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Parameter Convergence in systems with Convex/Concave Parameterization[1]

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Abstract

A large class of problems in parameter estimation concerns systems where parameters occur nonlinearly. In [1]-[5], a stability framework for identification and control of such systems has been established. We address the issue of parameter convergence in such systems in this paper. Sufficient conditions under which parameter estimates converge to their true values are derived and shown to be stronger than the standard persistent excitation requirements in linearly parameterized systems.

1 Introduction

The requirement for linear parameterization in adaptive control problems constrains its applicability since many of the dynamical systems in nature exhibit such behavior which can only be accurately captured and represented by nonlinearly parameterized models. Recently, a stability framework has been established for studying identification and control of nonlinearly parameterized (NLP) systems in [1]-[5]. In these papers various NLP systems were considered and the conditions for global stability, regulation and tracking were derived. In [6], conditions for parameter convergence in a discrete-time system were considered. In this paper we address the issue of parameter convergence in continuous time-systems. We derive conditions under which parameter estimates converge to their true values once a stable estimator has been established. These conditions are related to persistent excitation relevant for convergence in linearly parameterized systems, and are shown to be stronger, with the additional complexity being a function of the underlying nonlinearity. While the results are presented in the context of parameter identification in a first-order system, the same approach can be extended to all problems treated in [4].

2 Statement of the Problem

Our objective is to identify unknown parameters in one class of nonlinear systems. The dynamics of the system is

$$\dot{y} = -\alpha y + a(f(u,\omega))$$

where $a \in \mathbb{R}$, $\omega \in \mathbb{R}^m$ are bounded unknown parameters and $u \in \mathbb{R}^n$ are inputs. The function f is a scalar valued function given by $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, and $\theta = [a\ \omega^T]^T \in \Theta$ are bounded unknown parameters.

The parameter estimation algorithm that we propose is as follows:

$$\begin{aligned}
\dot{\hat{y}} &= -\alpha\hat{y} - \epsilon sat(\tfrac{\tilde{y}}{\epsilon}) + \hat{a}(f(u,\hat{\omega})) - a^* sat(\tfrac{\tilde{y}}{\epsilon}) \\
\dot{\hat{a}} &= -\tilde{y}_\epsilon f(u,\hat{\omega}) \\
\dot{\hat{\omega}} &= -\tilde{y}_\epsilon \phi^* \\
\tilde{y}_\epsilon &= \tilde{y} - \epsilon sat(\tfrac{\tilde{y}}{\epsilon}) \\
\tilde{y} &= \hat{y} - y,
\end{aligned}$$

where a^* and ϕ^* come from the solution of an optimization problem

$$\begin{aligned}
a^* &= \min_{\phi \in \mathbb{R}^m} \max_{\theta \in \Theta} g(\theta,\phi) &(1)\\
\phi^* &= arg \min_{\phi \in \mathbb{R}^m} \max_{\theta \in \Theta} g(\theta,\phi) \\
g(\theta,\phi) &= a sat\left(\frac{\tilde{y}}{\epsilon}\right)\left(f(u,\hat{\omega}) - f(u,\omega) - \phi^T(\widehat{\omega} - \omega)\right).
\end{aligned}$$

Suppose $\hat{\theta} = [\hat{a},\ \widehat{\omega}^T]^T$, our objective is to find sufficient conditions on u that are needed to ensure that $\hat{\theta}$ can converge to θ in a stable manner.

We define

$$\tilde{a} = \hat{a} - a, \qquad \tilde{\omega} = \widehat{\omega} - \omega, \qquad \tilde{\theta} = \hat{\theta} - \theta,$$

and we can rewrite the dynamics of the whole parameter estimation algorithm as

$$\begin{aligned}
\dot{\tilde{y}} &= -\alpha\tilde{y}_\epsilon + \hat{a}f(u,\hat{\omega}) - af(u,\omega) - a^* sat(\tfrac{\tilde{y}}{\epsilon}) \\
\dot{\tilde{a}} &= -\tilde{y}_\epsilon f(u,\hat{\omega}) &(2)\\
\dot{\tilde{\omega}} &= \dot{\widehat{\omega}} = -\tilde{y}_\epsilon \phi^*.
\end{aligned}$$

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Let $x = [\tilde{y}_\epsilon,\ \tilde{a},\ \tilde{\omega}^T]^T$. Our main objective is to show that the system (2) has uniform asymptotic stability with the origin at $x = 0$.

