MATLAB Tutorials

16.01 & 16.02 Unified Engineering - Fall 2006

Violeta Ivanova, Ph.D.
Educational Technology Consultant
MIT Academic Computing

violeta@mit.edu

Information Services and Technology

This Tutorial

- Class materials

  web.mit.edu/acmath/matlab/unified/linsys

- Topics
  - Basics Review
  - Linear Systems

Other References

- Mathematical Tools at MIT
  web.mit.edu/ist/topics/math
- Unified: Intro to MATLAB Tutorial
  web.mit.edu/acmath/matlab/unified/intro
- Other Course 16 MATLAB Tutorials
  web.mit.edu/acmath/matlab/course16/

MATLAB Basics Review

Toolboxes & Help

Vectors & Matrices

Operators & Functions

Information Services
and Technology

Help in MATLAB

- Command line help

  ```
  >> help <command>
          e.g. help eig
  >> lookfor <keyword>
          e.g. lookfor eigenvalue
  ```

- Help Browser
  - Help->Help MATLAB

MATLAB Toolboxes & Help

- MATLAB
  - + Getting Started
  - + Mathematics
    - + Matrices and Linear Algebra
      - + Solving Linear Systems of Equations
        Eigenvalues
    - + Polynomials and Interpolation
      - + Convolution and Deconvolution
  - + Programming
  - + Graphics

Variables & Data Types

- Data type detection

```
>> a = 5; a =' ok'; a = 1.3
```

- Numeric data types

```
>> x = 5
>> y = 5.34; x = round(y)
>> w = 5 + 6i
>> wr = real(w); wi = imag(w)
```

- Built-in constants: **pi i j Inf**
- Special characters

[] () {} ; % : = ... @

Vectors

- Row vector

```
>> R1 = [1 6 3 8 5]
>> R2 = [1 : 5]
>> R3 = [-pi : pi/3 : pi]
```

- Column vector

```
>> C1 = [1; 2; 3; 4; 5]
>> C2 = R2'
```

Matrices

- Creating a matrix

```
>> A = [1 2.5 5 0; 1 1.3 pi 4]
>> A = [R1; R2]
```

- Accessing elements

```
>> A(1,1)
>> A(1:2, 2:4)
>> A(:,2)
```

Matrix Operations

- Operators **+** and **-**

$$>> X = [x_1\ \ x_2];\ Y = [y_1\ \ y_2];\ A = X+Y$$

$$A =$$

$$x_1+y_1 \quad x_2+y_2$$

- Operators *****, **/**, and **^**

```
>> Ainv = A^-1
```

Matrix math is default!

- Operators **.***, **./**, and **.^**

$$>> Z = [z_1\ \ z_2];\ B = [Z.\hat{}2\ \ Z.\hat{}0]$$

$$B =$$

$$z_1^2 \quad z_2^2 \quad 1 \quad 1$$

Built-In Functions

- Matrices & vectors

```
>> [n, m]= size(A)
>> n = length(X)
>> M1 = ones(n, m)
>> M0 = zeros(n, m)
>> En = eye(n)
>> N1 = diag(En)
>> [evecs, evals] = eig(A)
```

- And many others …

```
>> y = exp(sin(t)+cos(t))
```

Graphics

- 2D linear plots: `plot`

```
>> plot (t, z, 'r-')
```

Colors: `b, r, g, y, m, c, k, w`

Markers: `o, *, ., +, x, d`

Line styles: `-, --, -., :`

- Annotating graphs

```
>> legend ('z = f(t)')
>> title ('Position vs. Time')
>> xlabel ('Time')
>> ylabel ('Position')
```

Multiple Plots

- Multiple datasets on a plot

```
>> p1 = plot(xcurve, ycurve)
>> hold on
>> p2 = plot(Xpoints, Ypoints, 'ro')
>> hold off
```

- Subplots on a figure

```
>> s1 = subplot(1, 2, 1)
>> p1 = plot(time, velocity)
>> s2 = subplot(1, 2, 2)
>> p2 = plot(time, acceleration)
```

