Adaptive Control of A Class of Time-delay Systems

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Abstract

The control of physical systems in the presence of time-delays becomes particularly challenging when parametric uncertainties are present. To cope with these ubiquitous uncertainties, we propose an adaptive controller in this paper that can accommodate both a time-delay and parametric uncertainties. The controller includes (a) a control architecture that is based on the plant relative degree rather than the plant order, (b) an integral implementation of the well known Posicast Controller so as to accommodate unstable plants, (c) high-order tuners for parameter adaptation, and (d) a Lyapunov-Krasvoskii functional that allows adaptive stabilization. The controller is shown to be semi-global in the time-delay τ and to result in asymptotic tracking. The implications of the adaptive controller are explored in the context of combustion control through simulation studies. Robustness properties of the controller are briefly discussed.

1 Introduction

Delay systems represent a class of infinite-dimensional systems where mechanisms related to transport, propagation, or other effects related to a significant time-lag are present. One such example is in combustion systems where recent modeling efforts have shown that one of the most challenging factors for successful control is the presence of large time-delays [1]. In addition to this, even small perturbations in the operating conditions introduce large and unpredictable changes in the system dynamics mandating a controller that can adapt to these uncertainties. The field of adaptive control has addressed parametric uncertainties in various kinds of dynamic systems including linear and nonlinear, single and multivariable, continuous and discrete, deterministic and stochastic systems.

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Very few of the results in this area pertain to problems where large time-delays are present. The main implication of this is that all results currently available are applicable to time-delay systems only when the delay values are small. The results in this paper help in bridging the gap between currently available results and practical needs of control problems.

A unique approach for controlling systems with a known time-delay was originated by Otto Smith in the fifties [2] by compensating for the delayed output using input values stored over a time window of $[t - \tau, t]$ and estimating the plant output using a model of the plant. In [3], this idea was extended to include unstable plants as well using finite-time integrals of the delayed input values thereby avoiding unstable pole-zero cancellations that may occur in Smith's controller. In [4, 5], pole-placement and adaptive versions of [3] were developed, and it was shown that the plant can be adaptively controlled in a stable manner in the large. More recently, in [6]-[8], an adaptive posi-cast controller has been proposed whose design is based on the relative degree of the plant to be controlled. While in [6], controllers were developed for plants with relative degree two, in this paper, we consider plants of arbitrary order and relative degree whose zeros are stable and whose high frequency gain is known.

The advantages of the controller proposed in this paper over those in [4, 5] are two-fold: Denoting $W_m(s)$ as the transfer function of the reference model that the plant in closed-loop is required to match, the first advantage is that in cases where $W_m(s)$ is unknown or too difficult to determine, the controller proposed here can be determined unlike those in [5] which require $W_m(s)$. The second is that the controller proposed here is much simpler, which is enabled by making use of properties of positive real transfer functions. We also note that the controller proposed here has been directly utilized in both simulation and experimental studies of a practical combustion system and has been shown to be successful in the presence of fairly large delays [7].

In Section 2, we state the problem. In Section 3, we consider the delay-free case, and present the controller structure, the adaptive laws, the proof of stability, and the robustness properties of the controller. Application to the combustion control problem is also treated in this section. In Section 4, we consider the case when time-delays are present, and present the requisite adaptive controller and its proof of stability. Section 5 contains a summary of the paper.

2 Statement of the Problem

The problem is the control of a plant given by the input-output description

$$y(t) \;=\; W_p(s)[u(t-\tau)], \;\; W_p(s) = \frac{Z_p(s)}{R_p(s)} \tag{1}$$

where $W_p(s)$ is the transfer function of a finite-dimensional system whose order n is unknown, relative degree m is known, zeros are in $\mathbb{C}^-$, and its high frequency gain is known.The time-delay τ is assumed to be known as well. The plant poles and zeros are unknown and it is assumed that all poles have multiplicity one. For ease of exposition, in what follows we assume that the high frequency gain is known and is unity.

It is required that the plant output follow the output of a reference model with a transfer function

$$y_m(t) \;=\; W_m(s)[r(t-\tau)], \; W_m(s) = \frac{1}{R_m(s)} \tag{2}$$

where R_m is a monic Hurwitz polynomial of degree m, and $k_m > 0$. Our goal is to determine a stable adaptive controller for this class of problems whose order depends on m and not on n. These controllers are motivated by problems where the order of the plant is large. In particular, in problems related to distributed parameter systems, the underlying system is truly infinite-dimensional, in which case any rational finite-dimensional approximations thereof inevitably leads to a large n. It is attractive in such cases to design a controller that depends on the number of integrations in the system, i.e. the relative degree, rather than the number of state-variables in the system, by making use of the stable zeros if the latter are present.

Since the control architecture that we propose is quite similar to that in the delay free case, we first present the controller for a plant with no delays.

3 The Adaptive Controller in the Delay-free Case

The problem that we address in this section is the control of

$$y(t) \;=\; W_p(s)[u(t)], \qquad W_p(s) \;=\; \frac{Z_p(s)}{R_p(s)} \tag{3}$$

where $W_p(s)$ is the transfer function of a finite-dimensional system whose order n is unknown, relative degree m is known, zeros are in $\mathbb{C}^-$, and its high frequency gain is unity. The plant poles and zeros are unknown and it is assumed that all poles have multiplicity one.

3.1 The controller structure

It is well known that any linear plant with stable zeros and a relative degree m can be stabilized by a controller of the form

$$u = k_c \frac{(s+z_c)^{m-1}}{p_c(s)} y \tag{4}$$

where $p_c(s)$ is a monic polynomial of degree $m-1$, for suitable values of k_c and coefficients of $p_c(s)$. In particular, we realize the controller in (4) in the following form.

$$u = -\frac{p(s)}{(s+z_c)^{m-1}} u - k_1 y \tag{5}$$

$$p(s) = k_{21} + k_{22} s + \ldots + k_{2(m-1)} s^{m-2} \tag{6}$$

The controller as in (5) and (6) together with the plant as in (3) results in a closed-loop transfer function of the form

$$W_{cl}(s) = \frac{(s+z_c)^{m-1} Z_p(s)}{R_p(s) p_c(s) + k_1 (s+z_c)^{m-1} Z_p(s)} \tag{7}$$

$$p_c(s) = (s+z_c)^{m-1} + p(s) \tag{8}$$

For a large k_1, the $n+m-1$ poles of $W_{cl}(s)$ can be shown, using Routh-Hurwitz arguments, to be close to the zeros of $(s+z_c)^{m-1} Z_p(s)$ and other m stable locations, for suitable values of k_{2i}, $i = 1, \ldots, m-1$ (see Appendix A and [9] for further details).

