A Model-Based Self-tuning Controller for Kinetically Controlled Combustion Instability

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Abstract

Combustion instability is one of the distinct characteristics in continuous combustion processes such as gas turbines, ramjet engines and afterburners. This combustion instability often occurs near blow limit and high thermal output conditions and generates large amplitudes of heat release and pressure oscillations. Active control has been used to suppress combustion instability, but due to complex dynamics of the combustion processes, modeling of the combustion system has often been limited the application of the active control. Recently, we developed a combustion model which is applicable to kinetically controlled combustion instability. In this paper, we first examine the sensitivity of the model characteristics, which show that the model parameters change by 100% for a 10% change in the operating conditions. Next, we propose a self-tuning controller, and show that it effectively suppresses the pressure oscillations in the presence of changing operating conditions.

1. Introduction

Combustion instabilities arise due to positive coupling between acoustic pressure waves and unsteady heat release. These instabilities are often observed in lean premixed gas turbine combustors, ramjet engines, afterburners etc. Pressure oscillations can become significant, leading to violent oscillations in the flow and mechanical vibrations of the system components. Passive control techniques have been used to suppress combustion oscillations. These involve modification to the fuel injection and distribution pattern or the combustor geometry [1]. In recent years, active control has received increasing attention because of its potential as a retrofit technology, and its adaptability over a wide range of operating conditions. Most active control designs use a simple time-delay controller with a fuel injector whose input is determined by adding an empirically chosen time delay to a filtered pressure signal [2]-[5].

An alternate approach is to develop model-based active control designs where one can either employ the underlying physics [6]-[8], or input-output data together with System Identification (SI) methods [9] to derive the model. In SI, suitable inputs and the corresponding pressure outputs are chosen to capture the dominant combustor dynamics, and these data are used to fit a particular system model structure. In this paper, we use active control which uses physically-based reduced-order modeling.

The instability conditions for a combustion system becomes unstable have been expressed in terms of the Rayleigh criterion [10], which states that the pressure and heat release oscillations become in-phase at unstable conditions and out-of-phase at stable conditions. Also, it is known that as operating conditions approach blow out limits or high volumetric heat-release, the combustion system transitions from stability to instability. This implies that the pressure and heat release which constitute the combustion dynamics exhibit varying phase relationships as operating conditions change. A good model of the combustion dynamics should therefore not only capture combustion instability but also the aforementioned stability characteristics that change with operating conditions.

In [11], we developed such a heat release model for kinetically controlled combustion which exhibited combustion instability near blowout limit and high thermal output conditions. We also showed using the model that the phase between pressure and heat release changes by changing operating conditions such as mass flow rate and equivalence ratio. With these properties, the heat release model corroborated the changing stability characteristics observed in several experimental studies [12]-[14] qualitatively.

In this paper, we investigate active control of combustion system using the same model proposed in [11]. We first include the effect of external actuation due to secondary-fuel injection and derive the model that accommodates the effect of the equivalence ratio modulation. Next, we examine sensitivity of parameters in the model by changing mean mass flow rate and mean equivalence ratio. We finally investigate the performance of a self-tuning controller and show that it effectively suppresses the pressure oscillations such changing operating conditions.

2. Modeling

It is well known that combustion instability is due to coupling interactions between heat release dynamics and acoustics. To carry out active model-based control, the effects of actuation of the overall process must also be modeled (See Figure 1). These components are described in this section.

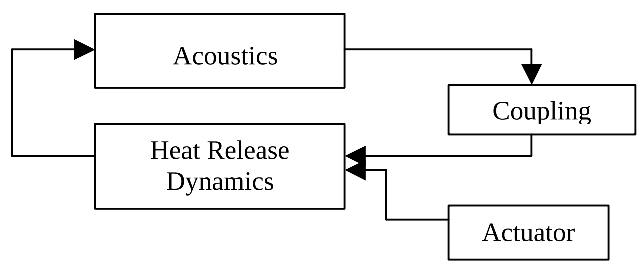

Figure 1 Schematic of the complete combustion dynamics

2.1 The Heat Release Dynamics

The heat release model presented in [11] use a Well-stirred reactor (WSR) [16]. WSR assumes perfect mixing inside a combustor thereby simplifying the system as ODEs. The highlights of this model are presented in Section 2.1.1 As the effect of actuation by control input has not been modeled in [11], we develop a heat release model which accommodates the actuation in Section 2.1.2.

