Mathlets: a product of the d'Arbeloff Interactive Mathematics Project

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The d'Arbeloff Interactive Mathematics Project

Personnel:

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IMP is a large project—the computer aided learning tools I will discuss today are just one part of the project.

Initial motivations:

(1) Differential equations is about functions. Textbooks deal with functions by means of formulas. Students come to the course thinking of functions in terms of point by point evaluation. A goal of the course is to get them to think of functions as a whole and in terms of their graphs: visually.

(2) Using chalk and slate, you have a choice: you can write down general formulas, or you can write down special cases. Students don't know in advance what your example is supposed to be an example of. Concepts come to life if they are exemplified, and examples come to life if you can see the whole family from which the example was taken.

(3) Students learn from hands on control of parameters.

IMP response:

Mathlets

`http://www-math.mit.edu/daimp`

We were lucky to have the example of

Interactive Differential Equations,

`http://www.aw-bc.com/ide/`

created Beverly West, Steven Strogatz, Jean Marie McDill, and John Cantwell, and designed by Hu Hohn.

Chronology:

Spring, 2000–Summer, 2002: Mathlets designed and built by Miller and Hohn.

Fall, 2002: Extensive videotaped sessions in which students enrolled in 18.03 (Differential Equations) worked through a script leading them through a Mathlet.

This amounted to usability testing and it led to many improvements in the Mathlets.

- Color coding

- Simplification of presentation

- Improvement of logic of placement

- Ways to get students to use the Mathlets to explore concepts

Spring, 2003: First use in homework in 18.03 for 640 students by Mattuck. Students were asked to explore ODE concepts using eleven of the Mathlets.

Exhaustive web-form questionnaires were used to learn how students used these tools.

Spring, 2004: Use as lecture demos and in homework in 18.03 for 620 students by Miller.

Fall, 2002 –: Porting into Java.

Some quotes from the questionnaire responses:

“I have always thought that the visuals were the most fun part of the problem set. It is always satisfying to see that the ”experimental” value is not too far off from the theoretical value.”

“I had no idea what damping was. Playing around with the damping visual I saw that without damping, the function just oscillated with the same amplitude. If someone had told me this, it wouldn’t have meant anything to me. I had to see it.”

“The concept of nth roots was abstract before the visual, but was clear afterwards.”

“Well, in the Complex Roots one I had a better visualization about complex numbers because I never thought of them with that circle model. So now when I think of complex roots I think about that circle.”

“I found my biggest problem was that I didn’t really understand what the graphs meant from class/reading. Thus, playing with the graph and having to use them really forced me to understand them, and I would DEFINITELY recommend continuing to use them. It’s funny that I didn’t even realize how much they helped until you are trying to think about it later and you remember the graph.”

“All of them without exception were more fun than the rest of the problem sets, and they were to a certain extent fun to fool around with.”

“I like the problems using the visuals. They are helpful. I don’t think that any more should be added, but the ones taht are currently in the problem sets are at teh correct level and take the right amount of time to get the point across.”

“It’s rewarding to see my theoretical values line up with my experimental ones.”

“I enjoyed the way the VibrationAmplitudePhase portrayed the external force and how it worked with varying frequencies. “

“The VibAmpPhase is really cute. My friend said he wanted to try to make it his personal screen saver.”

“Sometimes just calculating with something is not very effective in helping to understand it. I tink the visuals were very good, in general, in helping me understand concepts and think about concepts rather than just follow a standard method of solving a problem.”

“The convolution visuals really made the concept clear to me. Out of all the visuals, I’d say that I liked these ones the best.”

“Both [convolution visuals] really helped me in understanding the subjects.”

"I never really understood the concept of convolution (and the decay) until I saw the visual."

"Poles and Vibrations was great! Educational, and nice to look at. Well made in every respect."

"KEEP DOING THE VISUALS!! I always remember on tests pictures from visuals, plus it adds a dimension to learning."

"I feel like the time was well-spent since I learned a lot during the time."

"I spent more time on it that what was just required to complete the assignment because I found it interesting."

