Active control using fuel-injection of time-delay induced combustion instability

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Abstract

Active control using periodic fuel injection has the potential of suppressing combustion instability without radically changing the engine design or sacrificing performance. In this paper, we carry out a study of optimal model-based control of combustion instability using fuel injection. The model developed is physically based and includes the acoustics, the heat-release dynamics, their coupling, and the injection dynamics. A heat-release model with fluctuations in the flame surface area as well as in the equivalence ratio is derived. We show that area fluctuations coupled with the velocity fluctuations drive longitudinal modes to resonance caused by phase-lag dynamics, while equivalence ratio fluctuations can destabilize both longitudinal and bulk modes caused by time-delay dynamics. Comparisons are made between the model predictions and several experimental rigs. The dynamics of proportional and two-position (on-off) fuel injectors are included in the model. Using the overall model, two different control designs are proposed. The first is an LQG/LTR controller where the time-delay effect is ignored, and the second is a Posi-Cast controller which explicitly accounts for the delay. Injection at (i) the burning zone and (ii) further upstream is considered. The characteristics of fuel injectors including bandwidth, authority (pulsed-fuel flow rate), and whether it applies a proportional or a two-position (on-off) injection are discussed. We show that increasing authority and bandwidth result in improved performance. Injection at (ii) compared to (i) results in a trade-off between improved mixing and increased time-delay. We also note that proportional injection is more successful than on-off injection since the former can modulate both amplitude and phase of the control fuel.

1 Introduction

Combustion instability has been widely observed in premixed combustion at near-stoichiometric operation in high-power combustors [1]-[4] as well as at lean near-flammability conditions in low-emission combustors [5]-[7]. Active control has been recognized as a promising technology to abate combustion instability in practical systems [5]-[11]. Among the various methods, control using periodic fuel injection has been observed to have the maximum impact on instability using the smallest fraction of the system energy. A systematic active control design for pulsing fuel that can guarantee optimal and robust performance over a wide range of operating conditions is therefore highly desirable.

It has long been recognized that combustion instability is governed by strong coupling between the heat release and the acoustic field of the combustor chamber [12]. While acoustics is well known and can be modelled accurately [13], heat release dynamics remains a challenge. Most current designs are based on phase-shift algorithms which succeed over a small range of operating conditions where the frequency and phase characteristics do not vary significantly. In addition, their behavior is not optimal in terms of fuel consumption, settling time, and robustness. In order to design an efficient active controller that can deliver guaranteed performance, a model of the actuated combustor that includes the combustion instability and the underlying dominant interactions between acoustics and the heat release, the actuator dynamics such as its bandwidth, nonlinearities, authority, delay effects, and effects of operating conditions, actuator type, and actuator locations is highly desirable. In this paper, we develop a general model of the acoustics, heat-release, fuel-injector, and various interactions between these components.

The response of heat-release rate for various perturbations in the flow-field is slowly beginning to be understood. Of these, perturbations due to velocity and equivalence ratio are two mechanisms which appear to affect heat-release rate in a dominant way. The heat-release response to perturbations in the velocity through area fluctuations was first quantified in [14] to establish a reduced-order model of combustion instability [15] in a laminar flow, and later adopted in [16] and [17] to establish instability under more turbulent conditions. The heat-release response to perturbations in the equivalence ratio was observed experimentally in [18, 19, 20], with preliminary results related to their modeling reported in [21, 22]. While both mechanisms can destabilize longitudinal modes [9, 23, 19, 24], bulk modes are more strongly affected by the latter [8, 21]. In this paper, we derive a general heat-release model that captures the effect of both velocity and equivalence ratio perturbations. We use the same flame kinematics equation as in [14] and derive a reduced order model of the heat release dynamics assuming that we have conditions of high Damkohler number, weak to moderate turbulent intensity, and that it is a thin sheet separating reactants and products.

The acoustic modes are assumed to be either due to a bulk-mode or a longitudinal mode. Coupling the heat release model and the acoustics, we derive the instability conditions for the combustor. We show that two different perturbations can cause the instability, where the first is due to equivalence ratio fluctuations and a convective delay, while the second is due to velocity fluctuations and a propagative time-delay. The model predictions in the first category are compared to experimental rigs in [19, 20, 8, 25] while those in the second are compared to [26, 23, 27, 28, 9, 29].

The performance of the active controller is tightly correlated with the performance of sensors and actuators. While high-bandwidth devices of the first are available, e.g., pressure transducers and heat-release sensors, actuation by means of fuel injection consists of low bandwidth, limited authority, time-delays, and nonlinearities (in the form of dead-zone, saturation and on-off effects) [30]. It is therefore important to model these effects and include them as much as possible in the control design. The impact of ideal actuators on combustion dynamics was studied extensively in [31] where it was assumed that the actuator has a very high bandwidth and free of nonlinearities as well as time-delay. In this paper, we study the effect of all of the above deviations from the ideal case. The most dominant effect of the actuator dynamics is a time-delay,and occurs due to the distance of the injection location from the burning plane. This effect is modeled explicitly and taken into account in the control design. Using the combined model of the combustion dynamics and the fuel-injector model, two different control designs are proposed in this paper, which include (1) an LQG-LTR controller, and (2) a posi-cast controller. The former is utilized for the case when the injection is at the burning zone since the time-delays present are small, while the latter is relevant when the injector is upstream of the burning zone since it introduces significant delays. In all cases, the impact of bandwidth, authority, and nonlinearities is investigated.

2 Physically-Based Combustor Model

In this section, we model the combustor heat release, acoustics and inhomogeneity dynamics, and we investigate the coupling between these dynamics and the susceptibility to instability.

2.1 Heat Release Dynamics

Modeling of the heat release dynamics is a challenge that has been clouded by the intricacy of turbulent combustion. One way of alleviating the complexity is by modeling turbulent premixed combustion, at high Damkohler numbers and weak to moderate turbulence intensity, as wrinkled laminar flames [2], and first used in [14] for combustion instability modeling. We carry out a similar procedure below but with the exception of incorporating the effects of perturbations in the equivalence ratio as well. The following assumptions are made to derive our model: (i) The flame is a thin interface separating reactants and products and is insensitive to pressure perturbations [5]. (ii) The flame can model turbulent premixed combustion if conditions of high Damkohler number and weak to moderate turbulence intensity prevail [2]-[32]. (iii) The flame is weakly convoluted.

