VI

ON THE DIVISION OF MENTAL LABOUR

By CHARLES BABBAGE

Chapter XIX, *Economy of Manufactures and Machinery*

(241.)* WE HAVE already mentioned what may, perhaps, appear paradoxical to some of our readers,—that the division of labour can be applied with equal success to mental as to mechanical operations, and that it ensures in both the same economy of time. A short account of its practical application, in the most extensive series of calculations ever executed, will offer an interesting illustration of this fact, whilst at the same time it will afford an occasion for shewing that the arrangements which ought to regulate the interior economy of a manufactory, are founded on principles of deeper root than may have been supposed, and are capable of being usefully employed in preparing the road to some of the sublimest investigations of the human mind.

(242.) In the midst of that excitement which accompanied the Revolution of France and the succeeding wars, the ambition of the nation, unexhausted by its fatal passion for military renown, was at the same time directed to some of the nobler and more permanent triumphs which mark the era of a people's greatness,—and which receive the applause of posterity long after their conquests have been wrested from them, or even when their existence as a nation may be told only by the page of history. Amongst their enterprises of science, the French government was desirous of producing a series of mathematical tables, to facilitate the application of the decimal system which they had so recently adopted. They directed, therefore, their mathematicians to construct such tables, on the most extensive scale. Their most distinguished philosophers, responding fully to the call of their country, invented new methods for this laborious task; and a work, completely answering the large demands of the government, was produced in a remarkably short period of time. M. Prony, to

* [The numbers within parentheses identify the paragraphs of the original volume, *Economy of Manufactures and Machinery*.]

whom the superintendence of this great undertaking was confided, in speaking of its commencement, observes: "*Je m'y livrai avec toute l'ardeur dont j'étois capable, et je m'occupai d'abord du plan général de l'exécution. Toutes les conditions que j'avois à remplir nécessitoient l'emploi d'un grand nombre de calculateurs; et il me vint bientôt à la pensée d'appliquer à la confection de ces* Tables la division du travail, *dont les Arts de Commerce tirent un parti si avantageux pour réunir à la perfection de main-d'œuvre l'économie de la dépense et du temps.*" The circumstance which gave rise to this singular application of the principle of *the division of labour* is so interesting, that no apology is necessary for introducing it from a small pamphlet printed at Paris a few years since, when a proposition was made by the English to the French government, that the two countries should print these tables at their joint expense.

(243.) The origin of the idea is related in the following extract:

C'est à un chapitre d'un ouvrage Anglais,* justement célèbre, (I.) qu'est probablement due l'existence de l'ouvrage dont le gouvernement Britannique veut faire jouir le monde savant:—

Voici l'anecdote: M. de Prony s'était engagé, avec les comités de gouvernement, à composer, pour *la division centésimale du cercle, des tables logarithmiques et trigonométriques, qui, non-seulement ne laissassent rien à désirer quant à l'exactitude, mais qui formassent le monument de calcul le plus vaste et le plus imposant qui eût jamais été exécuté, ou même conçu.* Les logarithmes des nombres de 1 à 200,000 formaient à ce travail un supplément nécessaire et exigé. Il fut aisé à M. de Prony de s'assurer que, même en s'associant trois ou quatre habiles co-opérateurs, la plus grande durée présumable de sa vie ne lui suffirait pas pour remplir ses engagements. Il était occupé de cette fâcheuse pensée lorsque, se trouvant devant la boutique d'un marchand de livres, il aperçut la belle édition Anglaise de Smith, donnée à Londres en 1776; il ouvrit le livre au hasard, et tomba sur le premier chapitre, qui traite de *la division du travail*, et où la fabrication des épingles est citée pour exemple. A peine avait-il parcouru les premières pages, que, par une espèce d'inspiration, il conçut l'expédient de mettre ses logarithmes en *manufacture* comme les épingles. Il faisait, en ce moment, à l'école polytechnique, des leçons sur une partie d'analyse liée à ce genre de travail, *la méthode des différences*, et ses applications à *l'interpolation*. Il alla passer quelques jours à la campagne, et revint à Paris avec le plan de *fabrication*, qui a été suivi dans l'exécution. Il rassembla deux ateliers, qui faisaient séparément les mêmes calculs, et se servaient de vérification réciproque."†

* *An Enquiry into the Nature and Causes of the Wealth of Nations*, by Adam Smith.

† Note sur la publication, proposée par le gouvernement Anglais des grandes tables logarithmiques et trigonométriques de M. de Prony.—De l'imprimerie de F. Didot, Dec. 1, 1820, p. 7.