“Helped me understand what graphs look like when I have different e-values, and also put it nicely in terms of trace and determinant, so now I understand many ways of sketching a curve. Also the T vs D graph gave me a really quick way to visualize everything.”

“The Linear Phase Portraits visual was wonderful.”

“I used it to study for a test and I got a 97!”

Sample problem from 18.03
Spring, 2004

14. (M 8 Mar) [Frequency Response] This problem will use the Mathlet `FreqResponseOrder2`. Open the tool and play around with it. Animate the spring system by pressing the "$>>$" key. Chose various values of b and k and watch the results.

(a) There is a white line segment on the main window, joining a blue diamond to a yellow diamond. What does its length signify? What does it mean when the blue dot is above the yellow one? Below? when they coincide?

Click on the "Bode and Nyquist Plot" button at lower right. Three windows open up. The top two windows show the gain and the phase lag, as functions of the input frequency ω. (Actually $-\phi$ is graphed rather than ϕ.) Verify for yourself that for various values of ω the values of A and ϕ look right for the displayed graph of x. (You can get numerical readouts by positioning the cursor over either of these windows.) Fix b and k and drag ω from $\omega = 0$ to $\omega = 4$. You can see that the system response falls behind the input signal, and more so as ω increases.

(b) The lower right window displays the complex number $k/p(i\omega)$. Explain why the Exponential Response Formula shows that the modulus of $k/p(i\omega)$ is the amplitude of the system response (the "gain") and the argument of $k/p(i\omega)$ is the negative of the phase lag.

(c) Fix k at $k = 4$, so $\omega_n = 2$, and fix the input circular frequency at $\omega = 2$. Drag the damping constant slider from $b = 0.5$ to $b = 1.5$ and watch what happens. Especially, watch what happens to the plot indicating $k/p(i\omega)$. Explain the observed effect with equations. To put the matter differently, what is the phase lag if $\omega = \omega_n$?

(d) Professor Trumper aside, engineers don't generally like resonance, so getting rid of resonance is good; and minimizing the damping used to do so is particularly good. This part explores the possibity of eliminating the resonant peak. Fix k at $k = 2$ and drag b from $b = 1.5$ down to $b = 0.5$. The resonant peak in the graph of amplitude as a function of ω seems to move over to the vertical axis and disappear as b increases. Does this actually happen? What is the smallest value of b for which the maximum of $A(\omega)$ occurs at $\omega = 0$? Write b_0 for this value of b; it depends upon k.

(e) For given k and b, what is the "near resonant circular frequency" ω_r: that is, for what value of ω is $A(\omega)$ maximum? (Your answer will depend upon whether b is above or below b_0.) During Professor Trumper's classroom demonstration, it was remarked that the phase lag is at least approximately 90° at near resonance. Is it exactly 90°?

Sample problem from 18.03
Spring, 2004

26. F 9 Apr [Fourier series] **(b)** This will the Mathlet `FourierCoefficients`. Go to an Athena cluster and get this up. Press "Formula" to see the significance of the sliders. Move them around a bit and watch what happens. The yellow curve gives the sum, the white curve gives the sinusoidal function you are adding some of to the sum at the moment.

(i) With the settings on "Sine Series" and "All terms," select target A. Move the sliders around till you get the best fit you can eyeball. Record your results:
$b_1 = \ldots, b_2 = \ldots, \ldots.$

(ii) Now select target D and do the same. But then, before you record your results, select "Distance." This makes a number appear above the graph, which gives a measure of the goodness of fit of the partial

Fourier series you have built. (A somewhat more precise description can be found in the Supplementary Notes, §22.) Move the sliders from the top one to the bottom one to get the best fit you can. Record the results. Notice that you began with large period and then worked your way down to small period.

Now press "Reset," and do the same thing from the bottom up: you are putting in the best possible multiples of sin$(6t)$, then sin$(5t)$, and so on, in that order. Are the numbers you obtain the same as the ones you got going in the other direction? How do these values match up with what you computed in Part **(a)**? Do you suppose you would get different answers if you put in terms in some other more random order?

(iii) Finally, for each of the other targets declare which option among "Sine Series" and "Cosine Series," and among "All terms," "Odd terms," and "Even terms," gives the best fit.