Under these assumptions, the flame surface can be described by a single-valued function $\xi(r,t)$ which represents the instantaneous axial displacement of the flame[1], and the total heat release, Q, is proportional to the integral of this surface over an anchoring ring:

$$\frac{\partial \xi}{\partial t} = u - v\frac{\partial \xi}{\partial r} - S_u(\phi)\sqrt{\left(\frac{\partial \xi}{\partial r}\right)^2 + 1}, \tag{1}$$

$$Q = \kappa(\phi)\int_0^R \sqrt{1 + \left(\frac{\partial \xi}{\partial r}\right)^2}\, dr, \tag{2}$$

where S_u is the burning velocity, $\kappa(\phi) = 2\pi\rho_u S_u(\phi)\Delta h_r(\phi)$, ρ_u is the density of the unburnt mixture, and Δh_r is the heat of reaction. To derive a linear model, the effects of perturbations in both u and ϕ will be considered.

Assuming negligible velocity component in the radial direction, and linearizing around nominal values $\overline{u}$, $\overline{S}_u$, and $\overline{\xi}(r)$, denoting $\overline{(.)}$ and $(.)'$ as steady and perturbation, respectively, we get

$$\frac{\partial \xi'}{\partial t} = u' + \overline{S}_u\frac{\partial \xi'}{\partial r} + \frac{\partial \overline{\xi}}{\partial r}\left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}}\phi', \tag{3}$$

with boundary conditions[2]

$$\xi'(R,t) = 0 \;\; \forall\, t, \qquad \xi'(r,0) = 0 \;\; \forall\, r \tag{4}$$

[1] We consider here a flame stabilized over a perforated plate, R is the radius of the perforation.

[2] It should be noted that with the appropriate change in coordinates and boundary conditions, Eq. (3) can also represent flames stabilized behind a gutter [17], or a dump [16].

Similarly, the unsteady heat release can be linearized by considering, in addition to finite u', perturbations in Δh_r and S_u due to ϕ' to obtain the relation

$$Q'(t) \;=\; \overline{\kappa}\int_0^R \xi'(r,t)dr + d_\phi \phi', \tag{5}$$

where[3]

$$\overline{\kappa} = 2\pi\rho_u S_u \Delta\overline{h}_r, \text{ and } d_\phi = 2\pi\rho_u \left(\overline{S}_u \left.\frac{d\Delta h_r}{d\phi}\right|_{\overline{\phi}} + \Delta\overline{h}_r \left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}} \right) \left(\int_0^R r\overline{\xi}dr \right).$$

It is worth noting that the flame area fluctuation, A'_f, is given by $A'_f(t) = 2\pi\int_0^R \xi'(r,t)dr$. This with Eq. (3) shows that the flame area is affected by both u' and ϕ', and the area in turn impacts Q' as shown in Eq. (5). This also shows that ϕ' affects directly Q' and indirectly through the area fluctuations.

It should be noticed that in Eq. (3) the flame area (represented by ξ') is affected by both forcing in u' and ϕ', which in turn affects the heat release indirectly as shown in Eq. (5). This latter also shows that Q' is directly affected by ϕ' and indirectly through the effect of ϕ' on area fluctuations.

Equation (3) can be manipulated further and solved for ξ' in the Laplace domain as:

$$\xi'(r,s) \;=\; \left(\frac{u'(s)}{s} + \frac{\partial\overline{\xi}}{\partial r} \left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}} \frac{\phi'(s)}{s} \right) \left(1 - e^{-(R-r)\frac{s}{\overline{S}_u}} \right), \tag{6}$$

where s is the Laplace operator. Differentiating Eq. (5) with respect to time, and using Eq. (3), we get

$$\dot{Q}' \;=\; \overline{\kappa}\int_0^R \left(u' + \overline{S}_u \frac{\partial\xi'}{\partial r} + \frac{\partial\overline{\xi}}{\partial r} \left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}} \phi' \right) dr + d_\phi \dot{\phi}', \tag{7}$$

which is integrated over r, as

$$\dot{Q}' \;=\; \overline{\kappa}\left(Ru' - \overline{S}_u \xi'(0,t) + \int_0^R \frac{\partial\overline{\xi}}{\partial r} \left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}} \phi' dr \right) + d_\phi \dot{\phi}'. \tag{8}$$

Taking the inverse Laplace of Eq. (6) at $r = 0$, and substituting in Eq.(8), after some manipulations, we get

$$\dot{Q}' \;=\; d_0 u' + d_1 \left(u'_{\tau_p}(t) \right) + d_2 \left(\phi'_{\tau_p}(t) \right) + d_3 \phi' + d_\phi \dot{\phi}' \tag{9}$$

[3]The factor $\left.\frac{d\Delta h_r}{d\phi}\right|_{\overline{\phi}}$ is positive and $\left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}}$ is also positive when $\phi \leq 1$.

where

$$
\begin{aligned}
x_\tau(t) &\triangleq \int_{t-\tau}^{t} x(\zeta)d\zeta, \\
d_0 &= \overline{\kappa}R, \quad d_1 = -\overline{\kappa}\,\overline{S}_u, \quad d_2 = -\overline{\kappa}\overline{S}_u \left.\frac{\partial S_u}{\partial \phi}\right|_{\overline{\phi}} \left.\frac{\partial \xi(r)}{\partial r}\right|_0, \\
d_3 &= -\overline{\kappa}\frac{\partial \overline{S}_u}{\partial \phi}\overline{\xi}(0), \quad \overline{\kappa} = 2\pi\rho\Delta h_r \overline{S}_u, \quad \tau_f = \frac{R}{\overline{S}_u}
\end{aligned} \tag{10}
$$

τ_f represents the characteristic propagation delay of the flame surface into the reactants flow. Note that for the class of flames considered in the paper, the slope at the flame tip, which is typically conical, is zero, and therefore the third term on the right-hand-side of Eq. (9) can be omitted. We also note that if the dominant instability is due to the bulk-mode, it implies that the velocity fluctuations u' are zero. Hence, unsteady heat release can occur only if equivalence ratio perturbations are present.

2.2 Acoustics

The host oscillators responsible for the combustion instability, in most cases, are generated by resident acoustic modes. These are typically, Helmholtz-type, longitudinal, or transverse, with the type of mode determined by the geometry of the combustion chamber. Helmholtz-type combustion instabilities (also known as bulk mode instabilities) are characterized by low frequencies and no spatial dependence for the pressure, unlike longitudinal modes which resonate at higher frequencies and vary with the span of the combustor depending on the boundary conditions [24]. Combustors exhibit low frequency instabilities, e.g. [16, 8], blamed on bulk modes. The origin of a Helmholtz-type resonance [13] is the coupling between a compressible volume of gas in a large cavity creating a restoring potential energy for an oscillating mass of slug flow gas in a narrow neck attached to the cavity. The slug flow could occur either at the inlet or exit piping to the combustor chamber where the flame resides and which can be considered as the cavity (see Fig. 1).