We note that the controller in (5) and (6) can be represented by the state-variable form

$$\dot{\omega}_1 = \Lambda \omega_1 + \ell u \tag{9}$$

$$u = -k_2^T \omega_1 - k_1 y + r \tag{10}$$

where $\Lambda \in \mathbb{R}^{m \times m}$, (Λ, ℓ) is controllable, and

$$k_2^T (sI - \Lambda)^{-1} \ell = \frac{p(s)}{(s+z_c)^{m-1}}. \tag{11}$$

The above discussions also indicate that for a suitable value k_1^* and k_2^* of k_1 and k_2 respectively, the closed-loop transfer function is stable and is given by $W_{cl}(s)$.

3.2 The adaptive controller

The controller structure in (9) and (10) suggests that when the plant parameters are unknown, an adaptive controller with time-varying parameters of the following form can be used:

$$\dot{\omega}_1 = \Lambda\omega_1 + \ell u \tag{12}$$

$$u = k_2^T(t)\omega_1 + k_1(t)y + r \tag{13}$$

Expressing the control parameters as $k_1(t) = k_1^* + \widetilde{k}_1(t)$, $k_2(t) = k_2^* + \widetilde{k}_2(t)$, $\omega = [\omega_1^T, y]^T$, $\widetilde{k} = [\widetilde{k}_1^T, \widetilde{k}_2^T]^T$, the closed-loop system equations can be described as

$$y = W_{cl}(s)(\widetilde{k}^T\omega) + r. \tag{14}$$

$W_{cl}(s)$ is not strictly positive real (SPR), but has stable poles, stable zeros, and is of relative degree m. Due to these properties, it is reasonable to assume that one can find a strictly positive real transfer function of the form

$$W_m(s) = W_{cl}(s)(s+a)^{m-1}.$$

We note that there may be other ways of choosing W_m. For example, instead of adding $m-1$ zeros all at $-a$, they could be added at $m-1$ distinct locations, but for simplicity, let us assume the above.

To enable the realization of $W_m(s)$ in closed-loop, we choose the control input, instead of $k^T(t)\omega(t) + r(t)$, as follows:

$$u(t) = (s+a)^{m-1}[k^T\omega'(t) + r'] \tag{15}$$

$$\omega'(t) = \frac{1}{(s+a)^{m-1}}[\omega(t)] \tag{16}$$

$$r'(t) = \frac{1}{(s+a)^{m-1}}[r(t)]$$

This will lead to

$$y = W_m(s)(\widetilde{k}^T\omega' + r'). \tag{17}$$

Now, the problem is to realize (15) without explicitly differentiating any signal. Let $p = m-1$. Using binomial expansion and the chain rule for differentiation, we obtain that

$$\begin{aligned} u &= k^T d_0 + p\dot{k}^T d_1 + ... + (p_{C_i})k^{(i)T} d_i \\ &\quad + ... + pk^{(p-1)T} d_{p-1} + k^{(p)T} d_p + r \end{aligned} \tag{18}$$

where

$$d_i(t) = \left[\frac{1}{(s+a)^i}\right][\omega(t)], \qquad i = 1, \ldots, p.$$

Note that all terms involving k and d_i are realizable. So, the only remaining piece is the realization of derivatives of k to pth order.

The overall problem can be summarized as follows: Given the error model in Eq. (17) where ω' is given by (16), determine an adaptive law for adjusting k so that it is differentiable p times and all the signals in the loop are bounded. The time-domain representation of the error model in (17) is given by

$$\dot{e} \;=\; A_s e + b_s(k - k^*)^T \omega', \qquad e_1 = h_s^T e \tag{19}$$

where

$$h_s^T(sI - A_s)^{-1} b_s = W_m(s).$$

Since $W_m(s)$ is SPR, we have that

$$A_s^T P_s + P_s A_s \;=\; -Q \leq 0, \qquad P_s b_s = h_s \tag{20}$$

We note that ω' is differentiable p times. In what follows, ω_i' and k_i denote the ith element of a vector ω' and k, respectively.

Using the high-order tuners developed in [10], the following adaptive law is suggested for adjusting k:

$$\dot{k}' \;=\; -e_1 \omega' \tag{21}$$

$$\dot{x}_i \;=\; (Ax_i + bk_i') f(\omega_i'), \qquad f(x) = 1 + \mu x^2 \tag{22}$$

$$k_i \;=\; c^T x_i, \tag{23}$$

for $i = 1, \ldots, m$, where (c, A, b) are chosen so that

$$c^T(sI - A)^{-1} b \;=\; \frac{\alpha(0)}{\alpha(s)} \tag{24}$$

and $\alpha(s)$ is an arbitrary stable polynomial of degree p. The choice of k as in (21)-(24) guarantees that k is differentiable p times.

3.3 Proof of stability

We choose a Lyapunov function candidate of the form

$$V = e^T P_s e + (k' - k^*)^T (k' - k^*) + \delta \sum_{i=1}^{m} z_i^T P z_i$$

where

$$\begin{aligned} z_i &= x_i + A^{-1} b k_i' \qquad (25) \\ A^T P + PA &= -I \end{aligned}$$

Note that from (23) and (24), it follows that

$$\dot{z}_i = A z_i f(\omega_i') + A^{-1} b \dot{k}_i' \qquad (26)$$

$$k_i - k_i' = c^T z_i \qquad (27)$$

Also Eq. (19) can be expressed as

$$\begin{aligned} \dot{e} &= A_s e + b_s (k' - k^*)^T \omega' + b_s (k - k')^T \omega', \\ e_1 &= h_s^T e. \qquad (28) \end{aligned}$$

By choosing the parameters μ and δ appropriately, we will show that V is a Lyapunov function.

