Adaptive Low-order Posi-cast Control of a Combustor Test-rig Model

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Abstract

Recently, an adaptive posi-cast controller has been developed for dynamic systems with large time-delays. In this paper, we evaluate its performance in the context of a 4 MW combustor test-rig model that mimics many of the dynamic characteristics of an actual engine including a significant time-delay. Using closed loop input-output data and system identification, a model of the test-rig was derived. Using this model, adaptive posi-cast controllers were designed and detailed numerical simulation studies were carried out. These studies consisted of (i) the closed-loop performance of the adaptive controller, (ii) comparison of the adaptive controller with an empirical phase-shift controller, (iii) robustness with respect to parametric uncertainties, (iv) robustness with respect to unmodeled dynamics and uncertain delays, (v) performance in the presence of noise, and (vi) effect of saturation constraints on the control input amplitude. These studies show that the adaptive posi-cast controllers decrease pressure oscillations much faster than the phase-shift controller without generating peak splitting. It also showed that the adaptive controller was capable of stabilizing the plant in the presence of 20% change in the resonant frequency, an order of magnitude change in the damping ratio, and unmodeled dynamics. The studies showed that the performance of the adaptive controller improved as the magnitude of the saturation constraints on the control input increased.

1 Introduction

Dynamic instability in continuous combustion processes and its control have been studied intensely in the past few years [1]. These instabilities are often observed in applications such as lean premixed gas turbine combustors, ramjet engines, afterburners, and waste incinerators. Recent modeling efforts in the area of combustion instability and control have shown that one of the most challenging factors for successful control is the presence of large time-delays. These time-delays are present due to transport lags which occur predominantly due to the distance between the point where the control fuel is injected and the heat-release zone where the fuel burns. In some cases, the time-delay can be quite large, of the order of several multiples of the dominant time-constant in the system. As a result, the achievable performance can be severely limited and necessitates that the controller explicitly accommodate the time-delay effect in its design.

The presence of a time-delay is further exacerbated by the inevitable occurrence of parametric and modeling uncertainties. The field of adaptive control has addressed parametric uncertainties in various kinds of dynamic systems including linear and nonlinear, single and multivariable, continuous and discrete, deterministic and stochastic systems. Very few of the results in this area pertain to problems where large time-delays are present. The main implication of this is that most results currently available in the area of adaptive control are applicable to time-delay systems only when the delay values are small.

A unique approach for controlling systems with a known time-delay was originated by Otto Smith in the fifties [2] by compensating for the delayed output using input values stored over a time window of $[t-\tau, t]$ and estimating the plant output using a model of the plant. In [3], this idea was extended to include unstable plants as well using finite-time integrals of the delayed input values thereby avoiding unstable pole-zero cancellations that may occur in Smith's controller. In [4, 5], pole-placement and adaptive versions of [3] were developed, and it was shown that the plant can be adaptively controlled in a stable manner. More recently, in [6]-[10], an adaptive posi-cast controller has been proposed whose design is based on the relative degree of the plant to be controlled. While the order of the controller in [6] and [7] are equal to the order of the plant and address the adaptive and non-adaptive aspects of the design, in [8]-[10], a low-order adaptive controller is treated, for the cases of relative degree two and higher, and when saturation constraints are present in the control input, respectively.

One of the most important steps in the development of a controls technology is the demonstration of scalability. The controls method that is designed and tested in laboratory-scale experiments must eventually be evaluated in large-scale setups and shown to result in cost-effective and satisfactory solutions. In this paper, we evaluate the adaptive posi-cast controller developed in [6]-[10] in the context of an engine-replica test rig [11]. The test facilities reported in [11] were prepared in such a way that the same sub-components such as fuel nozzles and air swirlers used in an actual engine were utilized, and the configuration used was such that the same dominant longitudinal acoustic behavior as in an engine was exhibited by the test rig. Also the rig was designed so that it can be tested at typical engine operating conditions. In this paper, we use the model of such a rig to evaluate the adaptive posi-cast controller. A detailed numerical study is carried out in this paper, where its performance is evaluated over a range of operating conditions and perturbations, and compared with an empirical phase-shift controller. The results show that the adaptive posi-cast controller is capable of providing an order of magnitude improvement in the pressure suppression compared to other empirical controllers, and is robust to parametric and modeling uncertainties.

