

# A Habit-Based Explanation of the Exchange Rate Risk Premium

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## ABSTRACT

This paper presents a model that reproduces the uncovered interest rate parity puzzle. Investors have preferences with external habits. Counter-cyclical risk premia and pro-cyclical real interest rates arise endogenously. During bad times at home, when domestic consumption is close to the habit level, the pricing kernel is volatile and the representative investor very risk-averse. When the domestic investor is more risk-averse than her foreign counterpart, the exchange rate is closely tied to domestic consumption growth shocks. The domestic investor therefore expects a positive currency excess return. Since interest rates are low in bad times, expected currency excess returns increase with interest rate differentials.

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According to the uncovered interest rate parity (UIP) condition, the expected change in exchange rates should be equal to the interest rate differential between foreign and domestic risk-free bonds. The UIP condition implies that a regression of exchange rate changes on interest rate differentials should produce a slope coefficient of 1. Instead, empirical work following Hansen and Hodrick (1980) and Fama (1984) consistently reveals a slope coefficient that is smaller than 1 and very often negative. The international economics literature refers to these negative UIP slope coefficients as the UIP puzzle or forward premium anomaly.

Negative slope coefficients mean that currencies with higher than average interest rates tend to appreciate, not to depreciate as UIP would predict. Investors in foreign one-period discount bonds thus earn the interest rate spread, which is known at the time of their investment, plus the bonus from the currency appreciation during the holding period. As a result, the forward premium anomaly implies positive predictable excess returns for investments in high interest rate currencies and negative predictable excess returns for investments in low interest rate currencies. There are two possible explanations for these predictable excess returns: time-varying risk premia and expectational errors.

In this paper, I assume that expectations are rational, and I develop a risk premium explanation for the forward premium anomaly. Following Campbell and Cochrane (1999), my model's stand-in investor has external habit preferences over consumption. But I depart from Campbell and Cochrane (1999) in one key respect: in bad times, when consumption is close to the habit level and investors are more risk-averse, risk-free rates are low. In this case, UIP fails just as it does in the data. What is the intuition for this result? When markets are complete, the real exchange rate, measured in units of domestic goods per foreign good, equals the ratio of foreign to domestic pricing kernels. Exchange rates thus depend on foreign and consumption growth shocks. If the conditional variance of the domestic stochastic discount factor is large relative to its foreign counterpart, then domestic consumption growth shocks determine variations in exchange rates. When the domestic economy receives

a negative consumption growth shock, the exchange rate depreciates, lowering the return of a domestic investor long in foreign Treasury Bills. When the domestic economy receives a positive shock, the exchange rate appreciates, increasing the return of the same investor. As a result, exchange rates carry consumption growth risks, and the domestic investor expects a positive risk premium. This reasoning echoes Backus, Foresi, and Telmer (2001), who show that currency risk premia can always be written as the difference between the higher moments of foreign and domestic pricing kernels. When pricing kernels are conditionally log normal, as they are in this paper, risk premia boil down to differences in conditional variances. When the domestic pricing kernel has relatively high conditional variance, an investor who is long in foreign Treasury Bills will receive a positive risk premium.

In the habit model, the conditional variance of the pricing kernel is large in bad times, when consumption is close to the habit level and risk-aversion is high. To account for the UIP puzzle in this framework, real interest rates must be pro-cyclical, meaning low in bad times when risk-aversion is high and high in good times when risk-aversion is low. Under these conditions, domestic investors expect positive currency excess returns when domestic interest rates are low and foreign interest rates are high, thus resolving the forward premium anomaly. The habit model endogenously delivers such counter-cyclical risk-aversion and pro-cyclical real risk-free rates. Expected currency excess returns increase sharply with interest rate differentials, and this produces a negative UIP coefficient in frictionless asset markets.

The success of this model relies on counter-cyclical risk aversion and pro-cyclical real interest rates. In contrast, Wachter (2006) shows that counter-cyclical real interest rates imply an upward-sloping real yield curve and help match features of the nominal yield curve. It is thus clear that this model cannot reproduce both the forward premium anomaly and an upward-sloping real yield curve. But direct evidence on the slope of the real yield curve is inconclusive, and recent contributions in finance and monetary economics, particularly Ang, Bekaert, and Wei (2008) and Clarida, Gali, and Gertler (2000), conclude that real interest

rates are pro-cyclical. Moreover, there is ample evidence, from Harvey (1989) to Lettau and Ludvigson (2009), that expected excess returns are counter-cyclical. As a result, risk premia should be high when real interest rates are low. This negative relationship between risk premia and interest rates is clearly observed on currency markets for both nominal and real interest rates. On equity markets, the evidence pertains to nominal interest rates. In US data, the link between nominal interest rates and equity excess returns has been known since Fama and Schwert (1977). More recently, Campbell and Yogo (2006) show that this link is robust. To extend this result, I turn to equity portfolios of developed countries sorted by interest rates. I compute average stock market returns for the last fifty years (denominated in local currencies) for each portfolio. I find that countries that offer high currency excess returns to the US investor also offer low equity Sharpe ratios to local investors. The model delivers the same result: when foreign interest rates are high, foreign currencies offer high excess returns, but foreign stock markets do not.

To show that the model quantitatively reproduces the UIP puzzle, I rely on a simulation and an estimation exercise. For the simulation, I consider two endowment economies; in each economy, a representative agent has Campbell and Cochrane (1999) preferences calibrated to imply pro-cyclical real risk-free rates. I derive closed form expressions for currency excess returns and UIP slope coefficients when endowment shocks are uncorrelated across countries. In addition, I relax this assumption and simulate a version of the model that is calibrated to match the first and second moments of consumption growth and real interest rates, the cross-country correlation of consumption growth rates, and the maximal Sharpe ratio. The model reproduces the forward premium anomaly, delivering a negative UIP coefficient. The mean, standard deviation and autocorrelation of the consumption growth rate, the real interest rate, the price-dividend ratio, the return on the stock market, and the long-term real yield are in line with their empirical counterparts. The simulation, however, highlights two weaknesses of the model: the simulated real exchange rate is too volatile and too closely correlated with

consumption growth. I make some progress on these issues in a Supplementary Appendix available online. There, I derive the optimal foreign and domestic consumption allocations in a two-country model where agents are characterized by the same habit preferences as in this paper. Agents can trade, but incur proportional and quadratic trade costs. The model then replicates the empirical forward premium and equity premium puzzle and the interest rates and exchange rates' volatility.

I also estimate the model by focusing on the Euler equation of an American investor. His stochastic discount factor depends on US aggregate consumption. He invests in domestic equity and foreign currency markets. I consider two sets of currency excess returns: the investment opportunities in 8 other OECD countries, and the 8 portfolios of currency excess returns built in Lustig and Verdelhan (2007). Along with the CRSP value-weighted stock market return, I also consider first 6, and then 25 Fama-French equity portfolios sorted on book-to-market and size. Following Hansen, Heaton, and Yaron (1996), these estimations rely on a continuously-updating general method of moments (GMM). The estimates lead to reasonable preference parameters. The hypothesis that pricing errors are zero cannot be rejected at conventional confidence levels for most samples.