Using Eqs. (20)-(28), we obtain that

$$\dot{V} = -e^T Q e + 2 e_1 \sum_{i=1}^{m} \left(c^T z_i \right) \omega_i' - \delta \sum_{i=1} \|z_i\|^2 f(\omega_i') - 2\delta \sum_{i=1} z_i^T P A^{-1} b e_1 \omega_i'.$$

If we choose δ and μ as

$$\delta = \frac{\|c\|}{\|PA^{-1}b\|} \qquad \mu = \frac{4||h_s||^2 m \|c\|^2}{\epsilon\delta},$$

and ϵ to be the smallest eigenvalue of Q, we can show that

$$\dot{V} \le -\delta \sum_{i=1}^{m} \|z_i\|^2 - \sum_{i=1}^{m} \left(\sqrt{\frac{\epsilon}{m}} \|e\| - \sqrt{\delta\mu} \|z_i\| |\omega_i'| \right)^2.$$

Hence, $\dot{V} \le 0$. This implies that e, k' and z_i are bounded. Therefore x_i and k are bounded. Using Barbalat's lemma, it can be argued that $\lim_t e_1(t) = 0$.

3.3.1 Robustness properties

The controller proposed in this section can be viewed as a high-gain controller similar to those in [11]. Instead of choosing a search-algorithm, a high-order tuner is proposed to achieve stability. Despite this high-gain feature, we show in this section that the same fixes as in standard adaptive control such as σ-modification and dead-zone can result in a robust behavior.

The problem is to establish boundedness when an external disturbance d is present in the plant so that

$$y(t) \;=\; W_p(s)[u(t)+d(t)].$$

The underlying error model can be derived as

$$\dot{e} \;=\; A_s e + b_s(k-k^*)^T\omega' + b_s d, \qquad e_1 = h_s^T e \tag{29}$$

where

$$h_s^T(sI-A_s)^{-1}b_s = W_m(s)$$

and $W_m(s)$ is SPR. The adaptive law is chosen as

$$\dot{k}' \;=\; -e_1\omega' - \sigma_0 k' \qquad \sigma_0 > 0 \tag{30}$$

with x_i, k_i, and $\alpha(s)$ defined as in (22), (23), and (24).

Choose

$$V \;=\; e^T P_s e + (k'-k^*)^T(k'-k^*) + \delta\sum_{i=1}^{m} z_i^T P z_i$$

where z_i is defined as in (25). The time-derivative of V is of the form

$$\begin{aligned}\dot{V} \;=\; & -e^TQe + 2e_1\sum_{i=1}^{m}\left(c^Tz_i\right)\omega_i' - \delta\sum_{i=1}\|z_i\|^2 f(\omega_i') - 2\delta\sum_{i=1} z_i^TPA^{-1}be_1\omega_i' \\ & + 2e_1 d - \sigma_0 k'^T(k'-k^*) - 2\delta\sigma_0\sum_{i=1} z_i^TPA^{-1}bk'.\end{aligned}$$

If we choose δ as

$$\delta = \frac{\|c\|}{\|PA^{-1}b\|}$$

we obtain that

$$\begin{aligned}\dot{V} \;\leq\; & -\left[e^TQe + \sigma_0\|k'\|^2 + \delta\sum_{i=1}^{m}\left\|z_i^2\right\| + \delta\mu\sum_{i=1}^{m}\left\|z_i^2\right\|\omega_i'^2\right] \\ & + \left[2\|e\|\,\|h_s\|\,\|d_\nu\| + 4\|c\|\,\|e_1\|\sum_{i=1}^{m}\|z_i\|\,|\omega_i'| + 2\sigma_0\|k'\|\,\|k^*\| + 2\sigma_0\|c\|\sum_{i=1}^{m}\|z_i\|\,|k_i'|\right].\end{aligned}$$

Completing squares, defining ϵ to be the smallest eigenvalue of Q and $\epsilon' \in (0,1)$, and choosing μ as

$$\mu = \frac{4||h_s||^2 m \|c\|^2}{\epsilon\epsilon'\delta} \tag{31}$$

we can show that

$$\begin{aligned}
\dot{V} \;\; \leq \;\; & -\frac{\epsilon(1-\epsilon')}{m}\left(\|e\| - \frac{\|h_s\|\,\|d_\nu\|}{(1-\epsilon')\epsilon}\right)^2 - \left(\frac{1}{m} - \frac{\sigma_0}{\delta}\,\|c\|^2\right)\left(\|k'\| - \frac{\|k^*\|}{2m\left(\frac{1}{m} - \frac{\sigma_0}{\delta}\,\|c\|^2\right)}\right)^2 \\
& - \sum_{i=1}^{m}\left(\sqrt{\frac{\epsilon\epsilon'}{m}}\|e\| - \sqrt{\delta\mu}\|z_i\||\omega_i'|\right)^2 - \sum_{i=1}^{m}\left(\sqrt{\frac{\delta}{\sigma_0}}\|z_i\| - \|c\|\sqrt{\frac{\sigma_0}{\delta}}\,\|k'\|\right)^2 \\
& + \frac{1}{\epsilon(1-\epsilon')}\,\|h_s\|^2\, d^2 + \frac{\|k^*\|^2\,\sigma_0}{4\left(1 - \frac{m\sigma_0}{\delta}\,\|c\|^2\right)}.
\end{aligned}$$

Hence, if σ_0 is chosen such that

$$\frac{1}{m} - \frac{\sigma_0}{\delta}\,\|c\|^2 > 0$$

then $\dot{V} \leq 0$ in D^c where D is a compact set in the space $(e^T, (k-k^*)^T, z^T)^T$. This implies that e, k' and z_i are bounded. Therefore x_i and k are bounded, which establishes robustness.

3.4 Application to Combustion Control

Continuous combustion systems occur in several propulsion and power generation problems where a continuous heat source is present in a confined chamber. The unsteady heat release often couples in feedback with the acoustic modes of the chamber thereby causing the modes to be driven into resonance. This dynamic instability often occurs at operating points of interest where low emissions, high volumetric heat-release, and high efficiency are achievable. To help realize these desired objectives, active control technology has been shown to be an effective tool [12]. Recent results have shown that a systematic methodology that uses a model-based control strategy optimizes the performance of the combustion system [13]. We discuss one such model and its control below.