The governing equations of the Helmholtz-mode are derived formally below by using the following assumptions: (i) The flow is assumed one-dimensional and incompressible in the ducts. (ii) The volume of the combustor chamber is larger than that of each duct. (iii) The gas behaves as a perfect gas, and is inviscid.

Applying the mass and energy conservations in the combustor portrayed in Fig. 1, and using

Figure 1: Schematic diagram of a combustor exhibiting a Helmholtz-type resonance

the perfect gas state equation, the perturbation of the pressure in the combustor cavity around the steady mean can be evaluated as

$$\frac{dp'}{dt} = \frac{1}{V}\left(c_i^2\,\dot{m}'_i - c_e^2\,\dot{m}'_e + (\gamma-1)Q'_f\right), \tag{11}$$

where V is the volume of the cavity, $\dot{m}$ is the mass flow rate of gas, c is the speed of sound, and the subscripts i and e denote inlet and exit, respectively. Using momentum and mass conservation, the perturbed incompressible flow in the ducts satisfies

$$\frac{d\,\dot{m}'_j}{dt} = -A_j\frac{\partial p'_j}{\partial x}(L_j,t), \tag{12}$$

where A and L are the cross-sectional area and length of the slug flow in the j^{th} duct, and $j=i$ or e. By substitution in Eq. (11), we get

$$\frac{d^2p'}{dt^2} + \frac{1}{V}\left(c_i^2A_i\frac{\partial p'_i}{\partial x}(L_i,t) - c_e^2A_e\frac{\partial p'_e}{\partial x}(L_e,t)\right) = \frac{\gamma-1}{V}\frac{dQ'_f}{dt}. \tag{13}$$

Assuming that the inlet and the exit ducts are acoustically open to the atmosphere, i.e., the pressure distribution in the ducts is negligible, hence, $\frac{\partial p'_j}{\partial x}(L_j,t) = \frac{p'}{L_j}$, and this results in the following oscillator equation for the pressure in the combustor:

$$\frac{d^2p'}{dt^2} + 2\zeta\omega\frac{dp'}{dt} + \omega^2p' = \frac{\gamma-1}{V}\dot{q}'(x,t), \tag{14}$$

where $\omega = \sqrt{\frac{c_i^2 A_i}{L_i V} + \frac{c_e^2 A_e}{L_e V}}$ is the effective Helmholtz frequency [13] associated with a combustor connected to ducts. The passive damping in the combustor due to different dissipation sources, e.g. heat-loss and friction, is accounted for in the natural damping ratio, ζ.

The governing equations for a longitudinal mode can be derived in a straight-forward manner [24] and are of the form

$$\frac{\partial^2 p'}{\partial t^2} - \bar{c}^2 \frac{\partial^2 p'}{\partial x^2} = (\gamma - 1)\dot{q}'(x,t), \tag{15}$$

where p is the pressure, and $\bar{c}$ is the mean speed of sound.

Eqs. (14) and (15) denote the acoustic dynamics for a Helmholtz mode and a longitudinal mode, respectively. In what follows instabilities arising from either of these two modes will be considered. The same approach can be used for transverse modes as well (for example, screech modes in rockets [33]). In what follows, we also assume that flames are localized close to the anchoring plane, so that $q'(x,t) = q'(t)\delta(x - x_f)$.

Using an expansion in basis functions for both Eqs. (14) and (15) as

$$p'(x,t) = \bar{p} \sum_{i=0}^{n} \psi_i(x)\, \eta_i(t), \tag{16}$$

where ψ_0 is a constant, since it corresponds to the spatial variation in the bulk-mode, $\psi_i(x) = \sin(k_i x + \phi_{i0})$, $i = 1, \ldots, n$, and k_i and ϕ_{i0} determined from the boundary conditions. Performing a weighted spatial averaging, the modal amplitudes can be shown to follow [24]:

$$\ddot{\eta}_i + 2\zeta\omega_i\, \dot{\eta}_i + \omega_i^2 \eta_i = \sum_{i=1}^{n} \tilde{b}_i\, \dot{q}_f' \tag{17}$$

where $\tilde{b}_0 = \gamma - 1/V$, $\tilde{b}_i = \gamma a_o \psi_i(x_f)/E$ for $i = 1, \ldots, n$, $E = \int_0^L \psi_i^2(x)\, dx$, γ is the specific ratio, $a_o = \frac{\gamma - 1}{\gamma \bar{p}}$, ζ represents the passive damping ratio in the combustor[4], L is its length, $\omega_0^2 = \sqrt{\frac{A_n}{L_n V}}$, V is the volume of the combustor, A_n and L_n are the cross-sectional area and length of the inlet/outlet neck connected to the combustor. and $\omega_i = k_i \bar{c}$, $i = 1, \ldots, n$. Typically, $\omega_0 << \omega_i$, for $i = 1, \ldots, n$.

[4]Dissipation in a combustor can be caused by heat losses in the flame zone and friction due to viscous effects.

2.3 Coupling Dynamics

In Sections 2.1 and 2.2, we analyzed heat release and acoustics individually. Here, we investigate the physical coupling between the heat release and the acoustics which will drives them to resonance.

In the case when a Helmholtz-type resonance is triggered in the combustor, the acoustic velocity is very small in the bulk and the possible coupling between heat release fluctuations and acoustics is through the pressure. Under such conditions, we can assume that $u' \sim 0$[22]. However, coupling can be produced through perturbations in the equivalence ratio, which in turn can occur due to feedline dynamics [16]. In particular, if either the air or fuel-flow feeds is choked and the other feed is unchoked, ϕ can fluctuate. In general, the fuel nozzle is more likely to be choked than the air duct [16], and the instantaneous equivalence ratio ϕ_s at the exit of the fuel nozzle due to air flow fluctuations is determined as:

$$\phi_s = \frac{\overline{\phi}}{1 + u'_s/\overline{u}} \tag{18}$$

where u_s denotes the velocity at the fuel supply, and is similar to a relation used in [16]. When linearized, we obtain the following relation:

$$\phi' = -\frac{\overline{\phi}}{\overline{u}} u' \tag{19}$$

In addition, there is a convective delay τ_c due to transport lag from the supply to the burning plane of the flame, and hence,

$$\phi' = \phi'_s(t - \tau_c) \tag{20}$$

where $\tau_c = L/\overline{u}$.