2.1.1 Well-stirred Reactor Modeling

The governing equations of a well-stirred reactor are obtained using the conservation laws and a set of reaction-rate equations. The conservation equations of the mass, energy and species in the WSR are given by:

Mass : $\frac{dM}{dt} = \dot{m}_i - \dot{m}$, (1)

Energy : $\frac{dE}{dt} = \dot{m}_i \bar{h}_i - \dot{m}\bar{h} + \dot{Q}_r$, (2)

Species : $\frac{dM_k}{dt} = \dot{m}_i Y_{k,i} - \dot{m} Y_k - \dot{W}_k$, (3)

where M , E , and M_k are a total mass, energy and mass of species k inside the combustor, respectively, $\dot{Q}_r = \sum_k \dot{W}_k h_k(T_i)$ is the heat release rate due to the chemical reaction, $\dot{W}_k$ is a consumption rate of species k , $\dot{m}$ is the mass flow rate, h is the enthalpy per unit mass, Y is the mass fraction, and subscript i refer to the inlet condition. In Eq. (2), it is assumed that heat loss is negligible.

For a single-step mechanism, the source terms can be represented as function of Y and T [16] as follow:

$\dot{W}_f = A_f V (rY_f)^{n_f} (rY_{O_2})^{n_{o2}} \exp(\frac{-T_a}{T})$ and $\dot{Q}_r = \Delta h_r \dot{W}_f$ (4)

where A_f is the frequency factor, Δh_r is the enthalpy of reaction (measured per unit mass of fuel), $T_a = E_a / R$ where E_a is activation energy and R is the gas constant, and subscript f and O_2 refer to fuel and air, respectively. Equations (1)-(4) represent complete dynamics of the WSR, which shows that the underlying model is nonlinear.

2.1.2 Actuation

We now derive a model that represents the impact of the equivalence ratio modulation via fuel injection. First, we use the following assumptions:

(A1) The fuel line and air lines are choked;

(A2) Fuel injector is proportional, and has sufficient bandwidth and authority;

(A3) Combustion dynamics can be assumed as a WSR and no spatial variation by fuel injection.

Increased mass flow rate of the fuel will modulate mass ratio of the fuel, Y_f , and total mass flow rate. However, the impact of the equivalence ratio modulation is factor of 20 larger than that of mass flow rate, and hence we only consider modulation in Y_f .

If we define modulation fuel flow rate as $\dot{m}_{fm}$, then relation between the fuel mass ratio disturbance, Y_{fm} , and $\dot{m}_{fm}$ is

$$Y_{fm} = \frac{\dot{m}_{fm}}{\dot{m}_a + \dot{m}_f + \dot{m}_{fm}} \approx \frac{\dot{m}_{fm}}{\dot{m}_a + \dot{m}_f} \quad (5)$$

Therefore, one can get a linear relation between $\dot{m}_{fm}$ and Y_{fm} . While a convective time delay, t_c , due to transport lag from the supply to a burning zone, can also be present, we neglect the delay in this paper (See Ref. [17] for a time delay model and the impact of bandwidth and authority of a fuel actuator). Using assumption in (A2), a fuel injector dynamics can be approximated as [17]:

$$\frac{\dot{m}_{fm}}{E(s)} = \frac{k_v}{t_m s + 1} , \quad (6)$$

where t_m is the time constant and k_v is the gain of the injector.

2.1.3 Linearization

The discussion in Section 2.1.2 show that active control introduces another perturbation Y_{fm} in Y_f , and the effect of $\dot{m}'_{fm}$ in $\dot{m}'_i$ is negligible. Therefore, expressing $Y_f = \bar{Y}_f + Y'_f + Y_{fm}$ and $\dot{m}_i = \bar{\dot{m}}_i + \dot{m}'_i$, linearization of Eq. (4), together with Eqs. (1)-(3), yields the following linear heat release rate model[1] :

$$\dot{Q}'_r = \frac{b}{s+a}\dot{m}'_i + \frac{s}{s+a} Y_{fm} \quad (7)$$

$$a = \frac{\bar{\dot{m}}}{\bar{r}V}\left(1 + n\frac{(\bar{T} - T_i)}{\bar{T}} - \frac{(\bar{T} - T_i)}{\bar{T}^2} T_a + n\frac{(Y_i - \bar{Y})}{\bar{Y}}\right), \quad (8)$$