In section 2, we model the test-rig using closed-loop data obtained with the empirical phase-shift controller in the loop using system-identification. Using the identified model, a posi-cast and a low-order posi-cast controllers are designed in section 3, and we compare the performance of these posi-cast controllers to that of a phase-shift controller. Finally, in section 4, we examine the robustness of the adaptive posi-cast controller to parametric and modeling uncertainties and to external noise.

2 Dynamic Model

While both physically based models [12] and system-identification based models [13] have been suggested for modeling the combustion dynamics, we adopt the latter procedure in this paper. Since it was known from the uncontrolled combustor response that there were pronounced oscillations at a single frequency, and from the combustor configuration that there was a distinct transport delay between the time of application of the control input to the point of its effect on the combustion dynamics, we chose a model structure of the form

$$W(s) = \frac{k(s+a)}{s^2 + 2\zeta\omega s + \omega^2} e^{-\tau s}.$$

In order to determine the parameters of $W(s)$, we used closed-loop input-output data that was available with a phase-shift controller in the loop [14]. We first determined the time-delay τ using the power spectrum of the pressure response obtained in closed-loop (see Figure 1). Noting that periodic spikes occur at the frequencies of 112 and 183 Hz as well as 405 and 475 Hz for $\theta = -90°$, the total time-delay $\overline{\tau}$ in the forward loop is calculated approximately as

$$\overline{\tau} \;=\; \frac{1}{\omega_w}$$

where ω_w is the interval between two consecutive spikes. Since ω_w =70 Hz (see Figure 1), $\overline{\tau}$=14.3msec. Since the phase-shift controller that corresponded to the time-response included a time-delay of 2.7 msec for $\theta = -90°$, we obtain τ =11.6msec.

The remaining parameters of $W(s)$ are obtained using standard system-identification procedures [15]. In particular, using $V_c(t-\tau)$ and p', where V_c is the output of the phase-shift controller and p' is the pressure response with $\theta = -180°$, the parameters k, a, ζ, and ω were obtained using an ARMA model and prediction error methods, to result in a transfer function model

$$W(s) \;=\; \frac{-468s - 1.76 * 10^6}{s^2 + 13.0s + 2.97 * 10^6} e^{-0.0116s}. \quad (1)$$

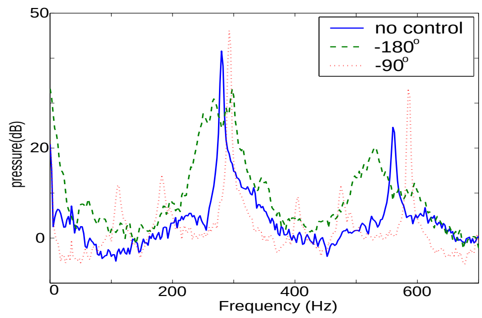


Figure 1: Pressure spectrum in a test-rig for uncontrolled and controlled cases. The controlled pressure response is shown for two different values, $\theta = -90°$ and $\theta = -180°$, of a phase-shift control parameter.

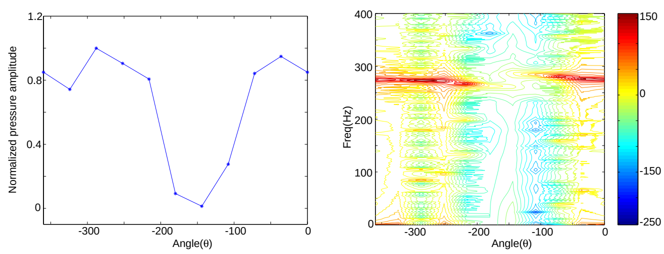
2.1 Model Validation

In order to verify the accuracy of the model in (1), the closed-loop performance using the phase-shift controller in [14] was simulated using $W(s)$. The phase-shift controller has two parameters that correspond to a gain k and a phase θ. With k set to 0.1, closed-loop responses were obtained at various ϕ, and are shown in Figure 2 (a) and (b). As can be seen in Figure 2(a), $\theta = -150°$ provides the optimal pressure response, which is corroborated by the experimental results in Figure 1, where it can be seen that the pressure response is lower when $\theta = -180°$ than when $\theta = -90°$. We note from Figure 2(b) that when θ=$-90°$ the resonant frequency shifts to a higher value and only a single peak is dominant around the resonant frequency. Also, between $-130°$ and $-180°$, two peaks are present on each side of the original resonant frequency. Both of these observations match with experimental data shown in Fig.1.