The equivalence ratio perturbations, in turn, can be related to the pressure perturbations in the combustor by considering the momentum conservation in the inlet duct,

$$\frac{\partial u'_i}{\partial t} + \frac{1}{\rho_i}\frac{\partial p_i}{\partial x} = 0. \tag{21}$$

where u_i and p_i denote the velocity and the pressure at the inlet duct, respectively. When the dominant acoustic modes are longitudinal, both perturbations in u and ϕ can induce instability. The coupling between u and p can be determined using the energy conservation equation as

$$\frac{\partial p'}{\partial t} + \gamma\overline{p}\frac{\partial u'}{\partial x} = (\gamma - 1)q' \tag{22}$$

Combining the acoustics, heat-release, and convective lag effects, we obtain the following equations:

$$\ddot{\eta}_i + 2\zeta\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \tilde{b}_i\left[d_0u' + d_1\left(u'_{\tau_f}(t)\right) + d_2\left(\phi'_{\tau_f}(t)\right) + d_3\phi'(t-\tau_c) + d_\phi\dot{\phi}'(t-\tau_c)\right] \quad (23)$$

This indicates that two different coupling mechanisms are possible excitations for the acoustics, one resulting from the velocity perturbations u' and the other from equivalence ratio perturbations ϕ'. Eq. (23) also indicates that two different time-delays, τ_f and τ_c can induce these excitations, one arising from propagation effects, and the other from convection. An additional point to note from (23) is that if the dominant pressure mode is that of a bulk-mode, then it can be excited only due to perturbations in the equivalence ratio. However, if longitudinal modes are the ones that are dominant, they can be excited either by u'-perturbations or by ϕ'-perturbations.

The complete combustion dynamics is therefore determined by (23) and the coupling relations given by (20), (21) and (22). For ease of exposition we assume that only one acoustic mode is present, and set $\eta_i = \eta$. The first or the latter two will be the governing equation depending on whether the dominant variations are in u' or in ϕ'. If the variations are mainly in u', then Eqs. (23) and (22) can be combined to obtain the relation

$$\ddot{\eta} + (2\zeta\omega - \gamma_1)\dot{\eta} + (\omega^2 + \gamma_2)\eta - \gamma_2\eta(t-\tau_f) = 0 \quad (24)$$

where

$$\gamma_1 = \overline{\kappa}R\tilde{b}\tilde{c}, \qquad \gamma_2 = \overline{\kappa}\,\overline{S}_u\tilde{b}\tilde{c} \qquad \tilde{c} = \frac{1}{\gamma k^2}\frac{d\psi}{dx}\overline{p}, \qquad \tilde{b} = \tilde{b}_1$$

and if they are due to ϕ', then Eqs. (23), (21), and (20) can be combined to obtain

$$\ddot{\eta} + 2\zeta\omega\dot{\eta} + \omega_i^2\eta - \beta_1\eta(t-\tau_c) + \beta_2\eta_i(t-\tau_c) - \beta_3\eta_{i_{\tau_f}} = 0, \qquad \eta_i(t) = \int_0^t \eta(\zeta)d\zeta \quad (25)$$

where

$$\beta_1 = 2\pi\rho_u\tilde{b}\frac{\overline{\phi}}{\overline{u}}\tilde{c}\gamma k^2\left(\overline{S}_u\left.\frac{d\Delta h_r}{d\phi}\right|_{\overline{\phi}} + \Delta\overline{h}_r\left.\frac{dS_u}{d\phi}\right|_{\overline{\phi}}\right)\left(\int_0^R r\overline{\xi}dr\right), \quad \beta_2 = 2\pi\rho_u\tilde{b}\frac{\overline{\phi}}{\overline{u}}\tilde{c}\gamma k^2\Delta\overline{h}_r\overline{S}_u\frac{d\overline{S}_u}{d\phi}\overline{\xi}(0)$$

$$\beta_3 = -\tilde{b}\frac{\overline{\phi}}{\rho\overline{u}}\tilde{c}\gamma k^2\overline{\kappa}\,\overline{S}_u\frac{d\overline{S}_u}{d\phi}$$

At the acoustic frequency, the impact of the second-term on the right-hand-side of Eq. (25) is typically smaller. Hence a simplified version of (25) can be analyzed in the form

$$\ddot{\eta} + 2\zeta\omega\dot{\eta} + \omega^2\eta - \beta_1\eta(t-\tau_c) = 0. \quad (26)$$

It is interesting to note that the structure of (26) is identical to that of (24) with the differences only due to the parameters. This implies that time-delay effects are present both in the presence of u'- and ϕ'-perturbations, with the distinction that with u', the time-delay is due to τ_f, which is due to flame propagation, and with ϕ', the delay is due to τ_c, a convection effect. The other distinction is in the damping effect; in the former, if γ_1 is positive, even in the absence of any time-delay, instabilities can be present. In the latter, on the other hand, instability is only due to the time-delay τ_c; the damping effect is stabilizing.

The above discussions indicate that the general class of models that describe the combustion instability are of the form of

$$\ddot{\eta} + 2\zeta_0\omega\dot{\eta} + (\omega^2 - k_1)\eta + k_2\eta(t-\tau) \quad = \quad 0 \tag{27}$$

with ζ_0, ω, k_1, k_2 and τ taking different values depending on whether the instability is due to u' or ϕ'.

3 Instability Properties

In [34], stability bounds for both second-order and third-order delayed ordinary differential equations of the form of (27) and (25), respectively, were investigated. The highlights of [34] that are relevant for our discussions here are briefly stated below.

For a system of the form of (27), three possible stability characteristics can occur in general: (i) the system is always stable for all τ, (ii) the system is stable for $\tau \in [0, \tau^*]$ and unstable for all $\tau \geq \tau^*$, and (iii) the system alternates between stability and instability as τ increases. The results of [34] outline conditions on the parameters ζ, ω, k_1, and k_2, for cases (i)-(iii) to occur. For case (iii) to occur, it is necessary for $k_1 < \omega^2$. In most of the combustors under consideration, it can be shown that the latter is satisfied, which implies that the model in (27) predicts bands of instability. It is interesting to note that most experimental results where ϕ'-fluctuations occur illustrate such bands. In the following section, we make more detailed comparisons between the bands predicted by our model and experimental results.

3.1 Comparison with Experimental Results

3.1.1 Instability due to ϕ' fluctuations

We now compare the instability characteristics predicted by the model in (27) with the four experimental results [19, 20, 8, 25]. In the first three cases, the feedline consisted of a choked fuel-line and an unchoked air-feed. In [20], they also studied the case when the fuel-line was unchoked and the air-feed was choked. In [25], the corresponding details are not given, though fluctuations were observed to be present at the outlet of the fuel-line. Such feedline characteristics in turn introduce ϕ'-fluctuations as shown in section 2.3. Below, we consider each of these four results and show how the instability characteristics as predicted by the model in (27) compare with the results reported in [19, 20, 8, 25].