The plant to be controlled is of the form [7]

$$P_{ref} \;=\; W(s)V_c \qquad \text{where } W(s) = W_0(s)e^{-s\tau}, \tag{32}$$

V_c is the voltage supplied to a fuel injector that modulates a secondary fuel source thereby affecting the unsteady heat release, and P_{ref} is an acoustic measurement from a reference location in the combustor. The transfer function $W_0(s)$ is given by

$$W_0(s) \quad = \quad \frac{F(s)G(s)W_{ac}(s)}{1 - G(s)H(s)} \tag{33}$$

where (see figure 1 for a schematic) $G(s)$ describes the acoustic response of the duct and is of the

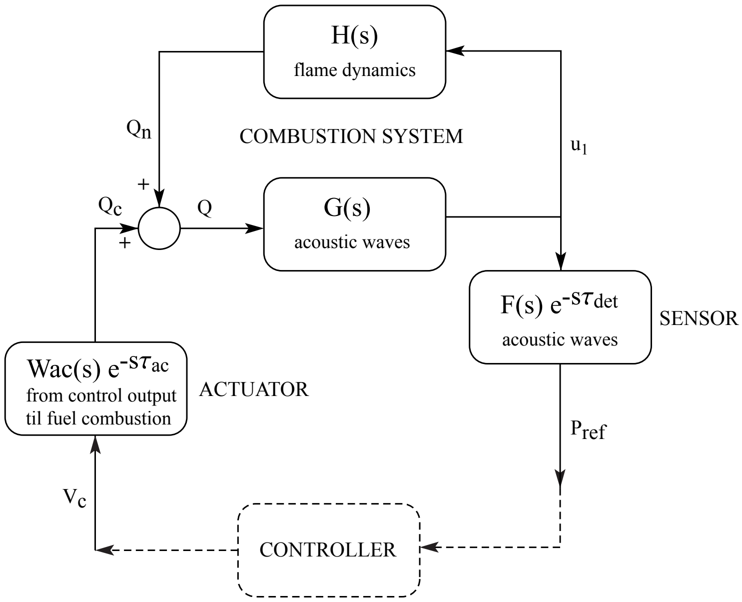

Figure 1: The control of a combustion system

form

$$G(s) \quad = \quad \frac{(R_d Y_{12} e^{-s\tau_d} - X_{12})(R_u e^{-s\tau_u} - 1)}{A\bar{\rho}_1 \bar{c}_1^2 \det(S)} \tag{34}$$

where

$$S \quad = \quad \begin{pmatrix} X_{11} - R_u Y_{11} e^{-s\tau_u} & X_{12} - R_d Y_{12} e^{-s\tau_d} \\ X_{21} - R_u Y_{21} e^{-s\tau_u} & X_{22} - R_d Y_{22} e^{-s\tau_d} \end{pmatrix} \tag{35}$$

R_d, R_u are pressure reflection coefficients at the upstream and downstream ends, respectively. X_{ij} and Y_{ij} are constants determined by the conservation equations, $\bar{\rho}_1$ is the density, and $\bar{c}_1$ is the speed of sound. τ represents the time-delay due to actuation and detection time-delay due to the location of the pressure measurement, τ_u and τ_d are time-delays associated with the acoustic wave propagation upstream and downstream of the combustion zone. $H(s)$ represents the combustion response whose precise structure varies with the nature of the flame stabilization mechanism in a given combustor. $F(s)$ represents the coupling relation between the pressure and the velocity and is of the form

$$F(s) \quad = \quad \bar{\rho_1}\bar{c_1}\frac{1 + R_u e^{\frac{-2s(x_u+x_{ref})}{\bar{c}_1(1-\bar{M}_1^2)}}}{R_u e^{-s\tau_u} - 1} e^{\frac{sx_{ref}}{\bar{c_1}-\bar{u}_1}} \tag{36}$$

where x_u and x_{ref} are the upstream end and sensor location, respectively, and M_1 and $\overline{u}_1$ are the Mach number and mean flow velocity in the cold section, respectively. $W_{ac}(s)$ is the transfer function of the fuel-injector.

As can be seen, $W_0(s)$ is an infinite-dimensional system. Using a Pade approximation, $W_0(s)$ can be approximated by a rational transfer function of order n. Using the underlying physics, for any n, one can derive the following properties of $W(s)$ [7]: (i) Since the flame is stable, $W_0(s)$ has stable zeros; (ii) The relative degree of $W_0(s)$ is equal to the relative degree of the actuator transfer function $W_{ac}(s)$ for simple flame models and can be larger for more complex flame models; (iii) The high frequency gain of $W_0(s)$ is positive. Since the order n in general depends on the level of approximation that is needed in a given problem, it cannot be assumed to be known. As a result, the controller proposed in this paper is necessary because its design depends on the plant relative degree and not on the plant order.

The above model was simulated using a Pade approximation of $W_0(s)$ where all poles and zeros less than 1100 rad/s were included in the control design, which yielded a relative degree of four. Of the poles, two pairs corresponded to unstable locations. Both the fixed version of the controller described in Eqs. (5) and (6) and the adaptive controllers described in (18), (21)-(23) was simulated in closed-loop. The resulting performances of the input u and the output y are shown in figure 2 for the controller parameters $k_1 = 9, k_{21} = 66100, k_{22} = 19000, k_{23} = -6085, z_c = 1000$. Both controllers yielded a satisfactory performance. It was also observed that the same system was not stabilizable using a controller with a lower order.

4 The Adaptive Controller in the Presence of a Delay

We now consider a plant with a time-delay as in Eq. (1). In [7], a low-order adaptive controller has been derived for plants with a time-delay for the case when the relative degree of the finite-dimensional part of the plant has a relative degree two. The stability proof consists, as in [6,

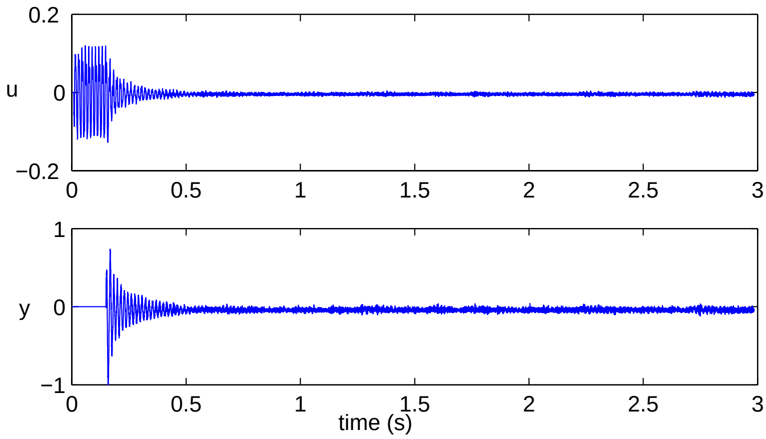

Figure 2: The performance of a relative degree four controller during the control of a combustion system in Eq. (32)