This paper adds to a large body of empirical and theoretical work on the UIP condition. On the empirical side, most papers test the UIP condition on nominal variables. Two recent studies, however, shift the focus to real variables. Hollifield and Yaron (2003) find that various measures of CPI inflation are unrelated to currency returns. Lustig and Verdelhan (2007) find that real aggregate consumption growth risk is priced on currency markets. On the theory side, numerous studies have attempted to explain the UIP puzzle under rational expectations, but few models reproduce the negative UIP slope coefficient. Successful attempts include Frachot (1996) in a Cox, Ingersoll, and Ross (1985) framework, Alvarez, Atkeson, and Kehoe (2005) when markets are segmented, Bacchetta and van Wincoop (2006) when inattention is rational, Bansal and Shaliastovich (2008) and Colacito (2006) in a Bansal

and Yaron (2004)'s long run risk model, and Farhi and Gabaix (2008) in a model with disaster risk.

The rest of the paper is organized as follows: section I outlines the two-country, one-good model. Section II reports simulation results on stock, bond, and currency returns. Section III estimates the model on currency and equity excess returns. Section IV concludes.

## I. Model

The model focuses on real risk, abstracting from money and inflation. It relies on habit-based preferences to reproduce the UIP puzzle when financial markets are complete.<sup>1</sup>

### A. Habit-based preferences

In the model, there are two endowment economies with same initial wealth and one good. In each economy, a representative agent is characterized by external habit preferences similar to Campbell and Cochrane (1999) but with time-varying risk-free rates. The agent maximizes:

$$E \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma},$$

where  $\gamma$  denotes the risk-aversion coefficient,  $H_t$  the external habit level and  $C_t$  consumption. The external habit level corresponds to a subsistence level or social externality. It depends on consumption through the following autoregressive process of the surplus consumption ratio, defined as the percentage gap between consumption and habit ( $S_t \equiv [C_t - H_t]/C_t$ ):

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g).$$

Lowercase letters correspond to logs, and  $g$  is the average consumption growth rate. The sensitivity function  $\lambda(s_t)$  describes how habits are formed from past aggregate consumption. I assume that in both countries idiosyncratic shocks to consumption growth are *i.i.d* log-normally distributed:

$$\Delta c_{t+1} = g + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d. N(0, \sigma^2).$$

‘Bad times’ refers to times of low surplus consumption ratios (when the consumption level is close to the habit level), and ‘negative shocks’ refers to negative consumption growth shocks  $u$ . The same features apply to the foreign representative agent. Foreign variables are denoted with a  $\star$  superscript. To obtain closed form solutions and present the main intuition, I here assume that the endowment shocks  $u_{t+1}$  and  $u_{t+1}^\star$  are independent across countries. I relax this assumption for the simulations.

The model delivers time-varying risk-aversion and time-varying real risk-free rates. Since each country’s habit level depends on domestic, not foreign, consumption, and on aggregate, not individual, consumption, the local curvature of the utility function, or local risk-aversion coefficient, is  $\gamma_t = -C_t U_{cc}(t)/U_c(t) = \gamma/S_t$ . When consumption is close to the habit level, the surplus consumption ratio is low and the agent very risk-averse.

To obtain risk-free rates, note that the pricing kernel, or stochastic discount factor (SDF), is:

$$M_{t+1} = \beta \frac{U_c(C_{t+1}, H_{t+1})}{U_c(C_t, H_t)} = \beta \left( \frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\gamma} = \beta e^{-\gamma[g + (\phi-1)(s_t - \bar{s}) + (1 + \lambda(s_t))(\Delta c_{t+1} - g)]}. \quad (1)$$

The sensitivity function  $\lambda(s_t)$  governs the dynamics of the surplus consumption ratio:

$$\lambda(s_t) = \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1, \text{ when } s \leq s_{\max}, 0 \text{ elsewhere,}$$

where  $\bar{S}$  and  $s_{\max}$  are respectively the steady-state and upper bound of the surplus-consumption ratio.  $\bar{S}$  measures the steady-state gap (in percentage) between consumption and habit levels. Assuming that  $\bar{S} = \sigma \sqrt{\frac{\gamma}{1-\phi-B/\gamma}}$  and  $s_{\max} = \bar{s} + (1 - \bar{S}^2)/2$ , the sensitivity function  $\lambda(s_t)$  leads to linear, time-varying risk-free rates:

$$r_t = \bar{r} - B(s_t - \bar{s}), \quad (2)$$

where  $\bar{r} = -\ln(\beta) + \gamma g - \frac{\gamma^2 \sigma^2}{2\bar{S}^2}$  and  $B = \gamma(1 - \phi) - \frac{\gamma^2 \sigma^2}{\bar{S}^2}$ . Interest rates are constant when  $B=0$ . For the UIP puzzle, this is obviously not an interesting case. When  $B < 0$ , interest rates are low in bad times and high in good times.<sup>2</sup>

When the interest rate is allowed to fluctuate, this model resembles the affine framework of Cox et al. (1985).<sup>3</sup> But this model does not correspond to a narrow definition of affine representations of the yield curve because the market price of risk is not linear in the state variable  $s$  (the log surplus-consumption ratio). The model belongs, however, to the generalized class of affine factor models because its market price of risk can be written as a linear function of the sensitivity function  $\lambda(s)$ .

## B. Real exchange rates and currency risk premium

I now turn to the definition of real exchange rates and currency risk premia.

**Real exchange rates** There are no arbitrage opportunities and financial markets are complete. The Euler equation for a foreign investor buying a foreign bond with return  $R_{t+1}^*$  is:  $E_t(M_{t+1}^* R_{t+1}^*) = 1$ . The Euler equation for a domestic investor buying the same foreign bond is:  $E_t(M_{t+1} R_{t+1}^* \frac{Q_{t+1}}{Q_t}) = 1$ , where  $Q$  is the real exchange rate expressed in domestic goods per foreign good. Because the stochastic discount factor is unique in complete markets, the change in the real exchange rate equals the ratio of the two stochastic discount factors

at home and abroad:

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^*}{M_{t+1}}. \quad (3)$$

**Exchange rate risk premium** The exchange rate risk premium is the excess return of a domestic investor who borrows funds at home, converts them to a foreign currency, lends at the foreign risk-free rate, and then reconverts his earnings to the original currency. Thus, in logs, the currency excess return  $r_{t+1}^e$  is:

$$r_{t+1}^e = \Delta q_{t+1} + r_t^* - r_t.$$

The domestic investor gains the foreign interest rate  $r_t^*$ , but has to pay the domestic interest rate  $r_t$ . He therefore loses if the dollar appreciates in real terms -  $q$  decreases - when his assets are abroad.