The stability of (2) follows since

$$V = \tilde{y}_\epsilon^2 + \tilde{a}^2 + a\tilde{\omega}^2$$

can be shown to be a Lyapunov function, and hence,

$$V(t_1) \leq V(t_0) \qquad \forall\, t_1 \geq t_0. \tag{3}$$

3 Parameter Convergence

The structure of the estimator in (2) implies that when $|\tilde{y}| \leq \epsilon$, adaptation stops and hence there is a dead-zone. This in turn implies that for parameter convergence to take place, conditions on u should be such that the trajectories leave the dead-zone sufficiently often. As a result, the conditions of "persistent excitation" are required to be stronger than in the case when unknown parameters occur linearly.

We now wish to show that under certain conditions on u, $\hat{a} \to a$ and $\hat{\omega} \to \omega$ uniformly. That is, we wish to show that for the system in (2), where ϕ^* and a^* are obtained by (1), the origin $x = 0$ is uniformly asymptotically stable (u.a.s). We define

$$\begin{array}{lll} \beta = & 1 & \text{if } af(\hat{\omega}, u) \text{ is convex;} \\ \beta = & -1 & \text{if } af(\hat{\omega}, u) \text{ is concave.} \end{array} \tag{4}$$

We assume in what follows that f is Lipschitz with respect to its arguments. That is, $B_\theta > 0$ exists such that

$$|\hat{a}f(u, \hat{\omega}) - af(u, \omega)| \leq B_\theta \|\tilde{\theta}\| \tag{5}$$

where $\tilde{\theta} = [\tilde{a},\ \tilde{\omega}^T]^T$. We also define constants B_f and B_ϕ which satisfies

$$|f(u(t), \hat{\omega}(t))| \leq B_f, \qquad |\phi^*(t)| \leq B_\phi,\ \ \forall\, t \geq t_0. \tag{6}$$

Before we proceed with the main result, a few properties of the estimator are stated in Section 3.1. The main result is stated and proved in Section 3.2.

3.1 Preliminaries

The key properties of the system in (2) are stated below. This is followed by a Lemma useful for the proof of parameter convergence.

Property 1: There exist positive values b_l and b_u, such that $b_l V(t) \leq \|x(t)\|^2 \leq b_u V(t)$.

Property 2: For a^* and β defined as in (1) and (4), respectively, the following holds:

$$\beta a^* \tilde{y} \leq 0$$

Lemma 1: For given systems

$$\begin{array}{rcl} \dot{x} & = & -\alpha x + z(t) \\ \dot{x}_m & = & -\alpha x_m + z_m \end{array} \tag{7}$$

where $\beta > 0$, and $|z(t)| \leq z_m$ for $t \geq t_0$,

(*i*) if $x(t_0) \leq x_m(t_0)$, then $x(t) \leq x_m(t), \forall\, t \geq t_0$;
(*ii*) if $x(t_0) < x_m(t_0)$, then $x(t) < x_m(t), \forall\, t \geq t_0$.

3.2 The Main Result

The sufficient condition for the u.a.s of system (2) is stated in the following theorem.

Theorem 1 If for every $t_1 > t_0$, there exist positive constants $T_0, \epsilon_0, \delta_0$, and a subinterval $[t_2, t_2 + \delta_0] \in [t_1, t_1 + T_0]$ such that

$$\beta \int_{t_2}^{t_2+\delta_0} (\hat{a}(t_2) f(u, \hat{\omega}(t_2)) - af(u, \omega))\, d\tau \geq 2\epsilon + \epsilon_0 \|\tilde{\theta}(t_2)\| \tag{8}$$

then the origin Ω defined by $\{\Omega : x \mid x = 0\}$ is uniform asymptotically stable(u.a.s).