20], of the construction of a Lyapunov-Krasovskii functional. The results of the previous section demonstrate that it is possible to derive a low-order controller for plants with an arbitrary relative degree in a stable manner. The question is if these two approaches can be combined somehow to guarantee any plants of the form of (1) where the only requirements regarding the plant are that the relative degree of $W_p(s)$ is known, it is minimum phase with a known high frequency gain, and that its delay is known. In this section, we present a controller structure, its adaptive version, and show that it can be stabilized for all initial conditions within a compact set and for all $\tau \leq \tau^*$, for a given τ^*.

4.1 The controller structure

Since the controller structure to be used for a plant with an arbitrary relative degree and a delay is quite similar to that for a plant with relative degree two, we present both of the cases below.

4.1.1 Case (i) $m = 2$

When the plant relative degree is equal to two, it was shown in [7] that the following controller suffices:

$$\begin{aligned} u(t) &= \left[\frac{k_2}{s+z_c}\right] u(t) + u_1(t) + k_1 y(t) \\ u_1(t) &= \int_{-\tau}^{0} \sum_{i=1}^{n} \alpha_i e^{-\beta_i \sigma} u(t+\sigma)\, d\sigma \end{aligned} \tag{37}$$

We note that this is possible since

$$u_1(t) \;=\; \left(\frac{n_1(s)}{R_p(s)} - \frac{n_2(s)}{R_p(s)}e^{-s\tau}\right)[u(t)],$$

where

$$\frac{n_1(s)}{R_p(s)} \;=\; \sum_{i=1}^{n}\frac{\alpha_i}{s-\beta_i}, \qquad \frac{n_2(s)}{R_p(s)} \;=\; \sum_{i=1}^{n}\frac{\alpha_i}{s-\beta_i}e^{\beta_i\tau} \tag{38}$$

For a small τ, as shown in [7], the controller stabilizes the plant. For implementation purposes, the control input in (37) is discretized as

$$u_1(t) \;=\; \sum_{i=1}^{N}\lambda_i^* u(t-i\Delta) \tag{39}$$

for a sampling interval of Δ.

4.1.2 Case (ii) $m \geq 2$

The controller structures in sections 3 and 4.1.1 imply that the following stabilizes a system with delay and arbitrary relative degree:

$$u \;=\; -k_2^T\omega_1 - k_1 y + u_1 + r \tag{40}$$

where k_2, k_1, and ω_1 are defined as in (9)-(11), and u_1 is given by (39). Using a combination of the proofs in appendices A and B, it can be shown that the above controller stabilizes the plant for a small τ, $k_1 = k_1^*$, $k_2 = k_2^*$, and λ^*, and leads to a closed-loop transfer function of the form

$$W_{cl}(s) \;=\; W_{cl_0}(s)e^{-\tau s} \tag{41}$$

where $W_{cl_0}(s)$ has stable zeros, and has a relative degree equal to that of the plant. This sets the stage for the adaptive controller design, described in the section below.

4.2 The adaptive controller

We introduce the controller parameter vector and the error parameter vector $\widetilde{k} = k - k^*$. We also denote

$$d(t)^T = [y(t), V_1(t), V_2(t), ...V_{m-1}(t), u(t-N\Delta), ..., u(t-\Delta)]$$

where

$$V_i(t) = \frac{s^{i-1}}{(s+z_c)^{m-1}}\left[u(t)\right], \qquad 1 \le i \le m-1.$$

Similar to the delay free case, the closed-loop transfer function $W_{cl}(s)$ is made to effectively have a relative degree unity by modifying the control signal u as

$$u(t) = (s+a)^{m-1}\left[k^T(t).d_a(t)\right] \tag{42}$$

where

$$d_a(t) = \frac{1}{(s+a)^{m-1}}\left[d(t)\right]. \tag{43}$$

Equation (42) can be rewritten as

$$u(t) = \sum_{i=0}^{m-1} \mathrm{C}_{m-1}^{i} k^{(i)^T}(t) d_i(t). \tag{44}$$

where C_i^j denotes the number of j-combinations of i elements,

$$\begin{aligned} k^{(i)^T}(t) &= \frac{d^i(k)}{dt^i}(t) \\ d_i(t) &= \frac{1}{(s+a)^i}\left[d(t)\right] \end{aligned} \tag{45}$$

As in the delay-free case, we express the control parameters as $k(t) = k^* + \widetilde{k}(t)$, to obtain the closed-loop system equations

$$y = W_{cl}(s)(\widetilde{k}^T\omega) + r \tag{46}$$

where $W_{cl}(s)$ is given by (41). Defining $e_1 = y_p - y_m$, we obtain that

$$e_1(t) = W_m(s)e^{-s\tau}\left[\widetilde{k}^T(t).d_a(t)\right] \tag{47}$$

where $W_m(s) = (s+a)^{m-1}W_{cl_0}(s)$ has relative degree unity and is SPR.

The overall problem can be summarized thus: given the error model (47) where d_a is given by Eq. (43), determine an adaptive law for adjusting k so that it is differentiable $m-1$ times and all signals in the loop are bounded. A time domain representation of Eq. (47) follows:

$$\begin{aligned} \dot{e}(t) &= A_s e + b_s \left(k(t-\tau) - k^*\right)^T .d_a(t-\tau) \\ e_1(t) &= h_s^T.e(t) \end{aligned} \tag{48}$$

where (h_s, A_s, b_s) is a state space representation of $W_m(s)$, that is, we have

$$h_s^T(sI - A_s)^{-1}b_s = W_m(s). \tag{49}$$

Since $W_m(s)$ is SPR, for any matrix Q_s symmetric strictly positive, there exists a matrix P_s symmetric strictly positive, such that

$$\begin{aligned} A_s^T P_s + P_s^T A_s &= -Q_s \\ P_s b_s &= h_s \end{aligned} \tag{50}$$

We note that d_a is differentiable $m-1$ times. In what follows, d_{a_i} and k_i denote the i^{th} element of the vectors d_a and k, respectively.