where

$$b = A'_f \Delta h_r \bar{r}^{n-1} \bar{Y}^n \exp(\frac{-T_a}{\bar{T}})(n\frac{(\bar{T} - T_i)}{\bar{T}} - \frac{(\bar{T} - T_i)}{\bar{T}^2} T_a + n\frac{(Y_i - \bar{Y})}{\bar{Y}}) \quad (9)$$

and $$s = A'_f \Delta h_r \bar{r}^{n-1} \bar{Y}^{n-1} \exp(\frac{-T_a}{\bar{T}}) \bar{\dot{m}}_i n \quad (10)$$

Equation (7) represents the actuated heat-release model, and has three parameters, a , b and s . a represents bandwidth and b and s are static gains of mass and equivalence ratio modulation inputs, respectively. a , b and s are functions of the mean residence time, the equivalence ratio, and the inlet temperature. a and b determines characteristics of feedback mechanism between acoustics and heat release dynamics where fuel and air lines are choked whereas s determines the effect of the external actuation by fuel injection.

2.2 Acoustics

We now present the model for the acoustics in a combustor. The acoustics is modeled assuming that the dynamics is predominantly one-dimensional, and that the burning zone is localized spatially.

Using the Galerkin method, we can express pressure perturbations as

$$p' = \bar{p} \sum_i y_i(x) h_i(t) \quad (11)$$

where p' denotes the pressure perturbation, $y_i(x) = \sin(k_i x + f_{i0})$ and k_i and f_{i0} are determined by the boundary conditions, the following finite-dimensional model can be derived [18]:

$$\ddot{h}_i + 2 w_i V_i \dot{h} + w_i^2 h_i = c_i \dot{q}' \quad (12)$$

$$p' = \bar{p} \sum_i d_i h_i(t) \quad (13)$$

$$u' = \sum_i b_i h_i(t) \quad (14)$$

[1] In the derivation, we also assume constant volume and pressure conditions and use the ideal gas law. In addition, we assume $(rY_{O_2})^{n_{o2}}$ ~const in Eq. (4) since we are interested in the system response near blow-out limit where Y_{O_2} does not change significantly. See Ref. [11] for more detail.

where V_i is the damping ratio in the acoustics due to passive effects, $c_i = \frac{(g-1)}{\bar{p}E} y_i(x_f)$, q' is the heat release perturbations per unit area, $d_i = y_i(x_s)$, $b_i = \frac{1}{gk_i^2}\frac{dy_i}{dx}(x_f)$, $E = \int_0^L y_i^2 dx$, L is the acoustic length of the combustor, k_i is the wave number, $w_i = \bar{c}k_i$, $\bar{c}$ is the speed of sound, x_f is the location of the burning zone, x_s is the sensor location.

2.3 The Complete Feedback Model

As mentioned earlier, the combustion instability occurs due to the interaction between heat-release dynamics and acoustics, both of which were modeled in Sections 2.1 and 2.2, respectively. The coupling between heat-release dynamics and acoustics is assumed to occur, primary through perturbations in the incoming mass-flow rate in this paper. This can therefore be described as

$$\dot{m}_i' = r_i A u'(x = x_f) \qquad (15)$$

where r_i is the density of the incoming mixture, A is the area of the combustor. Equation (7) and (12)-(15) determine the complete feedback model of the combustion dynamics and is shown in Figure 2 (The sensor dynamics is neglected since it typically has a much higher bandwidth than the combustion dynamics). We note that Y_{fm} is the control input, and $\dot{Q}_r'$ and p' are outputs that can be measured, with the transfer functions given by

$$\frac{p'}{Y_{fm}} = G_p(s) = \frac{\bar{p}ss\left[c_1 d_1(s^2 + 2V_2 w_2 s + w_2^2) + c_2 d_2(s^2 + 2V_1 w_1 s + w_1^2)\right]}{h(s)} \qquad (16)$$

and

$$\frac{\dot{Q}_r'}{Y_{fm}} = G_q(s) = \frac{s(s^2 + 2V_1 w_1 s + w_1^2)(s^2 + 2V_2 w_2 s + w_2^2)}{h(s)} \qquad (17)$$

where

$$h(s) = (s+a)(s^2 + 2V_1 w_1 s + w_1^2)(s^2 + 2V_2 w_2 s + w_2^2) - bb_1 c_1 s^2(s^2 + 2V_2 w_2 s + w_2^2) - bb_2 c_2 s^2(s^2 + 2V_1 w_1 s + w_1^2).$$