(244.) The ancient methods of computing tables were altogether inapplicable to such a proceeding. M. Prony, therefore, wishing to avail himself of all the talent of his country in devising new methods, formed the first section of those who were to take part in this enterprise out of five or six of the most eminent mathematicians in France.

First Section.—The duty of this first section was to investigate, amongst the various analytical expressions which could be found for the same function, that which was most readily adapted to simple numerical calculation by many individuals employed at the same time. This section had little or nothing to do with the actual numerical work. When its labours were concluded, the formulæ on the use of which it had decided, were delivered to the second section.

Second Section.—This section consisted of seven or eight persons of considerable acquaintance with mathematics: and their duty was to convert into numbers the formulæ put into their hands by the first section,—an operation of great labour; and then to deliver out these formulæ to the members of the third section, and receive from them the finished calculations. The members of this second section had certain means of verifying the calculations without the necessity of repeating, or even of examining, the whole of the work done by the third section.

Third Section.—The members of this section, whose number varied from sixty to eighty, received certain numbers from the second section, and, using nothing more than simple addition and subtraction, they returned to that section the tables in a finished state. It is remarkable that nine-tenths of this class had no knowledge of arithmetic beyond the two first rules which they were thus called upon to exercise, and that these persons were usually found more correct in their calculations, than those who possessed a more extensive knowledge of the subject.

(245.) When it is stated that the tables thus computed occupy seventeen large folio volumes, some idea may perhaps be formed of the labour. From that part executed by the third class, which may almost be termed mechanical, requiring the least knowledge and by far the greatest exertions, the first class were entirely exempt. Such labour can always be purchased at an easy rate. The duties of the second class, although requiring considerable skill in arithmetical operations, were yet in some measure relieved by the higher interest naturally felt in those more difficult operations. The exertions of the first class are not likely to require, upon another occasion, so much skill and labour as they did upon the first attempt to introduce such a method; but when the completion of a calculating-engine shall have produced a substitute for the whole of the third section of computers, the attention of analysts will naturally be directed to simplifying its application, by a new discussion of the methods of converting analytical formulæ into numbers.

(246.) The proceeding of M. Prony, in this celebrated system of calculation, much resembles that of a skilful person about to construct a cotton or silk-mill, or any similar establishment. Having, by his own genius, or through the aid of his friends, found that some improved machinery may be successfully applied to his pursuit, he makes drawings of his plans of the machinery, and may himself be considered as constituting the first section. He next requires the assistance of operative engineers capable of executing the machinery he has designed, some of whom should understand the nature of the processes to be carried on; and these constitute his second section. When a sufficient number of machines have been made, a multitude of other persons, possessed of a lower degree of skill, must be employed in using them; these form the third section: but their work, and the just performance of the machines, must be still superintended by the second class.

(247.) As the possibility of performing arithmetical calculations by machinery may appear to non-mathematical readers to be rather too large a postulate, and as it is connected with the subject of the *division of labour*, I shall here endeavour, in a few lines, to give some slight perception of the manner in which this can be done,—and thus to remove a small portion of the veil which covers that apparent mystery.

(248.) *That nearly all tables of numbers which follow any law, however complicated, may be formed, to a greater or less extent, solely by the proper arrangement of the successive addition and subtraction of numbers befitting each table*, is a general principle which can be demonstrated to those only who are well acquainted with mathematics; but the mind, even of the reader who is but very slightly acquainted with that science, will readily conceive that it is not impossible, by attending to the following example.

The subjoined table is the beginning of one in very extensive use, which has been printed and reprinted very frequently in many countries, and is called *a Table of Square Numbers*.

Any number in the table, column A, may be obtained, by multiplying the number which expresses the distance of that term from the commencement of the table by itself; thus, 25 is the fifth term from the beginning of the table, and 5 multiplied by itself, or by 5, is equal to 25. Let us now subtract each term of this table from the next succeeding term, and place the results in another column (B), which may be called first-difference column. If we again subtract each term of this first difference from the succeeding term, we find the result is always

Terms of the Table	A Table	B First Difference	C Second Difference
1	1		
		3	
2	4		2
		5	
3	9		2
		7	
4	16		2
		9	
5	25		2
		11	
6	36		2
		13	
7	49		

the number 2, (column C;) and that the same number will always recur in that column, which may be called the second difference, will appear to any person who takes the trouble to carry on the table a few terms further. Now when once this is admitted, it is quite clear that, provided the first term (1) of the Table, the first term (3) of the first differences, and the first term (2) of the second or constant difference, are originally given, we can continue the table of square numbers to any extent, merely by addition:—for the series of first differences may be formed by repeatedly adding the constant difference (2) to (3) the first number in column B, and we then have the series of numbers, 3, 5, 7, &c.: and again, by successively adding each of these to the first number (1) of the table, we produce the square numbers.