The following adaptive law is suggested for adjusting k:

$$\dot{k}'(t) = -e_1(t)d_a(t-\tau) \tag{51}$$

$$\dot{x}_i(t) = (Ax_i + bk_i')f(d_{a_i}(t-\tau)) \tag{52}$$

$$k_i = c^T.x_i \tag{53}$$

for $i = 1, ..., m+N$, where $f(\cdot)$ is defined as in (22) and (c, A, b) are chosen so that

$$c^T(sI - A)^{-1}b = \frac{\alpha(0)}{\alpha(s)} \tag{54}$$

and $\alpha(s)$ is a stable polynomial of degree $m-1$. The choice of k as in Eqs. (51)-(53) guarantees that k is differentiable $m-1$ times.

4.3 Proof of Stability

As in [6, 20], we shall introduce first a *model transformation* of (48) using an integration over one delay interval $[-\tau, 0]$, that is:

$$\begin{aligned} \dot{e}(t) &= A_s e + b_s\left(k(t) - k^*\right)^T .d_a(t-\tau) \\ &\quad - b_s \int_{-\tau}^{0} \dot{k}(t+\theta)^T d_a(t-\tau)d\theta. \end{aligned} \tag{55}$$

The next step is to introduce the following Lyapunov function candidate:

$$V = \sum_{i=1}^{4} V_i$$

$$
\begin{aligned}
V_1 &= e^T P_s e, \quad V_2 = (k' - k^*)^T (k' - k^*) \\
V_3 &= \delta \sum_{i=1}^{m} z_i^T P z_i, \\
V_4 &= \sum_{i=1}^{m} \int_{-\tau}^{0} \int_{t+\nu}^{t} \| c^T A z_i(\xi) \|^2 f(d_{a_i}(\xi - \tau))^2 d\xi d\nu \qquad (56) \\
z_i &= x_i + A^{-1} b k_i', \qquad A^T P + P^T A = -I \qquad (57) \\
\delta &= \frac{\|c\|}{\|P A^{-1} b\|}
\end{aligned}
$$

Note that Eq. (56) is similar to the delay free case, except that the Lyapunov-Krasovskii functional V_4 in Eq. (56)) has been added, as suggested by Burton [14] (for a second order example), and Niculescu [15] for dealing with time delays.

Using (57) and (52) in (55), it follows:

$$
\begin{aligned}
\dot{e}(t) &= A_s e + b_s \left(k(t) - k^* \right)^T .d_a(t - \tau) \\
&\quad - b_s \sum_{i=1}^{m} \left(\int_{-\tau}^{0} c^T A z_i(t+\theta) d_{a_i}(t-\tau) f(d_{a_i}(t+\theta-\tau)) d\theta \right).
\end{aligned}
$$

In equations (52) and (56), μ is a positive parameter that will be chosen so that $\dot{V} \leq 0$. Denoting

$$
c^T A z_i(t+\theta) f(d_{a_i}(t+\theta-\tau)) = a, \qquad e_1(t) d_{a_i}(t-\tau) = b
$$

it can be shown that

$$
\begin{aligned}
\dot{V} &\leq -e^T Q_s e - \delta \sum_{i=1}^{m} \|z_i\|^2 f(d_{a_i}(t-\tau)) - \sum_{i=1}^{m} \int_{-\tau}^{0} \left(a^2 - 2|a||b| + b^2 \right) d\theta \\
&\quad + 4 \sum_{i=1}^{m} \|c\| \|z_i(t)\| |e_1(t)| |d_{a_i}(t-\tau)| + \sum_{i=1}^{m} \int_{-\tau}^{0} \left[\|c^T A z_i(t)\|^2 f(d_{a_i}(t-\tau))^2 + b^2 \right] d\theta.
\end{aligned}
$$

Expressing $Q_s = Q_1 + Q_2$, where both Q_1 and Q_2 are positive-definite matrices, denoting ϵ as the minimum eigenvalue of Q_2, and choosing μ as

$$
\mu = \frac{4\|h_s\|^2 m \|c\|^2}{\epsilon \delta}
$$

we obtain that

$$
\begin{aligned}
\dot{V}(t) &\leq -e^T \left(Q_1 - \tau d_a^T(t-\tau) d_a(t-\tau) h_s^T h_s \right) e - \sum_{i=1}^{m} \left(\sqrt{\frac{\epsilon}{m}} \|e\| - \sqrt{\delta\mu} \|z_i\| |d_{a_i}(t-\tau)| \right)^2 \\
&\quad - \sum_{i=1}^{m} \left(\delta - \tau \|c^T A\|^2 (1 + \mu \mid d_{a_i}(t-\tau) \mid^2)^2 \right) \|z_i\|^2. \qquad (58)
\end{aligned}
$$

Thus, $\dot{V}$ is negative-definite if τ satisfies both of the inequalities

$$Q_1 - \tau d_a(t-\tau)^T d_a(t-\tau) h_s h_s^T \;>\; \alpha I \tag{59}$$

$$\delta - \tau \|c^T A\|^2 \left(1 + \mu d_{a_i}^2(t-\tau)\right)^2 \;>\; 0 \tag{60}$$

for some $\alpha > 0$. We show below that the conditions in Eqs. (59) and (60) can be replaced by *bounds* on states at time t_0 and over the interval $[t_0 - \tau, t_0]$ so that the domain of attraction over which $\dot{V} \leq 0$ can be delineated more precisely.