In [20], measurements of the pressure amplitude from different experiments where the position of the unchoked fuel inlet was changed, hence changing the convective delay τ_c, were collected as a function of τ_c/τ_{ac}. τ_{ac} is the acoustic mode time-constant, it follows that $\tau_{ac} = 2\pi/\omega$. The unstable regimes cluster around τ_c/τ_{ac} = 0.65 and 1.6 which is in agreement with our model. We neglected the effect of damping in all four cases, since it is small, and difficult to quantify. Therefore ζ was set to zero in Eq. (27). The results in [34] show that instability occurs when

$$\frac{n+1}{2} < \frac{\tau_c}{\tau_{ac}} < \frac{n+2}{2} \quad \text{for } n = 0, 2, 4... \tag{28}$$

which encompasses the unstable regions in [20]. In addition, in [19], instability occured at $0.5 < \tau_c/\tau_{ac} < 1.0$, when the fuel inlet was unchoked in perfect agreement with our model (see Fig. 2). It is worth noting that the acoustic mode was that of a quarter-wave in [19], and a half-wave in [20].

The instability mode in [8] corresponds to that of a bulk-mode with $\omega = 200$ Hz where ϕ'-fluctuations were observed to be present. Based on the experimental data provided in [8], we calculated $\tau_c = 3.1ms$, leading to $\tau_c/\tau_{ac} = 0.62$, which matches the model predictions from Eq. (28).

Our final comparison is with the results of [25], where the instability characteristics were observed to vary with L, the distance of the fuel-injector from the burning zone (see Fig. 7 in [25]). The paper shows that instability is also dependent on the value of $\overline{\phi}$. As the latter decreases, the

Figure 2: Instability band measured in [19] vs. our model predictions (shaded area).

instability vanishes. Our model can predict this behavior since $\overline{\phi}$, affects β_1 in Eq. (26). According to [34], as $\overline{\phi}$, and hence β_1 decreases, and below a critical value for the latter, the system will be stable irrespective of the amount of delay.

3.1.2 Instability due to u' fluctuations

In several of the experimental studies, the feedline dynamics was decoupled from the burning zone, but the pressure instabilities were still observed to be present. In [26, 23, 27], though the fuel inlet is choked and the air inlet is unchoked, the pressure fluctuations at the mixing section are small compared to those at the burning zone, which in turn suggests that the ϕ'-fluctuations are negligible as well. In [28], the fuel and air-streams are introduced using similar means with neither being choked. In [9], a premixer upstream of the cold-section of the combustor was introduced to ensure thorough mixing. In [29], a similar arrangement is made to decouple the feed-line perturbations. These observations indicate that a mechanism different from ϕ'-fluctuations is indeed responsible for introducing the resonance. One such possible candidate is u'-perturbations. In [24, 15], a model has been derived under such an assumption, and shown to compare favorably with experimental results in [23, 27, 35] for laminar flow conditions. On the other hand, the results in [28, 29, 10, 9], the flow velocities were in the turbulent regime. The question here is, whether the model in (27) can predict the instability reported in these four papers. In both [29, 9], the quarter-wave mode of the hot-section is driven into resonance. This implies that γ_1 in (24) is negative and hence ζ_0 in (27) is positive. But as indicated by the results in [34], instability can still occur. The model in (27) indeed predicts instability, which occurs due to the propagation delay τ_f. In particular, as τ_f increases, bands of instability occur, as indicated in Eq.(28). One of the quantities that can cause τ_f to change is $\overline{\phi}$, since $\tau_f = R/S_u$ and both R and S_u change with $\overline{\phi}$. It is interesting to note that in [9], it is observed (see Fig. 6 in [9]) that instability bands indeed occur as $\overline{\phi}$ changes. As pointed above, in this particular experimental set-up care was taken to decouple any ϕ'-perturbations from the burning zone.

It appears that similar mechanisms are present in [29, 10, 28] as well, though more experimental studies need to be carried out to support our hypothesis.

3.1.3 Instability with Diffusion flames

In a number of experiments, a diffusion flame rather than a premixed flame is present and still known to result in instability (for example, [36, 37, 11, 30]). Our model presented in this paper is not applicable due to the assumption that the flame is premixed. A more indepth study of a diffusion flame-model may be necessary in order to identify the possible mechanisms of instability.

Yet another assumption in our model is that the flame-perturbations are localized spatially. Therefore in rigs where the perturbations are present in a distributed form (as in [5]), the model presented here may not be appropriate. In [38] relevant models have been presented by assuming a certain spatial distribution of the flame.

4 Control

In this section, we investigate model-based control strategies for abating combustion instability using secondary fuel injection. A pulsating fuel injector delivers oscillations in the mass-flow rate in response to a voltage input. The injector dynamics is modeled from first principles, both in

Figure 3: A schematic of a typical injector.

the case when the mass-flow rate is proportional to the input voltage and in the case when the injector operates in an on-off mode. We assume that the pressure signal is the measured output, using a pressure transducer. The transducer dynamics is neglected since it typically has a much higher bandwidth than the combustion dynamics. We then develop active control strategies using the model developed in section 2 for the combustion dynamics and the fuel-injector model. We study the cases where an injector located at (i) the burning zone or (ii) further upstream. We assume that the combustion dynamics is determined by several coupled acoustic modes [24]. The control design is carried out for the case when the instability is primarily induced by fluctuations in the flame area coupled with the acoustics. We refer the reader to [21, 22] for investigations of instability caused by equivalence ratio fluctuations and its control.

4.1 Fuel Injector Dynamics

4.1.1 A Proportional Injector

The injector system consists of an electro-mechanical part and a fluidic part, where in the former, the input voltage generates an electro-magnetic field that causes a poppet to move against a spring (as seen in Fig. 3). The motion of the poppet controls the aperture of the injector allowing fluid to flow.

The electro-mechanical part relates the voltage E to the poppet position x through the electrical, electro-magnetic, and mechanical components, which can be modeled as

$$E = iR_e + L_e\frac{di}{dt} + V, \quad (29)$$

$$V = B_e l\frac{dx}{dt}, \quad (30)$$

$$F_m = B_e l i, \quad (31)$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_m, \tag{32}$$

where i is the current, F_m is the magnetic force, x denotes the motion of the armature in the direction of the magnetic force, R_e, L_e, B_e, and l denote, the resistance of the solenoid coil, the inductance of the coil, the magnetic flux density, and the length of the armature which moves orthogonal to the magnetic field, respectively. m, b, and k represent the effective mass, damping, and the stiffness of the armature/poppet system.