Proof of Theorem 1: In what follows, we assume that $\beta = -1$. The proof can be given in a similar manner if $\beta = +1$. When $\beta = -1$, (8) can be rewritten as

$$\int_{t_2}^{t_2+\delta_0} (\hat{a}(t_2) f(u, \hat{\omega}(t_2)) - af(u, \omega))\, d\tau \leq -2\epsilon - \epsilon_0 \|\tilde{\theta}(t_2)\|. \tag{9}$$

We define the following constants:

$$k_1 \leq \frac{\sqrt{b_l}\,\epsilon_0}{2 + \alpha\delta_0 + \epsilon_0 + B_\theta \delta_0^2 (B_f + B_\phi)}, \tag{10}$$

$$k_2 = \frac{k_1}{\alpha k_1 + B_\theta \sqrt{b_u}}., \tag{11}$$

$$k_3 = 2\alpha \frac{(k_1)^3}{3 B_\theta \sqrt{b_u}}, \tag{12}$$

$$\gamma_1 = 1 - \frac{2\alpha (k_1)^3}{3(\alpha k_1 + B_\theta \sqrt{b_u})},$$

$$p(\gamma_0) = 0, \qquad \text{where } p(\psi) = k_3 \psi^{\frac{3}{2}} + \psi - 1,$$

$$\gamma = \max\{\gamma_0,\ \gamma_1\},$$

$$k_4 = \frac{k_1 \gamma}{B_\theta \sqrt{b_u}}, \tag{13}$$

$$T = T_0 + \delta_0 + c_1 + c_2. \tag{14}$$

Using the above constants, it can be shown that γ satisfies the following inequalities:

$$1 > \gamma \geq 1 - \frac{2\alpha (k_1)^3}{3(\alpha k_1 + B_\theta \sqrt{b_u})}, \tag{15}$$

$$\gamma \geq 1 - k_3 \gamma^{\frac{3}{2}}. \tag{16}$$

We prove u.a.s by showing that

$$V(t_0 + T) \leq \gamma V(t_0), \qquad \text{for every } t_0. \tag{17}$$

Let $t_1 = t_0 + k_4$, $T_1 = T_0 + \delta_0$. We first prove that there always exists a $t_2 \in [t_1,\ t_1 + T_1]$ such that

$$|\tilde{y}_\epsilon(t_2)| \geq k_1\sqrt{V(t_2)} \tag{18}$$

Equation (18) is proved by contradiction. Let

$$|\tilde{y}_\epsilon(t)| < k_1\sqrt{V(t)} \qquad \forall\, t \in [t_1,\ t_1 + T_1]. \tag{19}$$

From (2), we have that

$$\tilde{y}(t_2+\delta_0)=\tilde{y}(t_2)+\int_{t_2}^{t_2+\delta_0}(-\alpha\tilde{y}_\epsilon(\tau)+\hat{a}\hat{f}(u,\hat{\omega})-af(u,\omega)-a^*sat(\tfrac{\tilde{y}}{\epsilon}))d\tau.$$

From (19) and (3), it follows that for $\tau \in [t_2, t_2 + \delta_0]$,

$$-\alpha\tilde{y}_\epsilon(\tau) \quad \leq \quad \alpha k_1\sqrt{V(\tau)} \leq \alpha k_1\sqrt{V(t_2)}. \tag{20}$$

From (5), (6), (9) and (19), it follows that

$$\begin{aligned}&\int_{t_2}^{t_2+\delta_0}[\hat{a}(\tau)f(u,\widehat{\omega}(\tau) - af(u,\omega)]d\tau \\ &\leq -2\epsilon - \epsilon_0\|\tilde{\theta}(t_2)\| + B_\theta\delta_0^2(B_f + B_\phi)k_1\sqrt{V(t_2)}\end{aligned} \tag{21}$$