Suppose the values of d_a over $[t_0 - \tau, t_0)$ are such that

$$\sup_{\theta \in [t_0 - \tau, t_0)} \|d_a(\theta)\|^2 \;\leq\; \gamma \tag{61}$$

for some real positive γ, and a delay value $\bar{\tau}_1$ is such that

$$\left\{ \begin{array}{l} Q - \bar{\tau}_1 \gamma h_s h_s^T > \alpha I \\ \delta - \bar{\tau}_1 \|c^T A\| (1 + \mu\gamma)^2 > 0. \end{array} \right. \tag{62}$$

Then using the *step-by-step* type argument for the construction of the solution of the associated FDE with persistent perturbation [16], it follows that combining both (61) and (62) on the interval $[t_0, t_0 + \tau)$, the following inequalities

$$\left\{ \begin{array}{l} Q - \tau d_a(\xi-\tau)^T d_a(\xi-\tau) h_s h_s^T > \epsilon I \\ \delta - \tau m \|c^T A\| \gamma (1 + \mu \mid d_{a_i}(\xi - \tau) \mid^2)^2 > 0. \end{array} \right. \tag{63}$$

are satisfied for all $\tau \in [0, \bar{\tau}_1(\gamma))$, and for all $\xi \in [t_0, t_0 + \tau)$. From the structure of the inequality in (58), it also follows that the Lyapunov-Krasovskii functional V is *non-increasing* on the interval $[t_0, t_0 + \tau)$, if the bound on the delay τ is given by $\bar{\tau}_1$. In addition, since V is a positive-definite function of e, we have that

$$\lambda_{min}(P_s) \|e(\zeta)\|^2 \leq V(\zeta) \leq V(t_0), \quad \forall \zeta \in [t_0, t_0 + \tau).$$

We note that $X = e + X_m$, where X and X_m are the overall states of the closed-loop system and the reference model, respectively, X_m is bounded, and that d is a sub-vector of X. Therefore, $d_a(t)$ is *bounded* on the (first) delay interval $[t_0, t_0 + \tau)$, and the corresponding bound is given by:

$$\sup_{\theta \in [t_0, t_0 + \tau)} \|d_a(\theta)\|^2 \;\leq\; \frac{V(t_0)}{\lambda_{min}(P_s)} + X_{mo} = \gamma_2(t_0) \tag{64}$$

where X_{mo} depends on the model initial conditions. Note that the bound $\gamma_2(t_0)$ includes information only with respect to the initial data of the system.

Let us consider now the derivative of V on the (second) delay interval $[t_0 + \tau, t_0 + 2\tau)$. Using the form of (62) and (64) on $[t_0, t_0 + \tau)$, it follows that the derivative of V is negative if the delay τ is bounded by $\tau < \bar{\tau}_2$, where $\bar{\tau}_2$ satisfies the inequalities:

$$\begin{cases} Q - \bar{\tau}_2 \gamma_2 h_s h_s^T > \alpha I \\ \delta - \bar{\tau}_2 \|c^T A\| (1 + \mu\gamma_2)^2 > 0. \end{cases} \tag{65}$$

By repeating the above process, it can be shown that the constructions above also hold on the next delay intervals $[t_0 + (k-1)\tau, t_0 + k\tau)$ for (any) positive integer $k \geq 2$. It therefore follows that if

$$\bar{\tau} \;=\; min\left\{\bar{\tau}_1(\gamma), \bar{\tau}_2(\gamma_2)\right\}. \tag{66}$$

where $\bar{\tau}_1$ and $\bar{\tau}_2$ satisfy Eqs. (62) and (65) respectively, then the inequalities in (58) are satisfied for all $t \geq t_0$. Hence, all the signals are bounded, and using the same arguments as in [17], it can be shown that $\lim\limits_{t\to+\infty} \|e(t)\| = 0$. This leads to our main result of this section:

Theorem 1 *Consider the plant in (1), the model in (2), the control input as in (44), and the adaptive law specified by Eqs. (51)-(53). Then for any d_a satisfying the inequality (61) on $[t_0 - \tau, t_0)$ where $\gamma > 0$, and for any* delay $\tau < \bar{\tau}$ *given by (66), it can be shown that*

(i) *the closed-loop system has globally bounded solutions, and*

(ii) $\lim_{t\to+\infty} \|e(t)\| = 0$,

Remarks:

1. The controller in (44) is of the order of $m - 1$ and is therefore independent of the order of the plant. Therefore, in all problems where $m << n$, the proposed solution results in a low-order controller leading to a stable performance. We also note that in problems where the goal is one of stabilization rather than tracking, no knowledge of the reference model $W_m(s)$ is required. The stability arguments presented also indicate that stabilization is enabled through a “high-gain” in the controller parameters. It is interesting to note that despite such a high-gain characteristic, the controller structure is still capable of accommodating large

time-delays in the plant. Recent numerical and experimental studies have reported that these controllers can be implemented successfully in practical applications. The same robustness properties as in Section 3.3.1 can be derived here as well by adding a term $-\sigma_0 k'$ to the adaptive law in Eq. (51).

2. The proof of stability is demonstrated in a straightforward manner through the use of Lyapunov functions. As a result, estimates of transient performance of the adaptive system can be derived quite easily. In this regard, the controller represents an improvement over those based on augmented errors [5] or recursive Lyapunov functions [18].

3. A combination of various tools has been incorporated in the proposed adaptive controller. The first is the introduction of a controller that is based on the relative degree of the plant, which exploits its high-gain properties for stabilization and the presence of stable plant zeros. The second is a variation of the pole-placement controller proposed in [4] and is modified in [9] so as to produce a reduced-order controller. The third is the utilization of high-order tuners proposed in [10]. While the details of the control laws differ from those in [10], the general idea behind the control laws was inspired by the results in [10]. The final component is the construction of the Lyapunov-Krasvoskii functional to demonstrate stability. All of the four components were used to demonstrate closed-loop stability.

4. Further improvements of the delay bounds can be obtained if V_4 in the Lyapunov candidate V is replaced by:

$$V_4(t) = \sum_{i=1}^{m} \int_{-\tau}^{0} \int_{t+\nu}^{t} \|c^T A z_i(\xi)\|^2 \left[1 + \mu^2 d_{a_i}(\xi - \tau)^4\right] d\xi d\nu.$$

Using the same computational scheme, the inequality in (60) will become:

$$\delta - \tau \|c^T A\|^2 \left(1 + \mu^2 d_{a_i}(t - \tau)^4\right) > 0 \tag{67}$$

which is less conservative than (60).

5. The adaptive controllers proposed in this paper for the case when $m = 2$ have been implemented experimentally both on a bench-top [7], and on a medium-scale combustion rig [19], and resulted in about 20 db reduction in pressure in the combustion system.

5 Summary

In this paper, the problem of adaptive control in the presence of large time-delays is considered. The control architecture proposed consists of a reduced order controller that depends on the relative degree of the plant rather than its order which is combined with a posicast control structure. This architecture is shown to be amenable to adaptation and to lead to stability within a bounded domain for a small time-delay. Stable adaptive laws that are implementable were generated by using high-order tuners and a Lyapunov-Krasvoskii functional and are in turn used to guarantee closed-loop stability.