The fluidic part can be modeled using the unsteady Bernoulli equation, and the conservation of mass across the injector assuming incompressible flow. The unsteady velocity, v, can be obtained from the former applied between the inlet to the valve which is connected to a pressurized tank (where the flow velocity is ≈ 0, and $p = p_o$), and the outlet to the combustor where $p = p_c$, and $\Delta p = p_o - p_c$, as

$$\rho L_i \frac{dv}{dt} + \frac{1}{2}\rho v^2 = \Delta p, \tag{33}$$

In case small perturbations in p_c affect the velocity out of the injector, the unsteady velocity out of the valve, v, is linearized as

$$\tau_{fluid}\frac{dv'}{dt} + v' \cong \frac{-p'_c}{\sqrt{2\Delta\overline{p}\rho}}, \tag{34}$$

where $\tau_{fluid} = L_i/\sqrt{2\Delta\overline{p}/\rho}$ is the fluidic time constant, and L_i is the distance between the tank and the valve's outlet. τ_{fluid} is negligible for conditions where $L_i << 1m$, $\Delta\overline{p}$ is large (which is expected), and ρ is small ($O(1kg/m^3)$ for most gaseous fuels)[5].

The mass flow rate, defined as $\dot{m}_f = \rho v A$, is perturbed, assuming oscillations in v (caused by the dynamics in Eq. (34)) and A (caused by the motion of the poppet x), as

$$\dot{m}'_f = \rho\overline{A}v' + \rho\overline{v}A', \tag{35}$$

where we assume $A' = k_o x$, and $k_o > 0$. Equations (34) and (35) describe the fluid dynamics due to perturbations in p'_c and/or x. In most practical cases, $\Delta\overline{p}$ is large to guarantee choked conditions in the injector's discharge, thus, the first term in the RHS of Eq. (35) can be neglected. Using

[5] In the case of liquid fuels, the time constant can be comparable to the acoustics time constant due to large ρ, $O(1000kg/m^3)$.

$\overline{v} = \sqrt{\frac{2\Delta\overline{p}}{\rho}}$ from Eq. (33), we simplify Eq. (35) as

$$\dot{m}'_f \cong k_o \rho \sqrt{\frac{2\Delta p}{\rho}} x. \quad (36)$$

Equations (29)-(32) and (36) determine the input-output relation between the fuel-injector input E and the output $\dot{m}'_f$ which is expressed in the Laplace domain as:

$$\frac{\dot{m}'_f(s)}{E(s)} = \frac{k_v}{(\tau_e s + 1)(ms^2 + bs + k) + B_e^2 l^2 / R_e s}, \quad (37)$$

where $k_v = B_e l k_o \rho \sqrt{\frac{2\Delta\overline{p}}{\rho}} / R_e$.

In most solenoid systems, the armature electric time constant, $\tau_e = L_e / R_e$, is negligible compared to the acoustics time constant [39]. In these valves, the stiffness of the spring, k, is large, for a fast closing of the valve, when the voltage is turned *off*. Also, the mass, m, of the armature is very small in many of the typical injectors to minimize inertia forces [39]. The damping term, b, contains the overall damping including stiction and friction, and typically is large. Thus, the mechanical system can be simplified as a first-order system; a damper-spring system [40]. The mechanical time constant usually limits the bandwidth of typical injectors to approximately 100 Hz. (Note that other effects, such as impact dynamics are not included here, since we expect them to be of higher frequencies than the combustor dynamics). Thus, Eq. (37) is simplified as

$$\frac{\dot{m}'_f(s)}{E(s)} = \frac{k_v \tau_m}{\tau_m s + 1}, \quad (38)$$

where $\tau_m = (b + B_e^2 l^2 / R_e)/k$.[6]

4.1.2 A Two-Position (on-off) Fuel injector

Some fuel injectors currently used for combustion control [11, 30, 37] operate only between two positions, *on* and *off*. Unlike the proportional injectors discussed above, the physical stops play a more prominent role in the dynamics. However, one can still model two-position injectors in the same manner as above by including the effect of the physical stops as a saturation block together with Eq. (38) as shown in Fig. 4. A two position injector is turned *on* after the voltage input

[6] For more advanced proportional injectors, internal feedback loops exist (using for example a position transducer for the armature) to guarantee accurate metering, and increase its bandwidth, e.g., a Moog DDV proportional valve has a bandwidth of 450 Hz [41].

Figure 4: Block diagram of a typical two-position injector.

overcomes a certain threshold, thus creating a dead-zone in the control input (see figure) which will be discussed further when control is implemented in Section 4.3.

An additional point to note is the distinction between the injector dynamics during transition from closing and opening. Typically, the injector is over-driven by a high voltage in the opening mode to ensure fast opening. The opening time constant is different than the closing one. This effect can also be included in the injector model, as seen in Fig. 4, by assuming that τ_m varies between two values τ_{m_1} and τ_{m_2} depending on whether the injector is transitioning from *off* to *on* or *on* to *off*. Note that $\tau_{m_1} = \tau_m$ (as defined before) and $\tau_{m_2} = b/k$. We refer the reader to [42] for more details regarding this model when validated against two different injectors, Parker 9-130-905, and 9-633-900.[7]

4.2 Actuated Combustor

Denoting the contribution of the fuel injector to the equivalence ratio as ϕ_c', we have that $\phi_c' = \frac{m_f'}{\overline{m}_a}/\alpha_o$, where $\overline{m}_a$ and α_o are the mean air mass flow rate, and the stoichiometric fuel to air ratio, respectively. We assume that ϕ_c' is uniform radially, and that perturbations are carried intact by the mean flow to the burning zone, after a time delay τ_c, where $\tau_c = L_c/\overline{u}$, L_c is the distance between the injector discharge and the burning zone, and $\overline{u}$ is the mean velocity of the reactants in the combustor.

From Eq. (23), the impact of ϕ_c' on the combustion dynamics can be taken into account as

$$\ddot{\eta}_i + 2\zeta\omega_i \, \dot{\eta}_i + \omega_i^2 \eta = \frac{\widetilde{b}_i}{A_c} \left[\overline{\kappa} R u' + d_1 \dot{\phi}_c (t - \tau_c) \right], \tag{39}$$

[7] It should be noted that when inertia forces in the armature are important, a second-order fuel-injector model, with high damping, is more appropriate. The experimental measurements in [37] for a General Valve Series 9 model show similar dynamics.

for $i = 1, 2, .., n$, where i denotes the mode number.

4.3 Control Design

4.3.1 Injection at the Burning Zone

Injection at the flame has shown success in several experimental facilities [11, 30, 37], and in a practical full-scale 170 MW gas-turbine combustor [10]. We carry out active control design assuming that the injection is at the flame, using the model in Eq. (39) while $\tau_c \approx 0$, together with the injection dynamics described by Eq. (38). The input-output model relation between the injector input voltage, E, and the pressure, p', is given by

$$p'(s) = W_p(s)E(s), \qquad W_p(s) = \frac{k_p Z_p(s)}{R_p(s)}, \tag{40}$$

where $W_p(s)$ is the transfer function of a finite-dimensional model of the combustor, k_p, $Z_p(s)$ and $R_p(s)$ are the corresponding gain, numerator and denominator, respectively.