MATLAB Linear Systems

Linear Equations & Systems

Eigenvalues & Eigenvectors

Convolution

Information Services
and Technology

Systems of Linear Equations

- Definition:

$$\left| \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1m}x_m = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2m}x_m = b_2 \\ \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nm}x_m = b_m \end{array} \right.$$

- Matrix form: Ax = b

$$\begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \ldots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ \ldots \\ b_m \end{bmatrix}$$

Solving Linear Equations

- Define matrix `A`

$$>> A = [a_{11}\ \ a_{12}\ \ \ldots\ \ a_{1m}$$
$$a_{21}\ \ a_{22}\ \ \ldots\ \ a_{2m}$$
$$\ldots$$
$$a_{n1}\ \ a_{n2}\ \ \ldots\ \ a_{nm}]$$

- Define vector `b`

$$>> b = [b_1;\ b_2;\ \ldots\ b_m]$$

- Compute vector x

```
>> x = A \ b
```

Differential Equations

- Ordinary Differential Equation (ODE)

$$y' = f(t, y)$$

- Linear systems -> State Equation

$$\begin{bmatrix} x_1' \\ \dots \\ x_n' \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix} \Rightarrow \boxed{\dot{\bar{x}} = A\bar{x}}$$

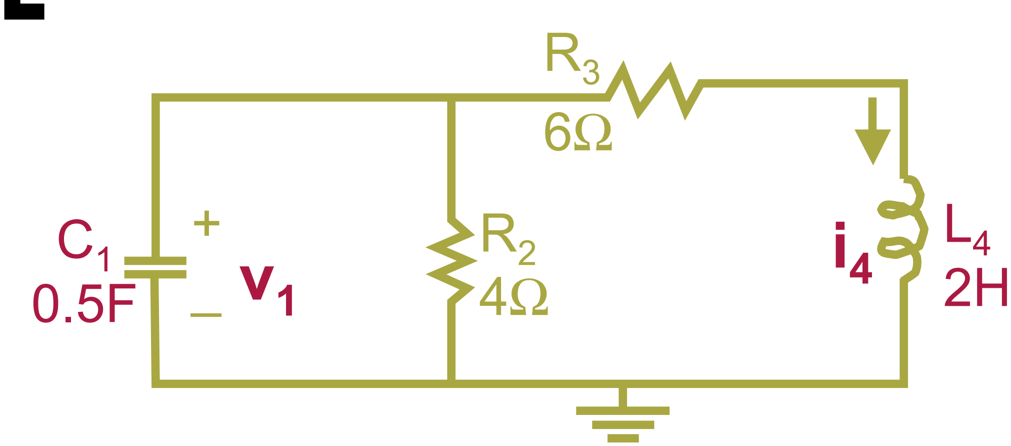
- Eigenvalues, λ_i, and eigenvectors, v_i, of a square matrix ***A***

$$\boxed{A\, v_i = \lambda_i\, v_i}$$

Linear Dynamic Networks

- Solution using **eigenvalue method**
  - Identify the network's states .... $\bar{x}$
  - Identify initial conditions .... $\bar{x}(0)$
  - Find the state equation .... $\dot{\bar{x}} = A\bar{x}$
  - Find eigenvalues & eigenvectors .... λ_i, v_i
  - The general solution is .... $\bar{x}(t) = \sum_i a_i \bar{v}_i e^{\lambda_i t}$
  - Solve for a_i .... $\bar{a} = \begin{bmatrix} \bar{v}_1 & \bar{v}_2 & \ldots & \bar{v}_n \end{bmatrix}^{-1} \bar{x}(0)$