From Property 2, we have that $a^*sign(\tilde{y}) \geq 0$. Since $x = [\tilde{y}_\epsilon,\ \tilde{\theta}^T]^T$, (19) implies that

$$\|x(t_2)\| \leq \|\tilde{\theta}(t_2)\| + k_1\sqrt{V(t_2)}, \tag{22}$$

Using these facts, and equations (20) and (21), we obtain that

$$\tilde{y}(t_2+\delta_0)\leq-\epsilon-\epsilon_0(\|x(t_2)\|+(1+\epsilon_0+\delta_0\alpha+B_\theta\delta_0^2(B_f+B_\phi))k_1\sqrt{V(t_2)}). \tag{23}$$

Using (22), property 1, the definition of k_1 in (10) and $\tilde{y}_\epsilon$ in (2), (23) can be further simplified as

$$\tilde{y}_\epsilon(t_2 + \delta_0) \leq -k_1\sqrt{V(t_2 + \delta_0)}$$

which proves (18). Equation (18) implies that there are two cases to be considered: Two cases now arise: $\exists t_3, t_4 \in [t_1,\ t_1 + T_1]$ so that $(i)\tilde{y}_\epsilon(t_3) \leq -k_1\sqrt{V(t_3)}$, $(ii)\tilde{y}_\epsilon(t_4) \geq k_1\sqrt{V(t_4)}$. The proof of *Theorem 1* is given by considering Case (i) and Case (ii) separately.

Case (i): $\tilde{y}_\epsilon(t_3) \leq -k_1\sqrt{V(t_3)}$.

Let $\Delta T = [t_3, t_3 + k_5]$. We focus on the behavior of $\tilde{y}_\epsilon(t)$ for $t \in \Delta T$, where k_5 is such that

$$\tilde{y}_\epsilon(t) < 0 \qquad\qquad \text{for } \ t \in \Delta T. \tag{24}$$

Such a k_5 exists since $\tilde{y}_\epsilon(t_3) < 0$ and $\dot{\tilde{y}}_\epsilon$ is bounded. From (24) and Property 2, we have that $a^* = 0$ for $t \in \Delta T$. From (2), it follows that for $t \in \Delta T$,

$$\dot{\tilde{y}}_\epsilon = -\alpha\tilde{y}_\epsilon + m(t)$$

where $m(t) = \hat{a}f(u,\hat{\omega})-af(u,\omega)$. From Property 1, Property 2 and (3), it follows that $\forall\ \tau \geq t_3$,

$$|m(\tau)| \leq B_\theta\|x(\tau)\| \leq B_\theta\sqrt{b_u}\sqrt{V(\tau)} \leq B_\theta\sqrt{b_u}\sqrt{V(t_3)}. \tag{25}$$

Let $y_m(t)$ be specified as the solution of the following differential equation for $t \geq t_3 + \tau$, $\tau \geq 0$.

$$\dot{y}_m = -\alpha y_m+B_\theta\sqrt{b_u}\sqrt{V(t_3)}, \qquad y_m(t_3) = -k_1\sqrt{V(t_3)}. \tag{26}$$

From (25) and (26), Lemma 2 implies that

$$\tilde{y}_\epsilon(t_3 + \tau) \leq y_m(t_3 + \tau), \qquad \text{for } \ \tau \geq 0. \tag{27}$$

We can compute $y_m(t)$ to be

$$y_m(t_3 + \tau) =(-\tfrac{B_\theta\sqrt{b_u}\sqrt{V(t_3)}}{\alpha}-k_1\sqrt{V(t_3)})e^{-\alpha\tau}+\tfrac{B_\theta\sqrt{b_u}\sqrt{V(t_3)}}{\alpha},$$

We note that $y_m(t_3+\tau)$ is a concave function of τ for $\tau \geq 0$. From properties of concave functions, it can be shown that $y_m(t_3 + \tau)$ satisfies the inequality

$$y_m(t_3 + \tau) \leq y_m(t_3) + \nabla_\tau y_m(t_3 + \tau)\ |_{\tau=0}\,. \tag{28}$$