Backus et al. (2001) show that expected foreign currency excess returns are equal to one half of the difference between the conditional variances of the two pricing kernels when stochastic discount factors are log-normal. To prove this point, note that real interest rates are defined as:

$$\begin{aligned} r_t &= -\log E_t M_{t+1} = -E_t m_{t+1} - \frac{1}{2} \text{Var}_t(m_{t+1}), \\ r_t^* &= -\log E_t M_{t+1}^* = -E_t m_{t+1}^* - \frac{1}{2} \text{Var}_t(m_{t+1}^*). \end{aligned}$$

The expected change in the exchange rate is:

$$E_t(\Delta q_{t+1}) = E_t(m_{t+1}^*) - E_t(m_{t+1}) = -r_t^* + r_t - \frac{1}{2} \text{Var}_t(m_{t+1}^*) + \frac{1}{2} \text{Var}_t(m_{t+1}).$$

As a result, the expected log currency excess return is equal to:<sup>4</sup>

$$E_t(r_{t+1}^e) = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^*). \quad (4)$$

Equation (4) shows that in order to obtain predictable currency excess returns, log SDFs must be heteroskedastic. The same equation also highlights the link between currency excess returns and other risk premia. When the SDF is lognormal, the maximal Sharpe ratio is approximately equal to the standard deviation of the log SDF. As a result, currency excess returns correspond to the difference in squared maximal Sharpe ratios obtained on any other assets. I investigate the link between currency excess returns and other risk premia in the next section, and focus now on the UIP puzzle.

### C. A solution to the UIP puzzle

To further simplify notations, I assume that the preferences of domestic and foreign investors are characterized by the same underlying structural parameters: the same risk-aversion coefficients ( $\gamma = \gamma^*$ ), the same persistence and steady-state values for the surplus-consumption ratio ( $\phi = \phi^*$  and  $\bar{S} = \bar{S}^*$ ), and the same mean and volatility for consumption growth rates ( $g = g^*$  and  $\sigma = \sigma^*$ ). In this set-up, I derive a closed form expression for the UIP slope coefficient.

In the model, the variance of the log stochastic discount factor is equal to:

$$Var_t(m_{t+1}) = \frac{\gamma^2 \sigma^2}{\bar{S}^2} [1 - 2(s_t - \bar{s})],$$

and equation (4) leads to the following expected currency excess return:

$$E_t(r_{t+1}^e) = E_t(\Delta q_{t+1}) + r_t^* - r_t = \frac{\gamma^2 \sigma^2}{\bar{S}^2} (s_t^* - s_t). \quad (5)$$

The real interest rate differential is:

$$r_t - r_t^* = -B(s_t - s_t^*).$$

As a result, the expected change in exchange rates is linear in the interest rate differential:

$$E_t(\Delta q_{t+1}) = \left[1 + \frac{1}{B} \frac{\gamma^2 \sigma^2}{\bar{S}^2}\right] [r_t - r_t^*] = \gamma \left(\frac{1 - \phi}{B}\right) [r_t - r_t^*]. \quad (6)$$

In this framework, the UIP slope coefficient no longer needs to be equal to unity, even if consumption shocks are simply *i.i.d.* Since the risk premium depends on the interest rate gap, the coefficient  $\alpha$  in a UIP regression can be below 1. *This means that accounting for the forward premium anomaly requires pro-cyclical interest rates, i.e.  $B < 0$ .*

What is the intuition behind this result? First, exchange rates covary with consumption growth shocks and command time-varying consumption risk premia. As mentioned earlier, this model implies that the local curvature of the utility function is equal to  $\gamma/S_t$ . A low surplus consumption ratio (when consumption is close to the habit level) thus makes the agent more risk-averse. Using equations (1) and (3), the change in the real exchange rate is:

$$\Delta q_{t+1} = k_t + \gamma[1 + \lambda(s_t)](\Delta c_{t+1} - g) - \gamma[1 + \lambda(s_t^*)](\Delta c_{t+1}^* - g),$$

where  $k_t$  summarizes all variables known at date  $t$ . In bad times, when the domestic investor is more risk averse than his foreign counterpart, the surplus consumption ratio is lower,  $s_t < s_t^*$ , and the sensitivity function is higher at home than abroad,  $1 + \lambda(s_t) > 1 + \lambda(s_t^*)$ . In other words, the conditional variance of the pricing kernel is higher at home than abroad. In this case, domestic consumption shocks dominate the effect of foreign consumption shocks on the exchange rate. As a result, when the domestic economy receives a negative consumption growth shock in bad times, the exchange rate depreciates, lowering domestic returns on

foreign bonds. When the domestic economy receives a positive consumption growth shock, the exchange rate appreciates, increasing domestic returns on foreign bonds. Thus, the exchange rate exposes the home investor to more domestic consumption growth risk when the domestic investor is more risk averse than his foreign counterpart. *The domestic investor therefore receives a positive currency excess return if he is more risk averse than his foreign counterpart.* When the domestic investor is less risk averse than the foreign investor, foreign consumption shocks dominate the exchange rate, and the foreign investor receives a positive excess return. Here the risk premium is perfectly symmetric, thus taking into account the fact that positive excess returns for the domestic investor mean negative excess returns for the foreign investor. The currency risk premium is time-varying because risk-aversion is time-varying too.

Second, *times of high risk aversion correspond to low interest rates.* In bad times, when consumption is close to the subsistence level, the surplus consumption ratio  $s_t$  is low, the domestic agent is very risk-averse, and domestic interest rates are low. As we have seen, a domestic investor expects to receive a positive foreign currency excess return in times when he is more risk-averse than his foreign counterpart. Thus the domestic investor expects positive currency excess returns when domestic interest rates are low and foreign interest rates are high. This translates to a UIP coefficient less than 1. It is negative because in times of high risk-aversion a small consumption shock has a large impact on the change in marginal utility. The stochastic discount factor has therefore considerable conditional variance  $Var_t(m_{t+1})$ , and risk premia are high. As a result domestic currency excess returns increase sharply with risk-aversion and thus interest rate differentials.