Next, we examine the effect of the gain of the controller. Figure 3 shows the effect of the gain in the pressure response with a fixed phase angle at at θ=$-150°$.

As shown in Figure 3, initially only one peak is dominant, but secondary peaks occur when the gain increases beyond 0.04 and the optimal gain is obtained at 0.08. Increasing the gain any further does not improve the performance of the phase-shift controller, but in fact increases the pressure amplitude.

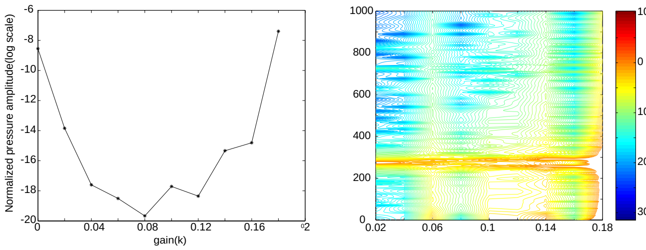
Both of the above show that the combustion model predicts the experimental observations quite well, especially around the unstable frequency of 275 Hz.



(a) Normalized peak pressure amplitude

(b) Pressure power spectrum

Figure 2: Closed-loop pressure power spectrum of a rig model with a phase-shift controller changing a phase angle



(a) Normalized peak pressure amplitude

(b) Pressure power spectrum

Figure 3: Closed-loop pressure power spectrum of a rig model with a phase-shift controller changing a gain with a fixed phase angle at $\theta = -150°$

3 Posi-cast Controllers

In this section, we briefly outline the posi-cast controllers presented in [6]-[10]. In section 3.1, we describe a fixed posi-cast controller whose order is $2n$, where n is the order of the plant being controlled. In section 3.2, we describe a low-order posi-cast controller and its adaptive counterpart (see [8, 10] for details).

3.1 Posi-cast Controller

In this section, we examine the performance of the posi-cast controller. First, we briefly describe the posi-cast controller presented in Ref. [16]. The model in Eq. (1), in the presence of a time-delay, τ, can be re-written as

$$p'(t) = W_p(s)[V_c(t-\tau_c)], \qquad W_p(s) = \frac{k_p Z_p(s)}{R_p(s)}. \quad (2)$$

The presence of the time-delay, τ, in the control input, motivates the use of an additional signal in the control input, $V_c(t)$, denoted as $V_{sc}(t)$ which anticipates the future output using a model of the system. The resulting controller structure is described as

$$\begin{aligned} V_c(t) &= \frac{c(s)}{\Lambda(s)} V_c(t-\tau_c) + \frac{d(s)}{\Lambda(s)} p'(t) + V_{sc}(t), \\ V_{sc}(t) &= \frac{n_1(s)}{R_p(s)} V_c(t) - \frac{n_2(s)}{R_p(s)} V_c(t-\tau_c), \end{aligned} \quad (3)$$

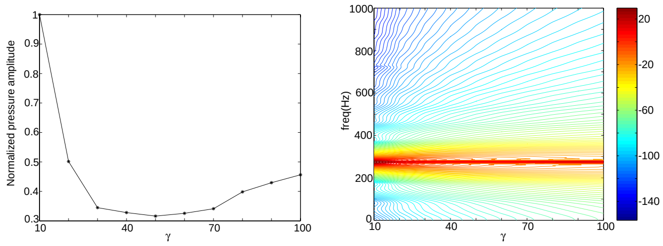
where $\Lambda(s)$ is a chosen stable polynomial of degree $n-1$, $d(s), n_1(s)$ and $n_2(s)$, are polynomials of degree $n-1$ at most, and $c(s)$ is of degree $n-2$ at most. For stability, these must satisfy the relations

$$c(s)R_p(s) + k_p d(s) Z_p(s) = \Lambda(s) n_2(s), \quad (4)$$
$$n_1(s) = R_p(s) - R_m(s), \quad (5)$$

where $R_m(s)$ is the desired characteristic equation, which is a stable monic polynomial of the same order as $R_p(s)$.