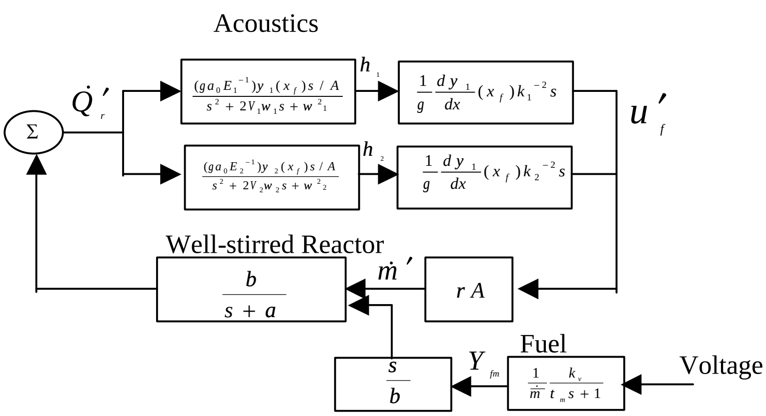

Figure 2 A linear model of the combustion dynamics with heat release

3. Sensitivity of the Heat Release Model

The transfer function, G_p and G_q have a number of physical parameters that vary with the operating conditions such as the mass flow rate, equivalence ratio, inlet temperature. Even though heat loss has not been included in Eq. (2), this may also cause significant change in the system. In this section, we examine sensitivity of the system parameters and the transfer functions, G_p and G_q.

3.1 Sensitivity of System Parameters

First, we determine parameters of the transfer functions in Eqs. (16) and (17) using the configuration in Ref. [12], which consists of a lean premixed combustor in which a flame was stabilized behind a rearward-facing step. The length of the combustor, L, is 2.5m, the location of the burning zone, x_f, is 1.2m, cross sectional area of the combustor, A, is 0.0114 m^2. The combustor is modeled assuming that the dynamics are predominantly one dimensional as described in Section 2.2 due to small cross sectional area compared to the length of the combustor. Using the mean speed of sound of $\bar{c}$ =430m/s, the longitudinal acoustic frequencies were calculated as w_1 =48 Hz for a quarter-wave mode and w_2 =129.7 Hz for a three-quarter mode. Other parameters that were used to formulate the acoustic equations, are given by, g =1.4, $\bar{p}$ =1atm, V_1 =0.001, and V_2 =0.001. Both g and $\bar{p}$ are directly determined by mixture properties and operating condition, but V_1 and V_2 are arbitrary values that were added to account for passive damping in the system. Using the above values, the values of b_1, c_1, b_2 and c_2 were determined. The sensor location was assumed to be sufficiently close to the burning zone, i.e. $x_s = 1.2$ m, which determines the values of d_1 and d_2. For the heat release model, using the chemical properties of C_2H_6, we set $A_f = 4.24 \cdot 10^8$, $n_f = 0.1$, $n_{O_2} = 1.65$ and $T_a = 15098K$ [16]

Then, we examine sensitivity of parameters by changing the operation conditions especially the mean mass flow rate and mean equivalence ratio. Same configurations in Ref. [12] were used and the mass flow rate was changed from 600 kg/m^3s to 700 kg/m^3s, and the equivalence ration was changed from 0.75 to 0.85.

Three parameters, s, a and b that can affect the model as shown in Eqs. (16) and (17) are examined and shown in Figures 3, 4 and 5. As shown in Figure 3, by changing the mass flow rate or the equivalence ratio about 10%, s changes more than 2 times. This change does not affect the locations of the poles and zeros, but it affects the gain of the closed systems directly. This implies that the gain of the controller should be able to access this gain change by self-tuning the gain of the controller.

In case of a and b, changes are more critical. As shown in Figure 4, the a experiences order of magnitude changes. The change is more serious for b. As shown in Figure 5, the sign of the beta changes from positive to negative values. As mentioned in the previous section, this sign change generates the mode switching. These changes in a and b, alter characteristics of the system by changing the locations of poles in Eqs. (16) and (17).