Example: RCL Circuit



$$\frac{dv_1}{dt} = -\frac{1}{2}v_1 - 2i_4$$

$$\frac{di_4}{dt} = \frac{1}{2}v_1 - 3i_4$$

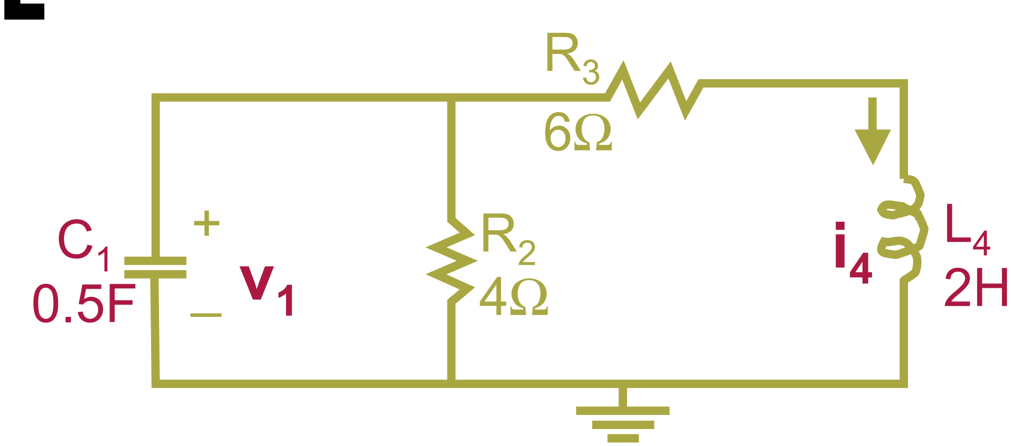
$$v_4(0) = 2$$

$$i_4(0) = 1$$

- State equation

$$\dot{\bar{x}} = \begin{bmatrix} x_1{}' \\ x_2{}' \end{bmatrix} = \begin{bmatrix} \dfrac{dv_1}{dt} \\ \dfrac{di_4}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2} & -2 \\ \dfrac{1}{2} & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ i_4 \end{bmatrix} = A\bar{x} \qquad \bar{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Example: RCL Circuit (continued)



$$\dot{\bar{x}} = A\bar{x}$$

$$Av_i = \lambda_i v_i$$

$$\bar{a} = \begin{bmatrix} \bar{v}_1 & \bar{v}_2 \end{bmatrix}^{-1} \bar{x}(0)$$

$$\bar{x}(t) = \sum_i a_i \bar{v}_i e^{\lambda_i t}$$

- Analytical solution

$$\bar{x}(t) = a_1 \bar{v}_1 e^{\lambda_1 t} + a_2 \bar{v}_2 e^{\lambda_2 t} \Rightarrow \left| \begin{array}{l} v_1(t) = \frac{4}{3}e^{-t} + \frac{2}{3}e^{-2.5t} \\ u_4(t) = \frac{1}{3}e^{-t} + \frac{2}{3}e^{-2.5t} \end{array} \right.$$

RCL Circuit: MATLAB Solution

$$\dot{\bar{x}} = A\bar{x}$$

$$\bar{x}(0)$$

$$Av_i = \lambda_i v_i$$

$$\bar{a} = \begin{bmatrix} \bar{v}_1 & \bar{v}_2 \end{bmatrix}^{-1} \bar{x}(0)$$

$$\bar{x}(t) = \sum_i a_i \bar{v}_i e^{\lambda_i t}$$

```
>> A = [a11 a12; a21 a22]
>> x0 = [v1,0; i4,0]
>> [V, L] = eig(A)
>> λ = diag(L)
>> a = V \ x0
>> v1 = a1*V11*exp(λ1*t)...
      + a2*V12*exp(λ2*t)
   i4 = a1*V21*exp(λ1*t)...
      + a2*V22*exp(λ2*t)
```

Linear Systems Exercises

- Exercise 1: `stateeig.m`
  - Matrices & vectors
  - Systems of linear equations
  - Eigenvalues & eigenvectors

- *Follow instructions in m-file …*

Questions?

