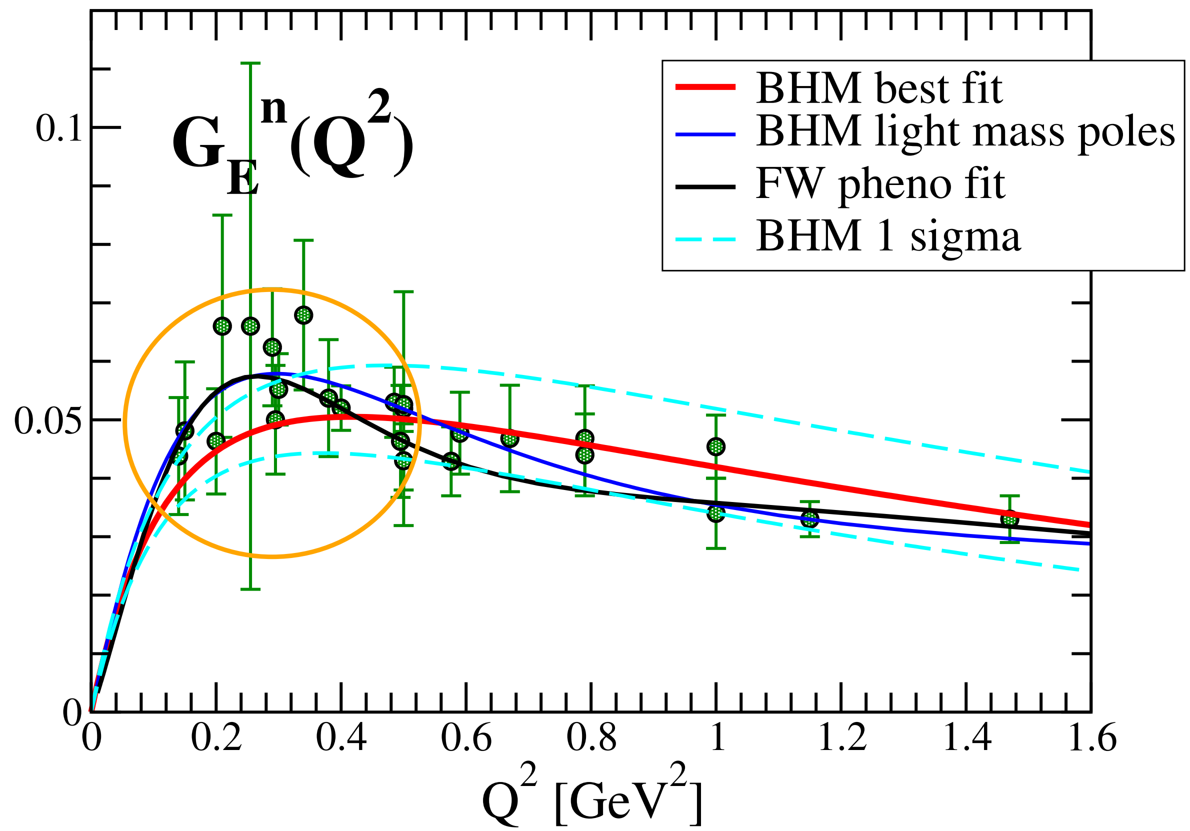


$$\rho_i^V(r) = \frac{1}{4\pi^2}\int_{4M_\pi^2}^{40M_\pi^2} dt\, \mathrm{Im}\, G_i^V(t)\, \frac{e^{-r\sqrt{t}}}{r} \qquad (i = E, M)$$

- much smaller pion cloud contribution for $r \geq 1$ fm compared to FW
- results independent of the contributions from $t > 40M_\pi^2$

$G_E^n(Q^2)$ w/ a BUMP-DIP STRUCTURE

• can one generate a bump-dip structure in the dispersive approach?



⇒ yes, but need **low-mass** poles: $M_S = 358$ MeV & $M_V = 558$ MeV

what shall these be? – not consistent w/ spec ftcs!