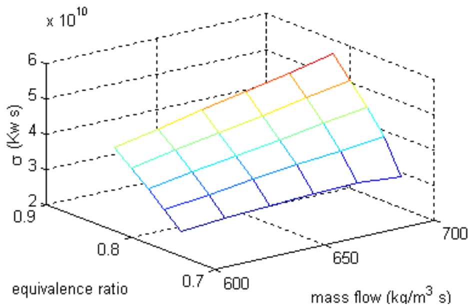

Figure 3 Effect of operation condition on s

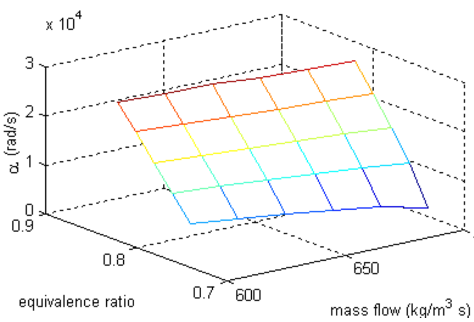

Figure 4 Effect of operation condition on a

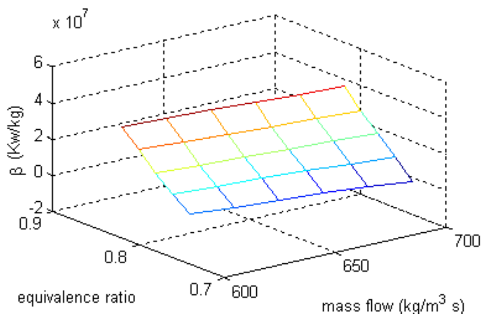

Figure 5 Effect of operation condition on b

3.2 Sensitivity of the Transfer Functions

As one can expect, the rapid change of system parameters alter the characteristics of the transfer functions, G_p and G_q significantly. Especially the changes of system parameters become maximum near the blow out condition, and therefore the transfer functions also experience drastic change in those conditions. Figure 6 shows the change of G_q near blow out limit. In this case, equivalence ratio is fixed at 0.76 and the mass flow rate, $\dot{m}/V$, is changed from 710 kg/m^3s to 800 kg/m^3s. The maximum heat release is achieved at $\dot{m}/V$ = 790 kg/m^3s and the blow out is at $\dot{m}/V$ = 805 kg/m^3s. In the figure, we observe a drastic change when $\dot{m}/V$ is changed from 770 kg/m^3s to 800 kg/m^3s which crosses the maximum heat release point. The reason of this drastic change at the maximum heat release is due to the sign change of b at this point as mentioned in Section 2.1.3.

We observe similar characteristics for $G_p(s)$ and therefore those graphs are not included in this paper.

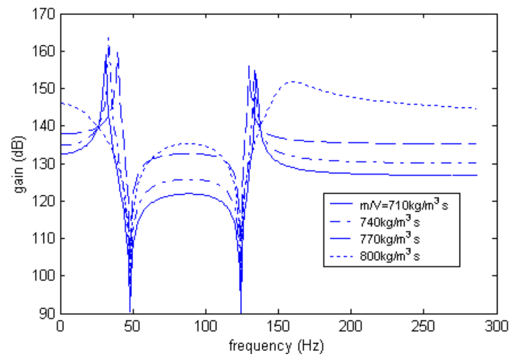

Figure 6 Gain Characteristics of $G_q(s)$ when $\dot{m}/V$ is changed from $710kg/m^3s$ to $800kg/m^3s$ at $f = 0.76$.

4. A Self-tuning controller

Due to stable zeros of the system, it is possible to stabilize the system, G_q and G_p by choosing an appropriate gain if we neglect the effect of time delay. However, due to drastic change of the system by changing the operating conditions, one needs to find control algorithms to self adjust the gain of the controller. In this section, we examine the feasibility of a fixed phase-lead controller and present a model based self-tuning controller.