Complex Eigenvalues

- Example: complex eigenvalues and eigenvectors from state equation

$$\bar{x}(t) == \begin{bmatrix} 0.5+0.5j \\ 0.5-j \end{bmatrix} e^{(-1+3j)t} + \begin{bmatrix} 0.5-0.5j \\ 0.5+j \end{bmatrix} e^{(-1-3j)t}$$

- Euler's Formula

$$e^{\alpha+\beta j} = e^{\alpha}\left(\cos\beta + j\sin\beta\right)$$

- Solution in terms of real variables

$$x_1(t) = \ldots = \left(\cos 3t - \sin 3t\right)e^{-t}$$

```
>> x1 =(cos(3*t)-sin(3*t))*exp(-t)
```

State Space

- State-space model of a linear system

$$\dot{X} = AX + Bu$$
$$Y = CX + Du$$

X, **u** & **Y**: state, input & output vectors
A, **B** & **C**: state, input & output matrices
D: usually zero (feedthrough) matrix

- Transfer functions

$$H(s) = \frac{Num(s)}{Den(s)} = C(sI - A)^{-1}B = D$$

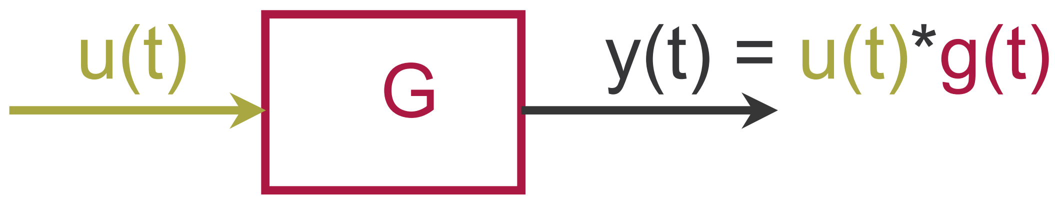
```
>> [Num, Den] = ss2tf(A, B, C, D)
```

Convolution

- Definition

$$g(t) \otimes u(t) = \int_{-\infty}^{\infty} g(t-\tau)u(\tau)\,d\tau$$

- Example: signals & systems



Polynomial Convolution

- Polynomials:

  $$x(s) = s^2 + 2s + 5; \quad y(s) = 2s^2 + s + 3$$

```
>> x = [1  2  5]; y = [2  1  3]
```

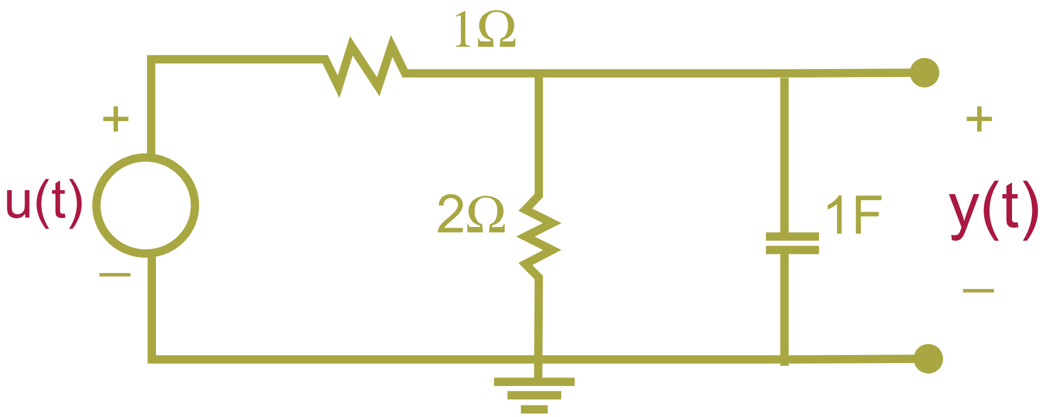
- Convolution and polynomial multiplication

```
>> w = conv(x, y)
>> w =
       2   5   15   11   15
```