An appropriate optimal control for the finite-dimensional system as in Eq. (40) is LQG/LTR [43]. Its success lies in its ability to generate satisfactory performance over a wide range of frequencies, unlike phase-shift controllers which can destabilize stable dynamics [44, 15]. Experimental validation of the LQG/LTR for combustion control has been demonstrated in [42, 35, 45]. In [35], a similar physically-based model was used with a loudspeaker as an actuator. In [42, 45], a system-ID approach based on subspace and ARMAX methods [46] was used to suppress pressure oscillations in a dump and a swirl-stabilized combustors, respectively, using pulsed injection. In this paper, we show through simulation studies that LQG/LTR can also be used successfully based on a physical model, using fuel injector as an actuator. For details of the LQG/LTR control design, we refer the reader to [15, 35].

Simulations of the LQG/LTR Controller: A fifth order combustor dynamics model including the first two modes, the flame dynamics, and the injector dynamics is considered. The combustor parameters and conditions are taken as in [15], these cause a three-quarter-mode instability which resonates at 500 Hz approximately, and has unsteady pressure amplitudes of $O(100Pa)$.

We choose first a proportional injector, as in [10], with a bandwidth of 300 Hz which is in the range of available high-speed injectors [41]. Figure 5 shows the time response of the pressure

Figure 5: Response of the controlled combustor with a proportional injector with a bandwidth of $\approx$ 300 Hz.

Figure 6: Response of the controlled combustor with an on-off injector set to deliver $\frac{\phi_c|_{max}}{\bar{\phi}} \approx 0.125$ and with a bandwidth $\approx$ 300 Hz.

and the control input, $\phi_c'/\overline{\phi}$ ($\overline{\phi} = 0.7$). Control is applied at $t = 50ms$. We note that although the bandwidth is lower than the system dynamics, the control with proportional injector is still capable of stabilizing the system, since the dynamics of the injector are taken into consideration in the design of the LQG/LTR. As seen in Eq. (38), the injector dynamics are first order, and the gain and phase introduced by the injector depend on its bandwidth. For smaller bandwidths cases, the phase increases while the control authority, i.e., the gain, is dramatically decreased around acoustic frequencies. The controller, however, has enough degrees of freedom to adjust the phase, and increase the voltage amplitude into the injector to produce the required control authority, $\phi_c'/\overline{\phi}$, around the acoustics frequencies thus guaranteeing stability.

While a low bandwidth proportional injector is capable of maintaining the system at vanishingly small pressure perturbations, a two-position injector may not be as effective [30, 37, 45]. Using a 300 Hz bandwidth injector, the LQG/LTR is still capable of stabilizing the system, but the pressure is suppressed to a small but finite amplitude limit cycle as seen in Fig. 6. The reason is that the injector has a threshold input voltage value at which it is activated. Thus, following the suppression of the instability, the injector stops pulsing at from $t \approx$105-138ms, as seen in the figure. Disturbances in the combustor force the pressure to grow, until the measured voltage by the

Figure 7: Response of the controlled combustor with on-off injector set to deliver $\frac{\phi_c|_{max}}{\overline{\phi}} \approx 0.27$, and with a bandwidth of 300 Hz.

Figure 8: Response of the controlled combustor with on-off injector with lower bandwidth ≈ 50 Hz (the acoustics unstable frequency is 500 Hz, approximately).

microphone reaches the threshold at which the injector starts to fire again. In the case simulated, this occurs at $t > 138ms$. This sequence is repeated indefinitely.

Increasing the control fuel-flow rate, and thus $\phi_c|_{max}$, the combustor is stabilized in a smaller settling time. As seen in Fig. 7, when $\phi_c'|_{max}/\overline{\phi}$ is doubled, the settling time diminishes by $\sim 80\%$. Moreover, the rms of the steady-state pressure is smaller. A similar effect has been observed in [30].

We also investigate the effect of bandwidth for a two-position injector. Limiting it to 50 Hz, as seen in Fig. 8, the pressure settles to a higher-amplitude limit cycle, and the injector is incapable of tracking the command from the controller; the injector stays open all the time. This shows that injector bandwidth is a serious problem [30, 45]. Different solutions have been proposed that include: (i) Developing faster injectors [41]. (ii) Use of multiple injectors which are fired alternatively to increase the apparent frequency of actuation [30]. A different approach that has shown promise regardless of high-bandwidth injectors is through fuel pulsing at low frequencies (much lower than the acoustics). This is demonstrated experimentally in [9, 28], and analytically in [47].

4.3.2 Injection Upstream the Burning Zone: Delay in the Control Input

While injecting fuel directly on the flame [10, 11, 30, 37] can avoid actuation delays, it introduces hot spots at the flame surface thus increasing emissions. In addition, if mixing is weak at the injection port, we run the danger of creating a secondary diffusion flame which can be completely decoupled from the main premixed flame, and hence become ineffective in suppressing the instability [8].

In this section, we study the effects of pulsed-fuel injection upstream the burning zone. This has been utilized in [8] where secondary injection was done at the primary fuel source. In [22], we presented a Posi-Cast control capable of working with an injector located at an arbitrary distance upstream the flame. In that case, a bulk mode was unstable. Here, we extend the analysis of the Posi-Cast control, and show that it is capable of stabilizing longitudinal modes as well.

4.3.3 Posi-Cast Control

A powerful approach for controlling systems with known time-delay was originated by Smith [48], known also as Posi-Cast for "positive forecasting" of future states. The idea is to compensate for the delayed output using input values stored over a time window equal to the delay time, i.e. $[t - \tau_c, t]$, and estimate the future output using a model of the combustor. Only stable systems were considered. An extension to include unstable systems was proposed in [49] using finite-time integrals of the delayed input values thereby avoiding unstable pole-zero cancellations which may occur. A frequency-domain pole-placement technique for unstable systems was first proposed in [50] and a similar technique will be presented here.

The model in Eq. (40), in the presence of a time delay, τ_c, can be re-written as

$$p'(t) = W_p(s)[E(t - \tau_c)], \qquad W_p(s) = \frac{k_p Z_p(s)}{R_p(s)}. \tag{41}$$

Due to the nature of the combustion system, not all states are accessible, only the system input, i.e. the voltage to the injector $E(t)$, and the output, p' in our case are measured. A standard pole-placement controller is required (for more information, see [50, 51]). The presence of the time-delay, τ_c, in the control input, motivates the use of an additional signal in the control input,

[8] This has been noticed in experiments at MIT and at UTRC [**?**].