From (27) and (28), we obtain

$$\tilde{y}_\epsilon(t_3 + \tau) \leq (-k_6 + k_7\tau)\sqrt{V(t_3)}, \qquad \text{for } \tau \geq 0, \tag{29}$$

where

$$k_6 = k_1, \qquad k_7 = \alpha k_1 + B_\theta\sqrt{b_u}.$$

If we choose $k_2 = \frac{k_6}{k_7}$ as in equation (13), equation (29) implies that

$$\tilde{y}_\epsilon(t) < 0 \qquad \forall\, t \in [t_3,\ t_3 + k_2],$$

and hence for $t \in [t_3,\ t_3 + k_2]$, $a^* = 0$. [1]

From (11) and (29), it follows that

$$\int_{t_3}^{t_3+k_2} |\tilde{y}_\epsilon(\tau)|^2 d\tau \geq \frac{k_6^3}{3k_7}V(t_3).$$

and that

$$V(t_3 + k_2) \quad \leq \quad V(t_3) - \tfrac{2\alpha(k_1)^3}{3(\alpha k_1+B_\theta\sqrt{b_u})}V(t_3). \tag{30}$$

From (15), equation (30) can be rewritten as

$$V(t_3 + k_2) \leq \gamma V(t_3)$$

Since t_3 and $t_3 + k_2$ belong to $[t_0,\ t_0 + T]$, and since $V(t)$ is nonincreasing, we have that

$$V(t_0 + T) \leq \gamma V(t_0),$$

and hence u.a.s follows.

Case (ii): $\tilde{y}_\epsilon(t_4) \geq k_1\sqrt{V(t_4)}$.

Let

$$t_5 = t_4 - k_4 \qquad \text{and} \qquad k_3 = B_\theta\sqrt{b_u}\sqrt{V(t_5)}, \tag{31}$$

For any $t_6 \in [t_5,\ t_4]$, we shall show by contradiction that

$$\tilde{y}_\epsilon(t_6) \geq k_3(t_6 - t_5)$$

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[1] This shows that k_2 in (13) is a more accurate and tight estimate of k_5, even though k_5 is guaranteed to exist.

Let

$$\tilde{y}_\epsilon(t_6) < k_3(t_6 - t_5) \qquad \text{for } t_6 \in [t_5,\ t_4]. \tag{32}$$

From (2), it follows that

$$\dot{\tilde{y}}_\epsilon = -\alpha \tilde{y}_\epsilon + m(t) \tag{33}$$

where $m(t) = \hat{a} f(u, \hat{\omega}) - a f(u, \omega) - a^* sat(\frac{\tilde{y}}{\epsilon})$. From Property 2, and since $\beta = -1$,

$$-a^* sat(\frac{\tilde{y}}{\epsilon}) \leq 0. \tag{34}$$

From Property 1, Assumption *A1*, (3) and (34), it follows similar to Case (i) that

$$|m(\tau)| \leq B_\theta \|\theta(\tau)\| \leq B_\theta \sqrt{b_u} \sqrt{V(t_5)} \qquad \forall\, \tau \geq t_5. \tag{35}$$

Let $y_m(t)$ be specified as

$$\dot{y}_m = -\alpha y_m + B_\theta \sqrt{b_u} \sqrt{V(t_5)} \qquad y_m(t_6) = k_3(t_6 - t_5). \tag{36}$$

Equations (33)-(36) can be used again as in *Case (i)* and combine Lemma 2 (ii) to show that

$$\tilde{y}_\epsilon(t_6 + \tau) < y_m(t_6 + \tau), \qquad \text{for } \tau \geq 0. \tag{37}$$

We can compute $y_m(t)$ to be

$$y_m(t_6 + \tau) = (-\tfrac{B_\theta \sqrt{b_u} \sqrt{V(t_5)}}{\alpha} + k_3(t_6 - t_5))e^{-\alpha\tau} + \tfrac{B_\theta \sqrt{b_u} \sqrt{V(t_5)}}{\alpha}, \tag{38}$$