Using the controller structure in Eq. (3) with the conditions in Eqs. (4) and (5), the closed-loop transfer function can be computed as

$$W_{cl}(s) = \frac{k_p e^{-s\tau_c}}{R_m(s)}. \quad (6)$$



(a) Normalized peak pressure amplitude

(b) Pressure power spectrum

Figure 4: Closed-loop pressure power spectrum of a rig model with a posi-cast controller changing the closed-loop pole damping.

A modification in the synthesis of $V_{sc}(t)$ in Eq. (3) was suggested by Manitius and Olbrot[3]. To avoid unstable pole-zero cancellations, $V_{sc}(t)$ must be generated as a finite integral of the form

$$V_{sc}(t) = \sum_{i=1}^{n} \left(\int_{-\tau}^{0} \alpha_i e^{-\beta_i \sigma} V_c(t+\sigma) d\sigma \right), \quad (7)$$

where β_i's are the eigen values of the combustor system, i.e. $R_p(s) = \Pi_{i=1}^{n}(s-\beta_i)$. Taking the Laplace transform of Eq. (7), one can show that

$$\frac{n_1(s)}{R_p(s)} = \sum_{i=1}^{n} \frac{\alpha_i}{s-\beta_i}, \frac{n_2(s)}{R_p(s)} = \sum_{i=1}^{n} \frac{\alpha_i}{s-\beta_i} e^{\beta_i \tau}. \quad (8)$$

Another condition for the successful use of the finite integral in Eq. (7) is that $R_p(s)$ has no repeated roots [4].

3.1.1 Performance of the Posi-cast Controller in the Test-rig Model: We evaluated the performance of the controller in (3) and (7) using the model in (1). We chose the filter poles $\Lambda(s) = s + 1.21 \cdot 10^4$. Denoting the ratio of the damping ratio of the closed-loop and open-loop poles to be γ, the resulting performance is shown in Figure 4(a) and (b), where the pressure amplitude is illustrated as γ was increased from 10 to 100. As can be seen from these figures, the optimal response is obtained when $\gamma = 50$.

In Figure 5 the performance of the posi-cast controller is compared with that obtained using the phase-shift controller. As one can see, the posi-cast controller outperforms the phase-shift controller with 20db more reduction in the pressure power spectrum.

3.2 Low-order Posi-cast Controllers

Often the order of the underlying dynamic model can become quite large [8], and as a result, it is more advantageous to derive a lower-order controller to facilitate design and implementation. Such a controller is given by Eq. (3), but with $\frac{c(s)}{\Lambda(s)} V_c(t-\tau_c)$ and $\frac{d(s)}{\Lambda(s)} p'(t)$ replaced by $\frac{k_2}{(s+z_c)}$ and $k_1 p'(t)$, respectively. Then, the closed-loop transfer function becomes

$$W_{cl}(s) = \frac{k_p(s+z_c)Z_p(s)e^{-s\tau}}{R_{cl}(s)} = W_{cl0}(s)e^{-s\tau} \quad (9)$$

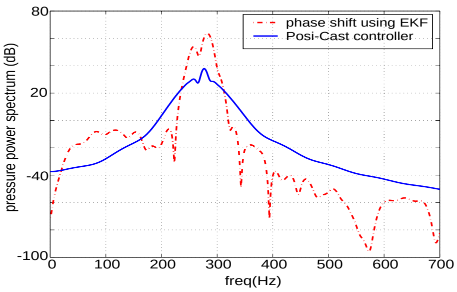


Figure 5: pressure power spectrum of a closed loop rig model with a phase-shift controller and a posi-cast controller

where the closed-loop poles are the zeros of

$$R_{cl}(s) \quad = \quad A(s) + B(s)e^{-s\tau} \qquad (10)$$

with

$$\begin{aligned} A(s) &= (s+z_c)(R_p(s) - n_1(s)) + k_2 R_p(s), \\ B(s) &= (s+z_c)(n_2(s) + k_1 k_p Z_p). \end{aligned}$$

It has been shown in [8] that for a small value of τ (ie. $\tau\omega < O(1)$) where ω is the resonant frequency, the above controller can stabilize a time-delay system with relative degree less than 2. For a large time-delay, bands of stability with a period of $2\pi/\omega$ are typically observed.