- Deconvolution and polynomial division

```
>> [y, r] = deconv(w, x)
```

Example: RC Circuit



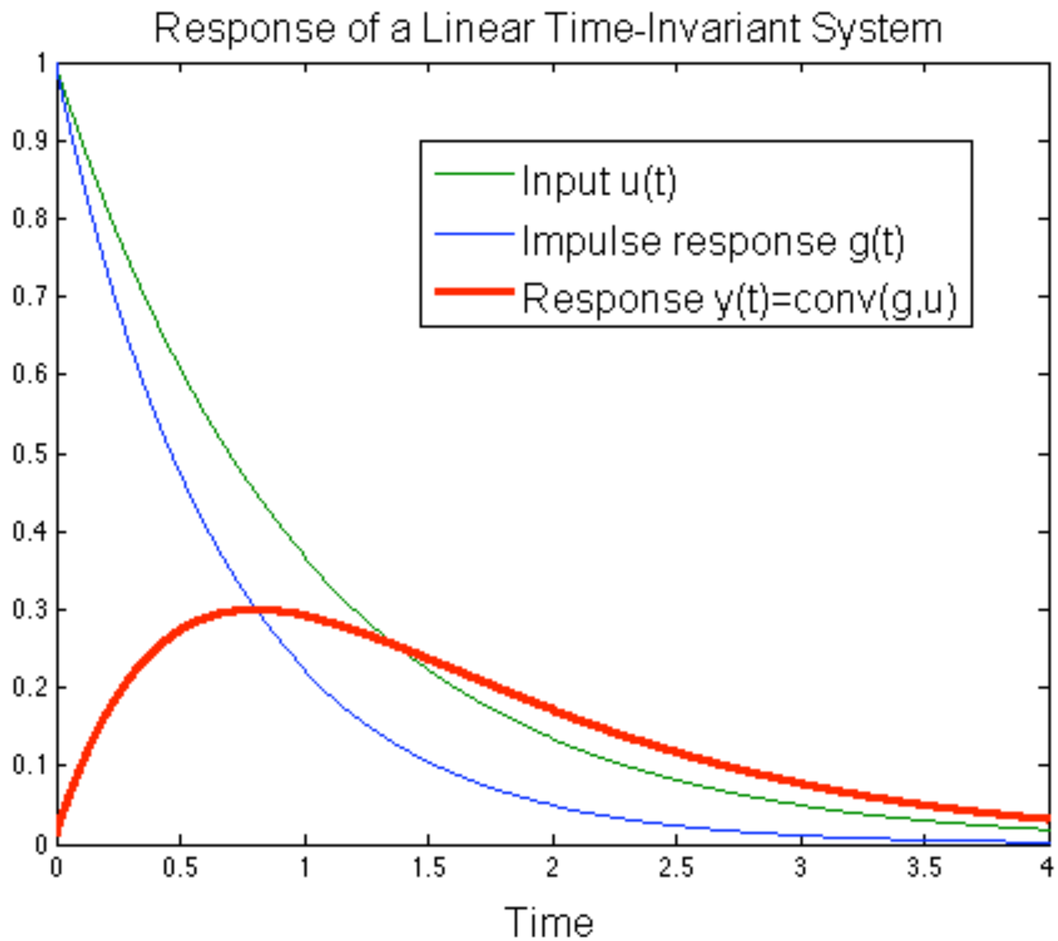
- Input:

$$u(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- Impulse response: $g(t) = \begin{cases} e^{-1.5t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

- System response: $y(t) = \begin{cases} 2e^{-t} - 2e^{-1.5t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Example: RC Circuit (continued)



RC Circuit: MATLAB Solution

$$0 \le t \le t_{max}$$

$$g(t) = e^{\alpha t}$$

$$u(t) = e^{\beta t}$$

$$y(t) = g(t) \otimes u(t)$$

```
>> t  = [0 : dt : tmax]
>> nt = length(t)
>> g  = exp(α*t)
>> u  = exp(β*t)
>> y  = conv(g, u)*dt
>> y  = y(1 : nt)
>> plot(t, u, 'b-', ...
        t, g, 'g-', ...
        t, y, 'r-')
```

Linear Systems Exercises

- Exercise 2: `convolution.m`
  - Matrices & vectors
  - Linear systems of equations
  - Convolution

- *Follow instructions in m-file …*

Questions?