We can reinterpret this result using Backus et al. (2001). They establish the following two necessary conditions on pricing kernels in order to reproduce the UIP puzzle: a negative correlation between the difference in conditional means and the half difference in conditional variances and a greater volatility of the latter. Let us check these two conditions. For the

first one, the difference in the conditional means of the two pricing kernels is here equal to  $\gamma(1 - \phi)(s_t - s_t^*)$ . The currency risk premium, which is the half difference in conditional variances of the two pricing kernels, is given in equation (5). The difference in conditional means and the half difference in conditional variances are clearly negatively correlated. For the second condition, the risk premium has a larger variance than the difference in conditional means if  $\gamma^2\sigma^2/\bar{S}^2$  is above  $\gamma(1 - \phi)$ , which is the case for pro-cyclical interest rates ( $B < 0$ ). This model therefore satisfies the Backus et al. (2001) conditions. Note that it also satisfies the conditions of Proposition 2, page 16 of Backus et al. (2001). As a result, it can reproduce the UIP puzzle, but only at the price of potentially negative real interest rates, which is clearly the case when  $B < 0$ . In a model with storable goods, negative real risk-free rates are an undesirable feature. Empirically though, I find that the model does not overestimate the frequency of negative real risk-free rates.

#### **D. Pro-cyclical real interest rates and counter-cyclical risk premia**

The model reproduces the UIP puzzle only if real interest rates are pro-cyclical and risk premia are counter-cyclical, eg bad times correspond to low risk-free rates and high risk premia. What is the evidence to support these assumptions? I will first review empirical findings on the US and then add new international evidence.

**US evidence** Current research in econometrics, finance and monetary economics agrees on the pro-cyclical behavior of real US interest rates. Challenging previous findings from Stock and Watson (1999), Dostey, Lantz, and Scholl (2003) show that ex-ante real interest rates are positively correlated to contemporaneous and lagged cyclical output. Likewise, Ang et al. (2008), who study the yield curve and inflation expectations, find that US real interest rates are pro-cyclical. The same conclusion appears in monetary economics. Following Taylor

(1993), an entire literature seeks to estimate monetary policy rules in which short term interest rates depend on inflation and output gaps, which are cyclical indicators. Clarida et al. (2000), for example, show that real interest rates increase with output gaps both in pre- and post-Volcker samples.

A large literature, from Harvey (1989) to Lettau and Ludvigson (2009), finds that risk premia are counter-cyclical. As a result, risk premia and real interest rates move in opposite directions. This is obviously true in currency markets: a UIP slope coefficient below unity implies that currency excess returns are higher when domestic interest rates are lower. The same results obtain on nominal and real variables. In equity markets, the evidence pertains to nominal interest rates. Fama and Schwert (1977) show that high nominal interest rates decrease future returns, a finding confirmed by Campbell and Yogo (2006). Using efficient tests of stock market predictability, they reject the null of no predictability for the nominal risk-free rate at monthly and quarterly frequencies over the 1952-2002 period.

**International evidence** I will now turn to the link between currency and equity risk premia across countries. In the model, domestic interest rates lower than those abroad imply high currency excess returns, and lower than usual domestic interest rates imply high Sharpe ratios.

To highlight the empirical link between currency and equity risk premia, I build portfolios of countries sorted on interest rates. Building these portfolios amounts to conditioning on foreign interest rates. Doing so is crucial because, in theory as in practice, unconditional country-by-country currency excess returns are zero in the long run. In theory, for similar countries, the purchasing power parity condition holds in the long run, and interest rate differentials and currency risk premia are on average equal to zero. In practice, country-by-country average currency excess returns are not significantly different from zero. By sorting countries on interest rates, one extracts non-zero risk premia from currency markets.

To illustrate this point, note that, using a first-order Taylor approximation, the ex-post currency excess return for country  $i$  is:

$$\begin{aligned} r_{t+1}^{ex-post,i} &\simeq -\gamma \left[ (\phi - 1)(s_t^i - s_t) + \frac{1}{\bar{S}}(u_{t+1}^i - u_{t+1}) \right] - B(s_t^i - s_t), \\ &\simeq E_t(r_{t+1}^{e,i}) - \frac{\gamma}{\bar{S}}(u_{t+1}^i - u_{t+1}). \end{aligned}$$

Currency portfolios bunch together countries with a similar level of risk-aversion (inversely related to the surplus-consumption ratio  $s^i$  and the interest rate  $r^i$ ). By taking averages of excess returns inside each portfolio, the idiosyncratic risks  $u_{t+1}^i$  cancel each other out, leaving only the expected currency excess returns.

To build these portfolios, I rank countries period by period using their interest rates at the end of the previous period. The first portfolio contains low interest rate currencies, and the last high interest rate currencies. Using this ranking, I allocate stock market excess returns (expressed in foreign currencies) into the same portfolios and compute mean excess returns for each of them. I consider only developed countries and build eight portfolios as in Lustig and Verdelhan (2007).<sup>5</sup> Table I reports the obtained average currency and stock market excess returns.

As expected from the UIP literature, and as shown in Lustig and Verdelhan (2007), low interest rate countries offer low currency excess returns, while high interest rate countries offer high currency excess returns. More interestingly, countries offering high currency excess returns for US investors offer low equity Sharpe ratios to their local counterparts. The spread between currency excess returns in the first and last portfolios is highly significant (with a mean value of  $-4.65\%$  and a standard error of 0.95). Likewise, the Sharpe ratio obtained on the spread between stock market excess returns in the first and last portfolios is significant (with a mean value of  $0.36\%$  and a standard error of 0.15). When the foreign interest rate is high, the foreign currency offers high excess returns, but the foreign stock market does

not. As a result, there seems to be a clear link between currency and equity risk premia, and interest rates appear to be the relevant risk indicator. Table I is clearly not a full test of the model. It does, however, encourage us to look further. To do so, I now turn to the calibration and simulation of the model.

[Table 1 about here.]

## II. Simulation

To calibrate the model, I assume that two countries - for example the United States and United Kingdom - share the same set of parameters ( $g$ ,  $\sigma$ ,  $\beta$ ,  $\gamma$ ,  $\phi$  and  $\bar{S}$ ) and that their endowment shocks are correlated across countries (with a correlation coefficient  $\rho$ ).

### A. Calibration

I fix the risk-aversion coefficient  $\gamma$  at 2. This is a common value in the real business cycle literature and also the value chosen by Campbell and Cochrane (1999) and Wachter (2006) in their simulations. To determine the remaining six independent parameters of the model, I target six simple statistics: the mean  $g$  and standard deviation  $\sigma$  of real per capita consumption growth rates, the cross-country correlation of consumption growth rates  $\rho$ , the mean  $\bar{r}$  and standard deviation  $\sigma_r$  of real interest rates, and the mean Sharpe ratio  $\overline{SR}$ . This calibration faces three difficulties. First, these moments determine the absolute value of  $B$ , but not its sign. I pick a negative  $B$  to ensure that Sharpe ratios are high when real interest rates are low, thus mimicking the data. Second, there is no simple closed-form expression for risk-free rate volatility. I rely on an approximation around the steady-state.<sup>6</sup> Third, there is a discrepancy between theoretical steady-state values and empirical averages. The mean of the state variable is above its steady-state value because the model predicts occasional

deep recessions not matched by large booms. The distribution of the surplus consumption ratio is therefore negatively skewed. This discrepancy between the mean and the steady-state implies that matching empirical averages to the model's steady-states values results in simulated moments that are slightly higher than those in the data.