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References

[1] J.P. Hathout, M. Fleifil, A.M. Annaswamy, and A.F. Ghoniem. Combustion instability active control using periodic fuel injection. *AIAA Journal of Propulsion and Power*, 18:390–399, 2002.

[2] O.J. Smith. “A controller to overcome dead time”. *ISA Journal*, 6, 1959.

[3] A.Z. Manitius and A.W. Olbrot. Finite spectrum assignement problem for systems with delays. *IEEE Transactions on Automatic Control*, AC-24 no. 4, 1979.

[4] K. Ichikawa. Frequency-domain pole assignement and exact model-matching for delay systems. *Int. J. Control*, 41:1015–1024, 1985.

[5] R. Ortega and R. Lozano. Globally stable adaptive controller for systems with delay. *International Journal of Control, Vol. 47, No. 1, pp 17-23*, 1988.

[6] S. I. Niculescu and A. M. Annaswamy. A simple adaptive controller for positive-real systems with time-delay. In *Proc. American Control Conference*, Chicago, IL, February 2000.

[7] S. Evesque, A.P. Dowling, and A.M. Annaswamy. Adaptive algorithms for control of combustion. In *NATO RTO/AVT Symposium on Active Control Technology for Enhanced Performance in Land, Air, and Sea Vehicles,*, Braunschweig, Germany, May 2000.

[8] S. Evesque, A.M. Annaswamy, S. Niculescu, and A.P. Dowling. Adaptive control of time-delay systems. In *Proceedings of the Eleventh Yale Workshop on Applications of Adaptive Systems Theory*, New Haven, Ct., 2001.

[9] S. Evesque. *Adaptive Control of Combustion Oscillations*. PhD thesis, University of Cambridge, Cambridge, UK, November 2000.

[10] A. S. Morse. High-order parameter tuners for the adaptive control of linear and nonlinear systems. In *US-Italy Joint Seminar on Systems, Models and Feedback: Theory and Applications*, Capri, Italy, 1991.

[11] B. Martensson. “The order of any stabilizing regulator is sufficient information for adaptive stabilization”. *Systems and Control Letters*, 6:87:91, July 1985.

[12] A.M. Annaswamy and A.F. Ghoniem. Active control in combustion systems. *IEEE Control Systems Magazine*, pages 49–63, December 1995.

[13] A.M. Annaswamy and A.F. Ghoniem. Active control of combustion instability: Theory and practice. *IEEE Control Systems Magazine*, 2002 (to appear).

[14] T. A. Burton. *Stability and periodic solutions of ordinary and functional differential equations*. Academic Press, Orlando,FL, 1985.

[15] S. I. Niculescu. *Time-delay systems. Qualitative aspects on the stability and stabilization (in French)*. Nouveaux Essais, Diderot, Paris, 1997.

[16] A. Halanay. *Differential equations: Stability, Oscillations, Time lags*. Academic Press, New York, 1966.

[17] K. S. Narendra and A. M. Annaswamy. *Stable Adaptive Systems*. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1989.

[18] M. Krstić, I. Kanellakopoulos, and P. V. Kokotović. Nonlinear design of adaptive controllers for linear systems. *IEEE Transactions on Automatic Control*, 39(4):738–752, April 1994.

[19] A.J. Riley, A.P. Dowling, and S. Evesque. Adaptive closed-loop control on an atmospheric gaseous lean-premixed combustor. In *Proceedings of the IGTI*, 2002 ASME Turbo Expo, Amsterdam, Netherlands, 2002.

[20] S. I. Niculescu and A. M. Annaswamy. An adaptive smith-controller for time-delay systems with relative degree $n^* \leq 2$. *Systems and Control Letters* (accepted for publication), 2002.

Appendix A

The characteristic polynomial of the closed-loop system in (8) can be rewritten as

$$R_{cl}(s) \;=\; R_p(s)p_c(s) + k_1(s+z_c)^{m-1}Z_p(s)$$

where

$$p_c(s) \;=\; s^{m-1} + \sum_{i=0}^{m-2} s^i \left(C^i_{m-1} z_c^{m-1-i} + k_{2(i+1)}\right)$$

for suitable constants C^i_{m-1}, $i = 0, \ldots, m-2$. For $k_1 > 0$ and large, $n-1$ roots of $R_{cl}(s)$ are close to the zeros of $(s+z_c)^{m-1}Z_p(s)$, and hence are stable. The remaining m zeros of $R_{cl}(s)$ are found at large s, and can be shown to be the roots of the polynomial

$$\overline{R}(s) \;=\; s^m + c_{m-1}s^{m-1} + \cdots + c_1 s + c_0$$

where

$$\begin{aligned} c_{m-1} &= \mathrm{C}^{m-2}_{m-1} z_c + k_{2,m-1} + b_{n-1} \\ c_{m-2} &= \mathrm{C}^{m-3}_{m-1} z_c^2 + k_{2,m-2} + b_{n-2} \\ &\quad + b_{n-1}\left(\mathrm{C}^{m-2}_{m-1} z_c + k_{2,m-1}\right) \\ \ldots &= \ldots \end{aligned}$$

$$
\begin{aligned}
c_i &= \mathrm{C}_{m-1}^{i-1} z_c^{m-i} + k_{2,i} + b_{n-m+i} \\
&\quad + \sum_{j=1}^{m-i-1} b_{n-j} \left(\mathrm{C}_{m-1}^{i-1+j} z_c^{m-j-i} + k_{2,i+j} \right) \\
\ldots &= \ldots \\
c_0 &= b_{n-m} + \sum_{j=1}^{m-1} b_{n-j} \left(\mathrm{C}_{m-1}^{j-1} z_c^{m-j} + k_{2,j} \right) \\
&\quad + k_0 k_1
\end{aligned}
$$

The above relations imply that for a suitable choice of k_{21}, k_{22}, ..., $k_{2(m-1)}$, coefficients $c_0, \ldots, c_{m-1}$ can be found such that $\overline{R}(s)$ is a Hurwitz polynomial. Therefore it follows that $R_{cl}(s)$ is a Hurwitz polynomial as well. •