As before, using properties of concave function functions, $y_m(t_6 + \tau)$ can be shown to satisfy the inequality

$$y_m(t_6 + \tau) \leq k_3(t_6 - t_5) + (\nabla_\tau y_m(t_6 + \tau) \mid_{\tau=0}) \tau. \tag{39}$$

From (38) and (31) and since $t_6 \geq t_5$, we have

$$\nabla_\tau y_m(t_6 + \tau) \mid_{\tau=0} = B_\theta \sqrt{b_u V(t_5)} - \alpha k_3(t_6 - t_5) \leq k_3.$$

Setting $\tau = t_4 - t_5$ in equation (39), we obtain that

$$y_m(t_4) \leq k_3 k_4,$$

and hence, from (37) and (31) that

$$\tilde{y}_\epsilon(t_4) < k_1 \sqrt{\gamma V(t_5)}. \tag{40}$$

We proceed to prove u.a.s in Case (ii) by using (40) and proof by contradiction. Suppose Eq. (17), which is a sufficient condition for u.a.s does not hold. This implies that

$$\gamma V(\tau_1) < V(\tau_2) \qquad \forall\, t_0 \leq \tau_1 < \tau_2 \leq T_0 + T. \tag{41}$$

Therefore (40) can be rewritten as

$$\tilde{y}_\epsilon(t_4) < \epsilon_2 V(t_4),$$

which contradicts our assumption of *Case (ii)*. Therefore, our assumption in (32) is false. As a result,

$$\tilde{y}_\epsilon(t) \geq k_3(t - t_5) \qquad \forall\, t \in [t_5,\ t_4]. \tag{42}$$

Integrating (42) over $[t_5, t_5 + k_4]$, it follows that

$$\int_{t_5}^{t_5 + k_4} |\tilde{y}_\epsilon(\tau)|^2 d\tau \geq \frac{k_3^2 k_4^3}{3}$$

and that

$$V(t_5 + k_4) \quad \leq \quad V(t_5) - 2\alpha \tfrac{k_3^2 k_4^3}{3} V(t_5). \tag{43}$$

From (16) and (43), we have that

$$V(t_5 + k_4) \leq \gamma V(t_5).$$

Since $t_4 \in [t_1, t_1 + T]$, it follows that $t_5 \in [t_0, t_0 + T_0]$. Combining this with the fact that $V(t)$ is nonincreasing, we have that

$$V(t_0 + T) \leq \gamma V(t_0)$$

This contradicts (41). This concludes the proof.

We shall denote the condition stated in eq. (8) as a nonlinearly-parameterized persistent excitation (NLP-PE) condition, which is distinct from its linear counterpart(LP-PE) as defined in [7]. When comparing NLP-PE and LP-PE two main differences can be observed. Both NLP-PE and LP-PE impose certain conditions on the values of the integrals of certain system signals. These conditions are required so that the only possible equilibrium points for the system are those where the parameter error is zero. The differences between the two cases concern the different requirements on the sign and magnitude of certain system integrals.

Unlike LP-PE, in the case of the NLP-PE, the sign of the integral is crucial. The sign does not solely depend on the sign of $f(u, \hat{\omega}) - f(u, \omega)$, but on the convexity/concavity of f as well. This coupling is introduced through the value of β which can take on either positive or negative value. The coupling arises from the features of the min-max adaptive algorithm and is required in order to enable the algorithm to escape the adaptation dead-zone specified by the region of the state-space where $|\tilde{y}| \leq \epsilon$. Also, the coupling between the sign of the integral and β is necessary, but not sufficient for the algorithm to leave the dead-zone. Therefore, a second difference between NLP-PE and LP-PE is introduced. Since the dead-zone has a finite size, the persistent excitation integral must have a large enough value to overcome the dead-zone. Because the dead-zone is finite, this value must be finite and bounded away from zero. That is the reason why there is a term containing ϵ, the size of the dead-zone, on the right hand side of (8).