The six target moments are measured over the period from 1947:II to 2004:IV for the US economy. Per capita consumption data on non-durables and services are from the BEA. US interest rates, inflation, and stock market excess returns are from CRSP (WRDS). The real interest rate is the return on a 90-day Treasury bill minus expected inflation. I compute expected inflation with a one-lag two-dimensional VAR using inflation and interest rates. The Sharpe ratio is the ratio of the unconditional mean of quarterly stock excess returns to their unconditional standard deviation. Table II summarizes the parameters used in this paper. They are close to the ones proposed by Campbell and Cochrane (1999) and Wachter (2006). The habit process is very persistent ( $\phi = 0.995$ ), and consumption is on average 7% above the habit level, with a maximum gap of 12% (respectively 6% and 9% in Campbell and Cochrane (1999)). The correlation between US and UK consumption growth rates is 0.15.

[Table 2 about here.]

With these parameters and 10,000 endowment shocks, I build the surplus consumption ratios, stochastic discount factors, interest rates in both countries, and the implied exchange rate. To compute moments on the price-dividend ratio (using the price of a consumption claim), stock market returns and real yields, I use the numerical algorithm developed by Wachter (2005).

## B. Results

The simulation delivers the moments reported in Table III. I first review the evidence on the UIP and equity premium puzzles and then turn to implied exchange rate volatility, the link between consumption growth and exchange rates, and simulated real yields.

[Table 3 about here.]

**UIP and equity premium puzzles** The calibration targets the first two moments of consumption growth, real interest rates, and equity Sharpe ratios; the simulation successfully reproduces their empirical counterparts. Let us now focus on moments not used in the calibration. First and foremost, the model delivers a UIP slope coefficient  $\alpha$  that is negative ( $-0.99$ ) and in line with its empirical value. Second, the model implies reasonable moments of equity returns, with a mean of 5.6% and a standard deviation of 8.7%. These values, however, remain lower than their empirical counterparts over the last fifty years. Third, the model reproduces the stark contrast between the low persistence of exchange rate changes and the high persistence of interest rate differentials. The model slightly underestimates the persistence of consumption growth and overestimates the persistence of risk-free rates. But it gives a close fit for exchange rates, market returns, price-dividend ratios, and real yields. To sum up, this framework simultaneously reproduces the first two moments of consumption growth and risk free rates and the UIP and equity premium puzzles. This is the main achievement of the model.

Before turning to the major shortcomings of the model, I note three minor discrepancies regarding the price-dividend ratio and the persistence of real interest rates. First, the mean log price-dividend ratio on equity is more volatile in the model than in the data, mostly because I use a moving average to smooth out the seasonality in quarterly dividends. Second, the model implies that price-dividend ratios predict equity excess returns with a positive sign.

The equity data, however, suggest either a positive sign, as in Lettau and Nieuwerburgh (2008), or no significant predictability at all, as in Campbell and Yogo (2006). Third, the model seems to overestimate the autocorrelation of real risk-free rates. But this empirical low autocorrelation reflects the high volatility of VAR-implied expected inflation and might be misleading. In the US, nominal interest rates are highly autocorrelated at both annual and quarterly frequencies; real VAR-implied interest rates are highly autocorrelated at annual frequencies; and real yields computed from inflation-indexed bonds are highly autocorrelated at quarterly frequencies.

**Exchange rate volatility and consumption growth shocks** This paper proposes a simple, fully developed model that replicates the UIP and equity premium puzzles. The model, because of its simplicity, has two major shortcomings: real exchange rates are too volatile and too closely linked to consumption shocks. Simulated real exchange rates vary here three times more than in the data. This result can be related to the definition of the exchange rate in complete markets found in equation (3), which implies that the variance of real exchange rate changes is equal to:

$$\sigma^2(\Delta q) = \sigma^2(m) + \sigma^2(m^*) - 2\rho(m, m^*)\sigma(m)\sigma(m^*).$$

In order to fit the equity premium, we know that the variance of the stochastic discount factor has to be high (Mehra and Prescott (1985) and Hansen and Jagannathan (1991)). We also know that the correlation among consumption growth shocks across countries is low. Power utility thus implies a low correlation of stochastic discount factors. Brandt, Cochrane, and Santa-Clara (2006) show that the actual real exchange rate is much smoother than the theoretical one implied by asset pricing models. The same tension is present here. When endowment shocks are uncorrelated across countries, standard deviations of

changes in exchange rates are proportional to Sharpe ratios.<sup>7</sup> Introducing some correlation in endowment shocks across countries weakens this link, but the real exchange rate remains too volatile.

Moreover, the model implies a strong and positive correlation between changes in exchange rates and consumption growth rates that is not apparent in the data. Backus and Smith (1993) find that the actual correlation between exchange rate changes and consumption growth rates is low and often negative. Chari, Kehoe, and McGrattan (2002), Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008) confirm their findings. Backus and Smith (1993) note that in complete markets and with power utility, the change in real exchange rates is equal to relative consumption growth in two countries multiplied by the risk-aversion coefficient ( $\Delta q_{t+1} = -\gamma[\Delta c_{t+1}^* - \Delta c_{t+1}]$ ). This implies a perfect correlation between consumption growth and real exchange rate variations. Habit preferences lead to a lower correlation than power utility does. But the model still implies too great a correlation between real exchange rates and consumption growth rates because a single source of shocks drives all variables.

The model needs to be refined. In a Supplementary Appendix, I study international trade in a similar environment. I show that reasonable proportional and quadratic trade costs reduce exchange rate volatility to empirical levels without endangering the results obtained on equity and currency markets. A complete solution to the Backus and Smith (1993) puzzle would certainly require a richer model with non-tradable goods and incomplete markets. I leave this question open for future research and turn to the model's implications for the real term structure.

**Real yields** Pro-cyclical real risk-free rates imply a slightly downward sloping average real yield curve, which is not unusual in consumption-based asset pricing models. Piazzesi and Schneider (2006), for example, find a downward sloping real yield curve using Epstein

and Zin (1989) preferences. More generally, Cochrane and Piazzesi (2008) note that the real yield curve should be downward sloping when inflation is stable. In that case, interest rate variations come from changes in real rates. Long-term bonds are safer investments for long-term investors because rolling over short term bonds encounters the risk of short-term interest rate changes.