(249.) Having thus, I hope, thrown some light upon the theoretical part of the question, I shall endeavour to shew that the mechanical execution of such an engine, as would produce this series of numbers, is not so far removed from that of ordinary machinery as might be conceived.* Let the reader imagine three clocks, placed on a table

* Since the publication of the Second Edition of this Work, one portion of the engine which I have been constructing for some years past has been put together. It calculates, in three columns, a table with its first and second differences. Each column can be expressed as far as five figures, so that these fifteen figures constitute about one-ninth part of the larger engine. The ease and precision with which it works, leave no room to doubt its success in the more extended form. Besides tables of squares, cubes, and portions of logarithmic tables, it possesses the power of calculating certain series whose differences are not constant; and it has already tabulated parts of series formed from the following equations:

$$\Delta^3 u_x = \text{units figure of } \Delta^{u}{}_x$$

$$\Delta^3 u_x = \text{nearest whole no. to } \left(\frac{1}{10{,}000}\Delta u_x\right)$$

side by side, each having only one hand, and each having a thousand divisions instead of twelve hours marked on the face; and every time a string is pulled, let them strike on a bell the numbers of the divisions to which their hands point. Let him further suppose that two of the clocks, for the sake of distinction called B and C, have some mechanism by which the clock C advances the hand of the clock B one division, for each stroke it makes upon its own bell: and let the clock B by a similar contrivance advance the hand of the clock A one division, for each stroke it makes on its own bell. With such an arrangement, having set the hand of the clock A to the division I., that of B to III., and that of C to II., let the reader imagine the repeating parts of the clocks to be set in motion continually in the following order: viz.—pull the string of clock A; pull the string of clock B; pull the string of clock C.

The table on the following page will then express the series of movements and their results.

If now only those divisions struck or pointed at by the clock A be attended to and written down, it will be found that they produce the series of the squares of the natural numbers. Such a series could, of course, be carried by this mechanism only so far as the numbers which can be expressed by three figures; but this may be sufficient to give some idea of the construction,—and was, in fact, the point to which the first model of the calculating-engine, now in progress, extended.

The subjoined is one amongst the series which it has calculated:

0	3,486	42,972
0	4,991	50,532
1	6,907	58,813
14	9,295	67,826
70	12,236	77,602
230	15,741	88,202
495	19,861	99,627
916	24,597	111,928
1,504	30,010	125,116
2,340	36,131	139,272

The general term of this is:

$$u_x = \frac{x.x-1.x-2}{1.2.3} + \text{the whole number in } \frac{x}{10} + $$
$$+ 10\Sigma^3\left(\text{units figure of } \frac{x.x+1}{2}\right)$$

Repetitions of Process	MOVEMENTS	CLOCK A — *Hand set to* I	CLOCK B — *Hand set to* III	CLOCK C — *Hand set to* II
		TABLE	*First difference*	*Second difference*
1	Pull A.	A. strikes . . . 1	. . . .	. . . .
	—— B.	The hand is advanced (by B.) 3 divisions .	B. strikes . . . 3	. . . .
	—— C.	. . . .	The hand is advanced (by C.) 2 divisions	C. strikes 2
2	Pull A.	A. strikes . . . 4	. . . .	. . . .
	—— B.	The hand is advanced (by B.) 5 divisions .	B. strikes . . . 5	. . . .
	—— C.	. . . .	The hand is advanced (by C.) 2 divisions .	C. strikes 2
3	Pull A.	A. strikes . . . 9	. . . .	. . . .
	—— B.	The hand is advanced (by B.) 7 divisions .	B. strikes . . . 7	. . . .
	—— C.	. . . .	The hand is advanced (by C.) 2 divisions .	C. strikes 2
4	Pull A.	A. strikes . . . 16	. . . .	. . . .
	—— B.	The hand is advanced (by B.) 9 divisions .	B. strikes . . . 9	. . . .
	—— C.	. . . .	The hand is advanced (by C.) 2 divisions .	C. strikes 2
5	Pull A.	A. strikes . . . 25	. . . .	. . . .
	—— B.	The hand is advanced (by B.) 11 divisions .	B. strikes . . . 11	. . . .
	—— C.	. . . .	The hand is advanced (by C.) 2 divisions .	C. strikes 2
6	Pull A.	A. strikes . . . 36	. . . .	. . . .
	—— B.	The hand is advanced (by B.) 13 divisions .	B. strikes . . . 13	. . . .
	—— C.	. . . .	The hand is advanced (by C.) 2 divisions .	C. strikes 2