4.1 A fixed controller

The purpose of the phase-lead controller is to add damping at the unstable frequency [19]. In its simplest form, the phase lead controller is as follows:

$$E(s) = G_c(s)y \tag{18}$$

where

$$G_c(s) = k_c \frac{s + z_c}{s + p_c} \tag{19}$$

with $z_c < w_u < p_c$ where w_u depends on the unstable frequency. z_c and p_c of the controller must be chosen such

$$p_c - z_c > 2q \tag{20}$$

where q is the growth rate at the unstable frequency. The value of k_c determines the exact phase added and therefore the damping at the unstable frequency. If we consider a case where we use Eq. (17), the system has 5 poles and 4 stable zeros. Including the injector dynamics and the fixed phase-lead controller increases the order of the system to 7th order system which has 7 poles and 5 stable zeros. Since we can locate asymptotic line in the LPH by appropriately positioning z_c and p_c, we can stabilize the system using high enough gain. However, the question is how much gain we need to stabilize the system if the system parameters such as a and b vary by the change of operating conditions more than a factor of 10. This implies that we need a self-tuned controller that can automatically update the gain k_c on-line to guarantee stability of the controlled system.

4.2 A self-tuning controller

Now, we show a self-tuning controller to address the parameter change in the model. A time varying gain $k_o(t)$ was added to accommodate for parameter change. The resulting controller is of the form,

$$E(s) = k_o(t)G_c(s)y \quad (21)$$

The turning rule for $k_o(t)$ must be selected so that the close-loop system is stabilized, and remain stable in the presence of model uncertainty. Assuming we use a heat release oscillation as control input, the closed-loop system has a transfer function

$$W_{cl} = \frac{G_q(s)G_l(s)}{1 - k_o G_c(s)G_q(s)G_l(s)} \quad (22)$$

where $G_l(s) = \frac{Y_{fm}(s)}{E(s)} = \frac{1}{\dot{m}}\frac{k_v}{t_m s + 1}$. By appropriately choosing k_o, $W_{cl}(s)$ can be made asymptotically stable. This can be shown theoretically for any dynamic system where $G_q(s)G_l(s)$ has relative degree two and stable zeros [20] and was shown experimentally for a bench top combustor rig [18]. Denoting $k_o(t) = k_o^* + \tilde{k}_o(t)$, it follows that

$$q' = G_q(s)G_l(s)[k_o^* G_c(s)q' + \tilde{k}_o(t)G_c(s)q'] \quad (23)$$
$$= W_{cl}(s)[\tilde{k}_o(t)X(t)]$$

where q' is the heat release rate per unit area measured by a heat flux sensor, where $W_{cl}(s)$ is obtained as in (22) with $k_o = k_o^*$ in $W_{cl}(s)$. The parameter $\tilde{k}_o$, which depends on the uncertainties in the model, is unknown. Eq. (23) is therefore purely for the purpose of analysis.

Suppose we generate a control strategy of the form

$$E(t) = k_o(t)X(t) + \dot{k}_o(t)X_f(t) \quad (24)$$

where $X_f(t)$ is the output of a first-order filter of the form

$$\dot{X}_f(t) + aX_f(t) = X(t) \quad (25)$$

and $X(t) = G_c(s)q'$. Using the control strategy in Eq. (24), the heat flux of the controlled combustor can be derived as in Eq. (23) and can be written as

$$q' = W_{cl}(s)[\tilde{k}_o(t)X(t) + \dot{k}_o X_f(t)]$$
$$= W_{cl}(s)(s+a)[\tilde{k}_o X_f(t)]$$

since s denotes the differential operator d/dt. Since $(s+a)W_{cl}(s)$ is of relative degree one with all its poles in the left-half plane, then it can be made SPR by appropriately choosing a if it has stable zeros. Thus, a tuning rule of the form

$$\dot{k}_o = -g_k q' X_f, \; g_k > 0,$$

will guarantee a stable behavior [21]. g_k is an adaptive gain that can be used to adjust the speed of adaptation.

5. Simulation Results

In this section, we evaluate the performance of the controllers, the fixed phase-lead controller and the self tuning controller, proposed in the previous section, using the same combustor rig. We change the operating conditions and examine the performance of the controllers over a range of operating conditions.