How do simulated real yields compare to the data? Table III reports moments of real holding period returns on a five-year nominal bond. The model underestimates this average return. Table IV reports additional evidence on the US and UK real yield curves, along with corresponding results on simulated series. In the model, the simulated real yield on a five-year note is 0.4 percentage points lower than the three-month real interest rate. Empirical evidence on the average slope of the real yield curve is unfortunately inconclusive. On the one hand, using UK inflation-indexed bonds from 1983 to 1995, Evans (1998) documents that real term premia are significantly negative ( $-2\%$ ). I extend his results using the Bank of England zero-coupon real yields and a Nelson and Siegel (1987) interpolation to obtain yields for the maturities in the model. I find a flat real yield curve from 1995 to 2006. On the other hand, J. Huston McCulloch's work on US TIPS contracts since 1997 shows an upward-sloping real yield curve. With short samples and potential liquidity issues, the empirical slope of the real yield curve obtained from inflation-indexed bonds remains an open question. Decomposing nominal yields into real and inflation-related components, Ang et al. (2008) find that the unconditional real rate curve remains fairly flat around 1.3%, which is close to the value of the five-year real rate in the model. As a result, in order to match the upward sloping nominal yield curve, the model would need an inflation risk premium that is at least 0.4 percentage points higher than the one estimated by Ang et al. (2008).

[Table 4 about here.]

**Actual data** Finally, I present two reality checks: a simulation with actual consumption series and a comparison to the results in Lustig and Verdelhan (2007). Figure 1 shows time-series for surplus consumption ratios, stochastic discount factors and local risk curvatures for an American investor. The simulation relies on the same set of parameters presented in the first column of Table II, but uses actual US consumption growth for the period from 1947:II to 2004:IV, instead of random shocks.

It appears that surplus consumption ratios vary between 4% and 12%. Thus, the local curvature, computed as  $\gamma/S_t$ , fluctuates between 15 and 60 and is much higher than the risk-aversion coefficient. The resulting stochastic discount factor is volatile in the mid-50s and then fluctuates around unity. Implied real interest rates are sometimes negative, reaching a minimum value of  $-0.4\%$ . But negative values do not happen more often than in US ex-ante real interest rates (computed as indicated in section II - A). This reality check also shows that habits are well defined. Ljungqvist and Uhlig (2003) argue that, in some cases, habit levels in Campbell and Cochrane (1999)'s model may decrease following a sharp increase in consumption. In fact, their model implies that an infinitesimal rise in consumption will always increase habit levels. With actual data, the case described by Ljungqvist and Uhlig (2003) never happens.

[Figure 1 about here.]

**Currency portfolios** The model and its simulation highlight the results in Lustig and Verdelhan (2007). They find that high interest rate currencies provide high excess returns because these currencies tend to depreciate in bad times for the American investor. The model naturally replicates this finding, which is at the core of any risk-based explanation of currency excess returns. As proof of this point, consider the following regression of changes in exchange rates on domestic consumption growth and domestic consumption growth mul-

tiplied by the interest rate differential:

$$\Delta q_{t+1} = \beta_0 + \beta_1 \Delta c_{t+1} + \beta_2 \Delta c_{t+1} (r_t^* - r_t) + \varepsilon_{t+1}.$$

Assume that the foreign interest rate is above its domestic counterpart. A positive coefficient  $\beta_2$  indicates that the exchange rate tends to appreciate ( $q$  increases) in good times for a domestic investor, increasing returns on foreign bonds. Likewise, the exchange rate tends to depreciate in bad times, decreasing returns on foreign bonds. As a result, when the foreign interest rate is above the domestic rate, the exchange rate means more consumption growth risk for the domestic investor. In the model,  $\beta_2$  is positive and significant.

### III. Estimation

The simulation exercise has shown that for parameter values close to the ones used in the habit literature, the model can reproduce the first two moments of consumption growth and interest rates as well as features of the equity and currency markets. In this section, I estimate preference parameters (risk-aversion  $\gamma$ , persistence  $\phi$ , average surplus consumption ratio  $\bar{S}$ ) that minimize the pricing errors of the Euler equations. I check that the estimated parameters imply pro-cyclical real interest rates and negative UIP coefficients.

#### A. Method

The estimation starts from the sample equivalent of an American investor's Euler equation:

$$E_T[M_{t+1}R_{t+1}^e] = 0,$$

where  $M_{t+1}$  is his SDF and  $R_{t+1}^e$  bunches all the test assets' excess returns. The SDF depends on US consumption growth shocks and preference parameters. The estimation relies on the continuously-updating estimator studied by Hansen et al. (1996). Hansen (1982)'s asymptotic theory determines standard errors for the three structural parameters. I apply the delta-method for the standard errors on the implied UIP coefficients.

I consider two measures of currency excess returns, using 8 individual currencies or 8 currency portfolios. For the individual currencies, I focus on investments in 8 OECD countries (Australia, Canada, France, Germany, Italy, Japan, Switzerland, and the United Kingdom). The sample period extends from 1971:I to 2004:IV, during which short term interest rates and exchange rates are available for all countries. As noted previously, the model predicts that average currency excess returns should be zero between similar countries. Thus, the estimation is run on conditional moments, using a constant and lagged domestic interest rates as instruments. This setup gives 16 moments. For the portfolios, I use the 8 currency excess returns that are proposed in Lustig and Verdelhan (2007). By taking into account many of the investment opportunities in currencies, these portfolios create a large cross-section of excess returns without imposing the estimation of a large variance-covariance matrix. The sample here ends two years earlier (from 1971:I to 2002:IV) because of data availability.

I estimate the preference parameters on these currency excess returns, and also on test assets that include equity excess returns. To do so, I use either 6 or 25 Fama and French (1993) equity portfolios sorted on book-to-market and size as well as CRSP value-weighted stock market returns. As a comparison, I also estimate the model using only equity excess returns. Overall, I consider eight different sets of test assets.

## B. Results

Estimation results confirm the model’s ability to account for the forward premium anomaly. The three structural parameters lie within their proposed ranges, and no corner solution is reached. Out of the eight different estimations, the model is rejected three times: twice when using only equity portfolios, and once using 42 moments (16 individual currency moments and 26 equity portfolios). In the five other cases, the model is not rejected. Table V reports  $p$ -values, which test the null hypothesis that pricing errors are zeros, ranging from 25% to 66%.

In analyzing the results, I focus now on cases where the model is not rejected. In these cases, risk-aversion coefficients  $\gamma$  vary between 6.2 and 10.0 and persistence parameters  $\phi$  vary between 0.81 and 0.99, with relatively high standard errors. Average surplus consumption ratios take values between 2.1% and 3.8%, which translate into habits ranging from 96% to 98% of consumption. All estimations imply negative values for  $B$ , meaning that real interest rates are pro-cyclical.<sup>8</sup> In simulations of a two-country symmetric model with *i.i.d* consumption shocks – similar to the one presented in Section II – these parameters deliver negative UIP coefficients.