The controller structure given in (9) includes fixed controller parameters k_1, k_2, α_i and β_i which need to be chosen based on the system parameters. Under uncertainties and variations in the operating conditions, and also to ensure that the STR design is independent of the details of the modelling, it is more appropriate to determine those control parameters adaptively. An adaptive controller has been developed for this purpose in [8], and we briefly describe the results here.

In the controller law in Eq. (9), the finite integral is approximated as follows:

$$V_{sc}(t) \quad = \quad \sum_{i=1}^{n} \lambda_i(t) V_c(t - idt). \qquad (11)$$

We also define the controller parameters and data vectors $\mathbf{k}$ and $\mathbf{d}$ as

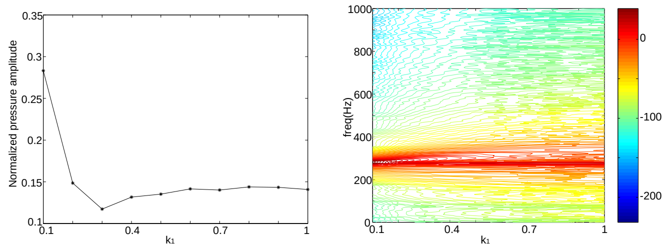
$$\begin{aligned} \mathbf{k}(t)^T &= [-k_1(t), -k_2(t), \lambda_n(t), ..., \lambda_1(t)] \\ \mathbf{d}(t)^T &= [-p(t), V(t), V_c(t - ndt), ..., V_c(t - dt)] \end{aligned}$$

where $V(t) = \frac{1}{s+z_c}[V_c(t)]$ This leads to the following underlying model:

$$p'(s) \quad = \quad W_m(s)e^{-s\tau}[\tilde{\mathbf{k}}^{\mathbf{T}}(t)\mathbf{d_a}](t) \qquad (12)$$

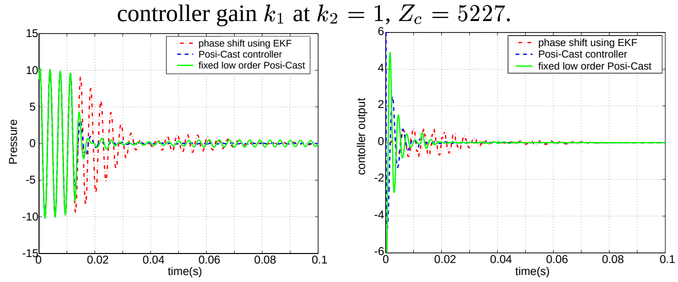
where $\mathbf{d_a}(t) = \frac{1}{s+a}[\mathbf{d}(\mathbf{t})]$ and $W_m(s) = (s+a)W^*{}_{cl0}(s)$ has relative degree 1 and is strictly positive real, and $\tilde{\mathbf{k}} = \mathbf{k} - \mathbf{k}^*$, where $\mathbf{k}^*$ denotes the desired parameter vector for which the closed-loop transfer function matches $W_m(S)$. It is shown in Ref.[17] that the stability of the system is guaranteed when one uses the following STR:

$$\begin{aligned} V_c(t) &= \mathbf{k}^{\mathbf{T}}(t)\mathbf{d}(t) + \dot{\mathbf{k}}^{\mathbf{T}}(t)\mathbf{d_a}(t) \qquad (13) \\ \dot{\mathbf{k}}^{\mathbf{T}}(t) &= -sign(k_p)p'\mathbf{d_a}(t-\tau) \qquad (14) \end{aligned}$$



(a) Normalized peak pressure amplitude (b) Pressure power spectrum

Figure 6: Closed-loop pressure power spectrum of a rig model with a fixed low-order posi-cast controller changing the controller gain k_1 at $k_2 = 1$, $Z_c = 5227$.



(a) Pressure time plot (b) control input

Figure 7: Closed-loop response of a rig model using a phase-shift, a posi-cast and a low-order posi-cast controller.