We used $k_c = 0.1$, $z_c = 180$, $p_c = 1200$ for the phase shift controller, $k_v = 1$, $t_m = 0.0004$ for injector dynamics (Proportional injectors that satisfy the above values are currently available [22]). The locations of the pole and zero and the gain of the controller is determined to suppress instability when $a = 6.35 \cdot 10^3$ rad/s, $b = 3.40 \cdot 10^6$ kW/kg which correspond a case where $f = 0.75$, $\dot{m}/V = 700kg/m^3 s$. The time constant of the fuel injector is chosen to have sufficient bandwidth. Then, we change the equivalence ratio from $f = 0.75$ to $f = 0.85$ linearly in 1 second and examine the performance of both controllers. Figure 7 shows the result of the simulation using the fixed phase-lead controller. As one can see, the controller is not able to suppress the instability when 13% change in the equivalence ratio is imposed. Only a small region, the fixed phase controller is able to suppress instability.

Next, we examine the performance of the self-tuning controller. We choose the controller parameters as $a = 2000$ and $g_k = 20000$ (a and g_k are chosen to make $(s+a)W_{cl}(s)$ to be SPR and the speed of adaptation is fast enough for system change). As shown in Figure 8, the self-tuning controller successfully stabilizes the system. Figure 9 shows the change of $k_o(t)$ in time and it shows that the gain decreases until 0.26 sec and maintains the value until 0.4 sec. Then the gain decreases again. It is interesting to note in Figure 8 that the heat release amplitude is minimum at the same point where $k_o(t)$ reaches steady-state and as the heat oscillations grow again at 0.4 sec, the gain also starts to tune the value again. Both Figures 8 and 9 clearly show that the self-tuning controller adjusts its gain on-line to adapt the varying combustion dynamics.

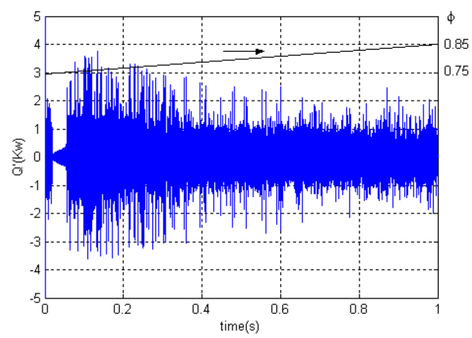

Figure 7 Closed loop response for a fixed phase-shift compensator when f changes from 0.75 to 0.85 in 1 sec.

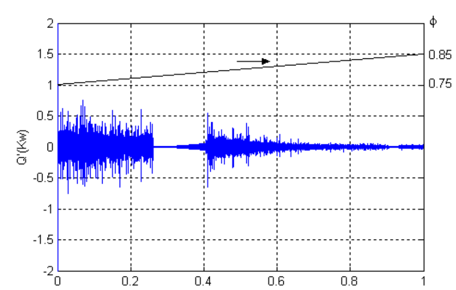

Figure 8 Closed loop response for a self-tuning compensator when f changes from 0.75 to 0.85 in 1 sec.

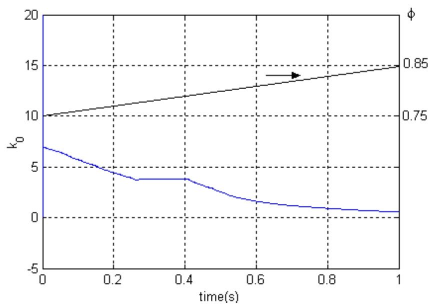

Figure 9 Tuning gain k_o of the self-tuning controller when f changes from 0.75 to 0.85 in 1 sec.

6. Summary

In this paper, we develop a complete combustion model which accommodates acoustics, heat release dynamics, actuation for combustion instability. The heat release dynamics is modeled as a WSR which is applicable for turbulent and kinetically controlled combustion. Modulation of the combustion is realized using a fuel injector and it affects the release rate directly. The WSR model is linearized and the model has three parameters, a , b and s which represent the time scale, gain of the mass oscillation, gain of the control input. These parameters are functions of operating conditions such as equivalence ratio and mass flow rate. By changing the operating condition by 10%, the parameters experience a minimum change of 100%. This rapid change in the parameters makes it necessary to use adaptive control algorithms that can automatically adjust its gain on-line. Both a fixed phase lead controller and a self-tuning controller are used to control combustion instability by varying operating conditions and it is shown that the self-tuning controller is able to satisfactorily suppress instability over a range of operating conditions in comparison to the fixed phase lead controller.

Acknowledgments

This work is sponsored by the National Science Foundation, grant no. ECS 9713415, and the Office of Naval Research, grant no. N00014-99-1-0448.

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