The estimated structural parameters seem reasonable and in line with the literature on domestic excess returns. Chen and Ludvigson (2008) estimate habit-based models without imposing the functional form of habit preferences. They conclude that in order to match moment conditions corresponding to Fama and French (1993) portfolios, habits should be equal to a large fraction of current consumption (97% on average). Using a simulation-based method, Tallarini and Zhang (2005) estimate Campbell and Cochrane (1999)’s model on US domestic assets (assuming a constant real risk-free interest rate). They find that the persistence coefficient  $\phi$  is above 0.9, the risk-aversion coefficient is equal to 6.3, and the model is rejected on equity returns. My results are similar. However, adding currency

excess returns leads to more precise estimations of model parameters, and the model is not always rejected. Currency returns are not spanned by the usual size and value factors and thus constitute an additional challenge. But they also provide an additional source of information on investors' risk characteristics.

[Table 5 about here.]

## IV. Conclusion

The empirical failure of the UIP condition implies that investors earn positive excess returns on high interest rate currencies and negative excess returns on low interest rate currencies. I show in this paper that Campbell and Cochrane (1999)'s habit-based preferences, which were designed to match some salient features of stock markets, are also consistent with stylized facts of currency markets.

The model has two key characteristics: time-varying risk aversion and pro-cyclical real interest rates. The domestic investor earns positive excess returns in times when he is more risk-averse than his foreign counterpart. In bad times, consumption is close to the habit level, risk-aversion is high, and interest rates are low. Thus, the domestic investor expects a positive risk premium when interest rates are lower at home than abroad. This mechanism reproduces the UIP puzzle. To verify this intuition, I present analytical results, simulations, and estimation exercises. Closed-form expressions for the UIP coefficient are easy to derive when consumption shocks are uncorrelated across countries. Relaxing this assumption, I rely on a simulation to show that the model implies negative UIP coefficients and sizable stock excess returns. Finally, I estimate the model on currency and equity excess returns and recover reasonable preference parameters that imply pro-cyclical real interest rates.

The main weakness of the model is that simulated exchange rates are too volatile and too

closely linked to consumption growth shocks. Moreover, the model starts from consumption allocations and does not explain where these allocations come from. In a Supplementary Appendix to this paper that is available online, I make further progress on these issues. Starting from endowment processes in both countries, I assume that agents can trade but incur proportional and quadratic trade costs. I then derive optimal international trade and consumption allocations when agents are characterized by habit preferences. The model still implies volatile stochastic discount factors and matches the first two moments of consumption growth, real interest rates, and equity risk premia. The model now reproduces the variance of changes in real exchange rates. The introduction of non-tradables loosens the link between consumption and exchange rates, but it does not fully solve the Backus and Smith (1993) puzzle.

Further work is needed because many models in international macroeconomics and international finance still do not produce time-varying risk premia. As a result, in these models, exchange rates and interest rates satisfy the UIP condition, even if it is overwhelmingly rejected by the data. These models assume that high interest rate currencies depreciate, even if they appreciate on average. This paper offers an alternative starting point that could be incorporated into larger models with production, investment, and savings decisions.

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# Notes

<sup>1</sup>Some examples of habit preferences in one-country models are Sundaresan (1989), Constantinides (1990), Abel (1990), Jermann (1998), Campbell and Cochrane (1999), Lettau and Uhlig (2000), Boldrin, Christiano, and Fisher (2001) and Chan and Kogan (2002). Shore and White (2003) and Gomez, Priestley, and Zapatero (2006) study international portfolio holdings under external habit preferences in multi-country models.

<sup>2</sup>Habit preferences with time-varying real interest rates have been used by Campbell and Cochrane (1995), Wachter (2006) and Buraschi and Jiltsov (2007) to model the US yield curve, and by Menzli, Santos, and Veronesi (2004) to study cross-sections of US assets.

<sup>3</sup>Frachot (1996) shows that a two-country version of Cox et al. (1985) produces a negative UIP slope coefficient for certain parameter values. His framework, however, offers no obvious economic explanation for currency risk premia. The UIP slope coefficient is equal to  $(1 - e^{-\lambda}) / (1 - \frac{\partial A^d(1)}{1 + \frac{\alpha}{2} A^s A^d(1)})$  where  $\lambda$ ,  $\alpha$ , and  $A^s$  are diffusion parameters, and  $A^d$  satisfies a unidimensional Riccati differential equation.

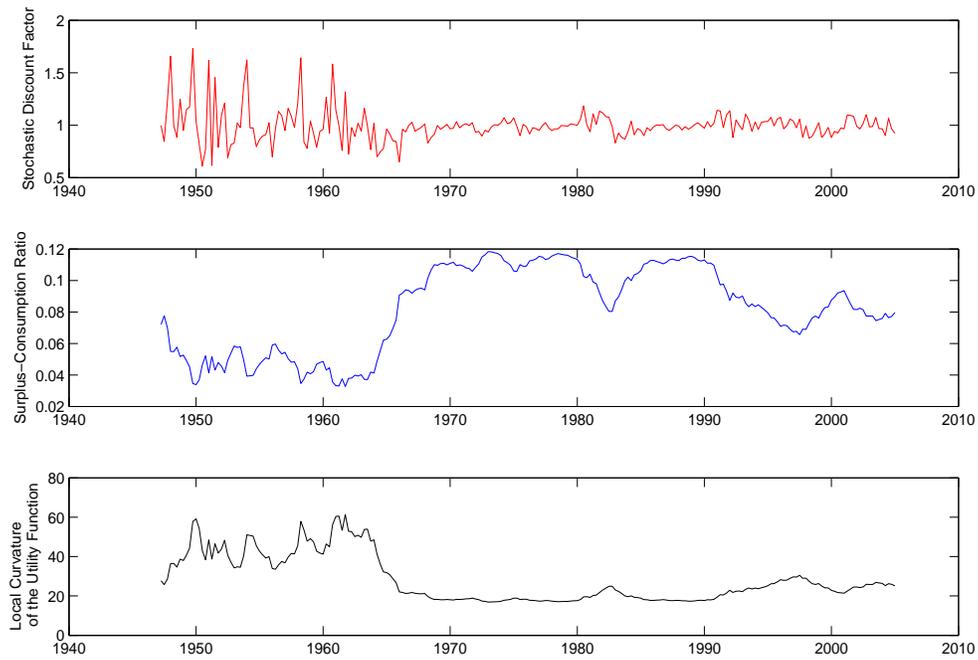
<sup>4</sup>Lustig and Verdelhan (2007) focus on log expected currency excess returns, instead of expected log excess returns, but the two expressions are consistent.

<sup>5</sup>Details about the portfolios' construction are available in Lustig and Verdelhan (2007).

<sup>6</sup>To approximate risk-free rate volatility, I assume that  $\lambda(s_t)$  remains equal to its steady-state value ( $\lambda(\bar{s}) = [1 - \bar{S}]/\bar{S}$ ). In that case, the variance of the interest rate is close to  $\sigma^2 B^2 [1/\bar{S} - 1]^2 / [1 - \phi^2]$ , where  $\bar{S}$  is defined in terms of  $\sigma$ ,  $\gamma$ ,  $\phi$  and  $B$ .