4 The NLP-PE Condition

The NLP-PE condition (8) in Theorem 1 specifies certain requirements on f in order to achieve parameter convergence. Since f depends on the time-varying signal u, the persistent excitation conditions ultimately are translated to requirements on u. For a given f, theorem 1 does not state how u should behave over time in order to satisfy the requirements, or even whether such a u is possible. In this section, we first state some observations about how u should behave in order to satisfy NLP-PE for a generic function f that satisfies assumptions (A1)-(A6). Next, we apply these observations to a specific case of f, and give an example of u that enables parameter convergence.

When examining NLP-PE as given in eq.(8), it can be considered that it consists of two separate components. The first component is that the magnitude of the integral in (8) must be large. The second component is that the integral must have the same sign as β over $[t_2, t_2 + \delta_0]$. We denote the integrand in (8) as $\tilde{f}$. The first component of NLP-PE states that for a large parameter error, there must be a large error in $\tilde{f}$. It is straightforward to demonstrate that this condition is equivalent to LP-PE. This is essentially an identifiability condition, since it is not possible to estimate the parameter values exactly if there doesn't exist a u such that, for a large parameter error, it produces a noticeable error in the system output. That is, parameter values for which all possible values of u produce an equivalent output are, for all practical purposes, equivalent and indistinguishable from each other.

The second component of NLP-PE states what the sign of $\tilde{f}$ should be in relation to the convexity/concavity of f. In case that f is convex, $\tilde{f}$ should be positive, and conversely, in case f is concave, $\tilde{f}$ should be negative. This implies that the system should periodically move to the region of the phase-plane where the gradient information is used for updating the parameter estimates. Hence, the min-max feature of the algorithm is necessary to ensure stability, but is not sufficient to guarantee parameter convergence. Parameter convergence is ensured by repeatedly turning on the gradient component of the min-max algorithm.

The coupling of convexity/concavity and the sign of the integral of $\tilde{f}$ has the following practical implications. Suppose that u is such that f is always identifiable, and that for a certain value of $\tilde{\theta}$, the integral in (8) is negative. To ensure parameter convergence, u must be such that one of the following occurs:

(a) For the given $\tilde{\theta}$, u must change in such a way that the sign of $\tilde{f}$ is reversed, while keeping the convexity/concavity of f the same, *or*

(b) For the given $\tilde{\theta}$, u must reverse the convexity/concavity of f, while preserving the sign of $\tilde{f}$

For u to be persistently exciting, it must be able to achieve either *(a)* or *(b)* for any combination of $\hat{\theta} \in \Theta$ and $\theta \in \Theta$.

We illustrate the above comments with a discussion of persistent excitation for the following specific example of f:

$$f = e^{-u^T\theta} \tag{44}$$

where $u : \mathbb{R} \to \mathbb{R}^n$, $\theta \in \Theta \subset \mathbb{R}^n$. It can be checked that f given in (44) is always convex with respect to θ for all u. Therefore, option *(b)* is not possible. Hence, u must be such that $\tilde{f}$ can switch sign for any $\tilde{\theta}$ as required by option *(a)*. The following definition states the desired property of the probing signal u.

Definition 1 *Let $w \in \mathbb{R}^n$ be any unit vector. A bounded function $u : \mathbb{R} \to \mathbb{R}^n$ is said to belong to class K^n if for any t_1 there exist positive constants ϵ_u, δ and T, and a subinterval $[t_2, t_2 + \delta] \in [t_1, t_1 + T]$ such that*

$$u^T(\tau)\, w \;\geq \epsilon_u \qquad \forall \tau \in [t_2, t_2 + \delta]$$

Definition 1 states that, periodically, the vector u should have a *positive* component along every w in $\mathbb{R}^n$, an example of which is $u = [\sin \omega t,\ \cos \omega t]^T$. This is more restrictive than the linear case, [7, 8] where u must have only a nonzero component periodically along every vector in $\mathbb{R}^n$. We now state the condition on u for NLP-PE.

Lemma 1 *Let $h(\epsilon)$ be a constant on the order of ϵ. For f defined as in eq. (44), and for $\tilde{\theta} > h(\epsilon)$, $u \in K^n$ implies that u is NLP-persistently exciting.*

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