3.2.1 Performance of the Low-order Posi-cast Controller in the Test-rig Model:

In the fixed low-order controller, k_1 is the most sensitive to the performance of the controller and this value determines the amount of damping. As shown in Figure 6, the pressure amplitude reaches a minimum at k_1=0.3 and stays at this value even as k_1 increases. It is worth noting that the low-order controller does not generate secondary peaks that were observed with the phase-shift controller. Next, we compare the closed-loop pressure response obtained using the low-order posi-cast controller with both the full-order posi-cast controller and the phase-shift controller (see Figure 7). As can be seen in this figure, the pressure response of the fixed low order controller is comparable to the full order posi-cast controller, though the control input corresponding to the low order controller is larger than that of the posi-cast controller.

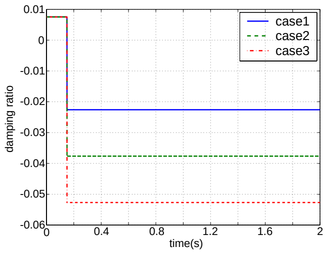
4 Robustness Studies

In this section, we carry out extensive robustness studies with respect to parametric uncertainties, unmodeled dynamics and uncertain delays, performance in the presence of noise, and effect of saturation constraints on the control input amplitude.

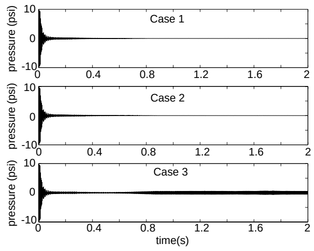
4.1 Effect of Parametric Uncertainties

We examine the effect of changes in both the damping ratio and the unknown frequency in this section.

The damping ratio of the model can be changed due to two reasons: 1) As the operating conditions change, the level of instability varies or even transition from instability

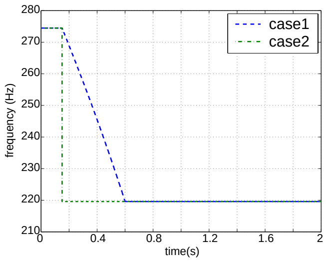


(a) Changes of damping ratio

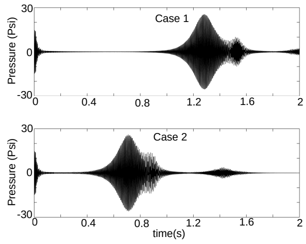


(b) Pressure time response

Figure 8: Pressure response of the closed-loop system using an adaptive low-order posi-cast controller with changes in damping ratio of a rig model



(a) Changes of a resonant frequency



(b) Pressure time responses

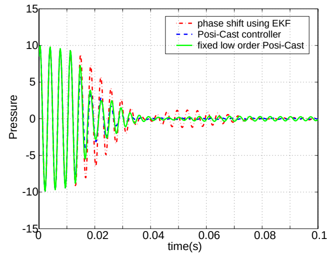
Figure 9: Pressure response of the closed-loop system using an adaptive low-order posi-cast controller with changes in frequency of a rig model

to stability occurs; (2) as the size of the limit cycle decreases due to control, the linear unstable term begins to dominate. We changed the damping ratio of the test-rig model in Eq. (1) (from an original value of $3.76 \cdot 10^{-3}$) as shown in Figure 8 (a). As shown in Figure 8 (b), the adaptive posi-cast controller was able to suppress instability with 500% decrease in the damping ratio. In case of increased damping ratio, it was observed that an increase of up to 2000% in the damping ratio was allowable.

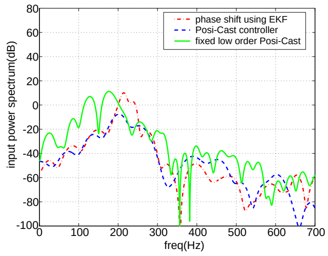
The resonant frequency varies as the temperature of a combustor changes. As a result, at a start up or during thrust change, it is expected that the resonant frequency can change. In Ref. [14], about 20% of resonant frequency change in 9 seconds was observed during warming up of an experimental combustor. Therefore, it is essential to compensate for the change in the resonant frequency so as to be able to use the controller in a wide range of operating condition. In the simulation, the resonant frequency was changed by 10% as shown in Figure 9 (a) and the performance of the adaptive low-order posi-cast controller was tested. Figure 9 (b) shows that regardless of the speed of the frequency change, the controller was able to stabilize the combustor. It was observed that the adaptive posi-cast controller was capable of handling up to a 20% change in the resonant frequency.