$E(t)$, denoted as $E_1(t)$ which anticipates the future output using a model of the system [22]. The resulting controller structure is described as

$$\begin{aligned} E(t) &= \frac{c(s)}{\Lambda(s)} E(t-\tau_c) + \frac{d(s)}{\Lambda(s)} p'(t) + E_1(t), \\ E_1(t) &= \frac{n_1(s)}{R_p(s)} E(t) - \frac{n_2(s)}{R_p(s)} E(t-\tau_c), \end{aligned} \tag{42}$$

where $\Lambda(s)$ is a chosen stable polynomial of degree $n-1$, $d(s), n_1(s)$ and $n_2(s)$, are polynomials of degree $n-1$ at most, and $c(s)$ is of degree $n-2$ at most. For stability, these must satisfy the relations

$$c(s)R_p(s) + k_p d(s) Z_p(s) = \Lambda(s) n_2(s), \tag{43}$$

$$n_1(s) = R_p(s) - R_m(s), \tag{44}$$

where $R_m(s)$ is the desired characteristic equation, which is a stable monic polynomial of the same order of $R_p(s)$.

Using the controller structure in Eq. (42) with the conditions in Eqs. (43) and (44), the closed-loop transfer function can be computed as

$$W_{cl}(s) = \frac{k_p e^{-s\tau_c}}{R_m(s)}. \tag{45}$$

The control input law, in Eq. (42), introduces additional dynamics including non-minimum phase zeros having the same eigen-values of $R_p(s)$. Obviously, these lead to unstable pole-zero cancellations since the combustor model is open-loop unstable (i.e., $R_p(s)$ has unstable eigen values). Unstable pole-zero cancellations are known to cause problems concerning observability and controllability of the plant (see [39] for more details). As a result, a modification in the synthesis of $E_1(t)$ in Eq. (42) was suggested by [49]. To avoid unstable pole-zero cancellations, $E_1(t)$ must be generated as a finite integral of the form

$$E_1(t) = \sum_{i=1}^{n} \left(\int_{-\tau_c}^{0} e^{-\lambda_i \sigma} E(t+\sigma) d\sigma \right), \tag{46}$$

where λ_i's are the eigen values of the combustor system, i.e. $R_p(s) = \Pi_{i=1}^{n}(s-\lambda_i)$. Taking the Laplace transform of Eq. (46), one can show that

$$\frac{n_1(s)}{R_p(s)} = \sum_{i=1}^{n} \frac{\alpha_i}{s-\lambda_i}, \frac{n_2(s)}{R_p(s)} = \sum_{i=1}^{n} \frac{\beta_i}{s-\lambda_i}, \tag{47}$$

Figure 9: Response of the controlled combustor with a time-delay of $100ms$ in the input signal, proportional injector. Note: only the envelope of the response is shown for clarity, since the scale of the plot does not permit seeing individual cycles.

where $\beta_i = \alpha_i e^{\lambda_i \tau_c}$. Another condition for the successful use of the finite integral in Eq. (46) is that $R_p(s)$ has no repeated roots [50].

The controller described in Eqs. (42) and (46) is sufficient to stabilize the combustor provided that an accurate description of the plant and the time delay are available. This controller has been shown to provide robustness to uncertainties in the plant including the time delay [49]. Adaptive versions of the same controller have been investigated [52, 53], and have shown to extend the robustness of the controller to parameter uncertainties.

Simulations of the Posi-Cast Controller: The controller in Eqs. (42) and (46) is implemented for injection at a distance of $\sim 3cm$ upstream the burning zone. τ_c is estimated to be $100ms$, which is about 50 times the time constant of the unstable frequency.

The closed-loop simulation is illustrated in Fig. 9. Although control is switched *on* at $t = 50ms$, the pressure keeps increasing for an additional $t = \tau_c = 100ms$ (from $t = 50 - 150ms$), then stalls for another 100 ms (from $t = 150 - 250ms$) before decaying. The reason for the former delay is physical and is due to the time taken for the pulsed-fuel to reach the burning zone. The latter is due to a computational delay in the controller. Specifically, the finite integral in Eq. (46) outputs incorrect values for a period of τ_c. This is because the computation of the finite integral relies on a stored window of the past values of the control input of the size of τ_c. When control is switched *on*, the window consists of control inputs proportional to p' which has not yet "felt" the effect of control due to the physical delay τ_c (the values of p' are still those of the open-loop combustor). It requires therefore $t = 2\tau_c$ to start forming a window of integration with control input corresponding to closed-loop values. This confirms observations in [49].

Figure 10: Response of the controlled combustor with a time-delay of $100ms$ in the input signal, on-off injector. Note: only the envelope of the response is shown for clarity, since the scale of the plot does not permit seeing individual cycles.

In Fig. 10, a two-position injector is used. The control design is based on the linear model, and its parameters are fine-tuned to handle the nonlinearities. As discussed earlier, the control is switched *on* at $50ms$, and stabilizes the system. The injector stays *on* as long as the voltage signal into the injector is greater than a threshold, as discussed before in Sec. 4.1.2.

It should be noted that when combustion instability is caused by ϕ'_s fluctuations, as discussed in Sec. 2, the characteristic equation will look different than in Eqs. (40) and (41). $R_p(s)$ will have terms which are delayed, due to the convective delay, τ_s, carried by ϕ'_s. Hence, $R_p(s)$ becomes infinite dimensional. To circumvent this, a Padé approximation [54] is used to get a finite dimensional description of $R_p(s)$, and thus the LQG/LTR and Posi-Cast controllers as described in Secs. 4.3.1 and 4.3.2, respectively, can similarly be used for this case.

5 Summary

In this paper, a complete model of the combustion dynamics leading to instability is developed. A model encompassing the acoustics, the heat release, the coupling, mixing and injector dynamics is presented. The heat release is derived using flame kinematics for flames with high Damkohler numbers and moderate turbulence, and the effect of forcing in the velocity and the equivalence ratio is illustrated, this latter being introduced for the first time in a kinematic study of the flame. Different stability criteria are discussed for the different dynamics resulting from the coupling of these with the acoustics. While velocity perturbations cause instability of a phase-lag nature, equivalence-ratio oscillations introduce a time-delay instability. Injection at and upstream the burning zone is implemented. An LQG/LTR control is used for the former while a Posi-Cast control is developed and used for the latter. Both proportional and two-position (on-off) pulsed injection are examined. Proportional injectors show superior performance to two-position ones. Performance is shown to be reduced as the authority and/or bandwidth decreases. Some of the short-term fixes proposed for this problem include the use of multiple injectors [30] and low frequency pulsing [9, 28, 47]. A long-term solution is clearly the design of a high-speed, high-authority injector.

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