4.2 Unmodeled Dynamics

The posi-cast controllers presented in Section 3 were designed using the model in Eq. (1). As mentioned in Section 2, Eq. (1) was determined using input-output data with the input excitation concentrated mainly around the unstable frequency of 275 Hz. In general, the combustion dynamics can have unmodeled dynamics due to various mechanisms such as hydrodynamics, entropy, heat transfer, vaporization, or atomization. A low frequency dynamics at 150 Hz and a high frequency dynamics at 450 Hz were included as second order oscillator with damping ratios ζ_1 and ζ_2, respectively, in the simulation. The adaptive low order controller was developed neglecting these low and high frequency dynamics. From the simulation, it was observed that for values of ζ_1 up to 0.22 and ζ_2 up to 0.034 , the adaptive controller exhibited a satisfactory response.



(a) Pressure time responses



(b) Power spectrum of control input

Figure 10: Pressure time plot of a closed-loop using a phase-shift, a posi-cast and a fixed low-order posi-cast controller in a rig model with control input saturation level as $\pm$ 1.5 $Volt$.

4.3 Uncertain Delay

The impact of a change in the time-delay on the performance of the adaptive low-order posi-cast controller was also evaluated. Such a change can occur with a change in the flow rate or a change of the burning zone location. An additional delay of $\tau_\triangle$ was included in the simulation. For all $\tau_\triangle \approx \pm 0.45\tau_{ac}$, where τ_{ac} is the acoustic time-constant $(1/275s)$, the controller was able to stabilize the pressure oscillations.

4.4 External Noise

The unmodeled dynamics in a combustor is often in the form of a broad-band noise. We therefore studied the performance of adaptive low-order posi-cast controller in the presence of external noise. In the simulation, white noise, at a sampling rate of 2 kHz, was included and supplied with the control signal to the test-rig model in Eq. (1). The signal to noise ratio between the the control signal and the white noise was -0.5dB. It was observed that the adaptive low-order posi-cast controller is able to reduce pressure oscillations by half in a pressure time plot compared to the phase-shift controller.

4.5 Saturation

The simulation results show that both the full-order and low-order posi-cast controllers initially require large control input and then the input magnitudes are decreased rapidly as the pressure amplitudes decrease. Hence, the impact of saturation of the control input needs to be examined. The saturation level of the control signal in the closed-loop experiment shown in Figure 1 was ± 1.5 $Volt$. With this level, the closed-loop pressure response was obtained using different control algorithms. Figure 10 shows the pressure time plot of the closed-loop systems with saturation level ± 1.5 $Volt$. Compared to Figure 7, which has no saturation, the performance of the posi-cast controller and the adaptive low-order controller are worsened, but their performance is still better than the phase-shift controller. As seen in Figure 10 (b), the bandwidth requirement of the control input in posi-cast controllers are less than that of the phase-shift controller.

5 Summary

In this paper, we investigated the performance of an adaptive low-order posi-cast controller on a 4MW combustor test-rig model which has a large time-delay. First, the combustor model was acquired from the closed loop input-output data using system-identification, and represented as a second order oscillator with a time-delay. The simulation results of the phase-shift controller with the test-rig model show that the model predicts the frequency shifting, peak splitting and the optimal phase angle, which means that the model is in good agreement around the resonant frequency. Simulation results with both the full-order posi-cast and the low-order posi-cast controllers show that these "forecasting controllers" decrease pressure oscillations much faster than the phase-shift controller without generating secondary peaks which limit the performance of the phase-shift controller. Next, we examined the robustness of an adaptive low-order posi-cast controller changing resonant frequency, damping ratio, time-delay, and including unmodeled dynamics and noise. It was shown that the adaptive low-order controller was able to stabilize the combustor model within a wide range, for example, 20% change of resonant frequency and an order of magnitude change in the damping ratio, significant unmodeled dynamics away from the resonant frequency with damping ratios between 0.2-0.034. The saturation of a control input was also investigated and the results show that by increasing the saturation level, the performance of the posi-cast controllers can be improved.

6 Acknowledgements

This work is sponsored in part by the National Science Foundation, grant no. ECS 9713415, and in part by the Office of Naval Research, grant no. N00014-99-1-0448. We gratefully acknowledge the assistance of Prabir Barooah and Andrzej Banaszuk at UTRC, and John de Laat, NASA Glenn, in providing us with the data related to the test-rig.

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