<sup>7</sup>The variance of real exchange rate appreciation is here at the steady-state:  $\langle \text{Var}_t(\Delta q_{t+1}) \rangle_{\text{Steady-state}} = 2(\gamma\sigma/\bar{S})^2 = 2\bar{S}\bar{R}^2$ .

<sup>8</sup>When using only 7 equity portfolios, the estimated coefficients imply a positive value for  $B$ . However, the model is clearly rejected in this case, and the standard errors on the coefficients are three to ten times larger than in the other cases.



**Figure 1.** Reality check. Stochastic discount factor, surplus consumption ratio and local curvature of the utility function of an American investor, computed with actual US consumption data only over the 1947:II-2004:IV period using the parameters presented in the first column of Table II.

**Table I**  
**Currency and Equity Risk**

This table presents average currency excess returns  $E(r^e)$  for an American investor and equity Sharpe ratios  $SR$  for foreign investors in foreign stock markets. The excess returns correspond to 8 portfolios of developed countries sorted on interest rates. The first portfolio contains low interest rate countries, and the last portfolio contains high interest rate countries. Data are quarterly and were taken from Global Financial Data. The period is 1953:I-2002:IV. Currency excess returns are computed using Treasury Bill yields and exchange rates, with the United States as the domestic country. Sharpe ratios are computed using ex-post stock market excess returns, expressed in foreign currencies, and the foreign equivalents of Treasury Bill yields. All moments are annualized. Standard errors are reported between brackets. They are obtained by bootstrapping estimations 10,000 times (i.e drawing with replacement under the assumption that excess returns are *i.i.d.*).

Portfolios	1	2	3	4	5	6	7	8
$E(r^e)$	-1.59 [1.52]	0.78 [1.29]	0.63 [1.49]	0.91 [1.31]	0.63 [1.44]	2.02 [1.17]	1.60 [1.21]	2.69 [1.19]
$SR$	0.58 [0.15]	0.52 [0.16]	0.29 [0.13]	0.38 [0.14]	0.51 [0.14]	0.39 [0.13]	0.28 [0.16]	0.24 [0.14]

**Table II**  
**Calibration Parameters**

This table presents the parameters of the model and their corresponding values in Campbell and Cochrane (1999) and Wachter (2006). Data are quarterly. The reference period is here 1947:II-2004:IV (1947-1995 in Campbell and Cochrane (1999), 1952:II-2004:III in Wachter (2006)). Per capita consumption is taken from the BEA web site. Interest rates and inflation data are from CRSP (WRDS). The real interest rate is the return on a 90-day Treasury bill minus the expected inflation, which is derived from a one-lag, two-dimensional VAR using inflation and interest rates. The UIP coefficient corresponds to US-UK exchange rates and interest rate differentials. UK consumption (1957:II-2004:IV), population, interest rates, inflation rates, and exchange rates are from Global Financial Data.

	My parameters	Campbell and Cochrane (1999)	Wachter (2006)
<b>Calibrated parameters</b>			
$g(\%)$	0.53	0.47	0.55
$\sigma(\%)$	0.51	0.75	0.43
$\bar{\pi}(\%)$	0.34	0.23	0.66
$\gamma$	2.00	2.00	2.00
$\phi$	0.99	0.97	0.97
$B$	-0.01	-	0.01
$\rho$	0.15	-	-
<b>Implied parameters</b>			
$\beta$	1.00	0.97	0.98
$\bar{S}$	0.07	0.06	0.04
$S_{\max}$	0.12	0.09	0.06

**Table III**  
**Simulation Results**

In the first panel, this table presents the mean, standard deviation, and autocorrelation of consumption growth  $\Delta c$ , risk-free interest rates  $r^f$ , changes in real exchange rates  $\Delta q$ , log price-dividend ratio  $pd$ , stock market risk return  $r^m$ , and real holding period return on a 5-year bond  $hpr^5$ . All moments are annualized. In the second panel, this table reports the correlation between stock market excess returns and log dividend price ratios  $\rho_{r_{t+1}^m - r_t^f, pd_t}$ , the correlation between stock market excess returns and risk-free rates  $\rho_{r_{t+1}^m - r_t^f, r_t^f}$ , the correlation between consumption growth differentials and changes in real exchange rates  $\rho_{\Delta q_t, \Delta c_t^* - \Delta c_t}$ , and UIP slope coefficients  $\alpha_{UIP}$ . Standard errors are reported in brackets. In both panels, the last three columns correspond to actual data for the US and the US-UK exchange rate over the 1947:II-2004:IV period (1951:I-2006:IV for holding-period returns on 5-year US government bonds).

	Simulation Results			Actual Data		
	Mean (%)	Std (%)	Autoc.	Mean (%)	Std (%)	Autoc.
$\Delta c$	2.13	1.04	0.01	2.10	1.35	0.17
$r^f$	1.65	2.54	0.99	1.40	1.99	0.76
$\Delta q$	8.44	42.69	0.00	0.38	10.29	0.06
$pd$	344.17	48.60	0.99	327.71	26.78	0.95
$r^m$	5.63	8.72	-0.01	8.63	16.70	-0.04
$hpr^5$	1.02	3.72	0.77	2.28	5.60	0.05
	<b>Coef.</b>	<b>s.e.</b>		<b>Coef.</b>	<b>s.e.</b>	
$\rho_{\Delta q_t, \Delta c_t^* - \Delta c_t}$	0.78	[0.01]		-0.04	[0.13]	
$\rho_{r_{t+1}^m - r_t^f, pd_t}$	0.12	[0.01]		-0.14	[0.08]	
$\rho_{r_{t+1}^m - r_t^f, r_t^f}$	-0.13	[0.01]		-0.03	[0.07]	
$\alpha_{UIP}$	-0.99	[0.35]		-1.29	[0.64]	

**Table IV**  
**Real Yield Curve**

The table reports average, standard deviation and autocorrelation of real yields in actual data and in the model. Panel A reports evidence obtained on inflation-indexed bonds in the UK and the US for different maturities. Data for the UK come from Evans (1998) and the Bank of England's website. Missing data points are obtained using a Nelson and Siegel (1987) interpolation. Data for the US come from J. Huston McCulloch's website. Panel B reports equivalent results obtained with the model.

	2 years	3 years	4 years	5 years	10 years
<b>Panel A: Data</b>					
UK - 1983:1-1995:11 - Monthly - Evans (1998)					
<i>Mean</i>	6.12	5.29	4.62	4.34	4.12
<i>Volatility</i>	1.83	1.17	0.70	0.53	0.45
<i>Autocorrelation</i>	0.63	0.66	0.71	0.77	0.85
UK - 1995:IV-2006:IV - Quarterly					
<i>Mean</i>	2.39	2.50	2.46	2.43	2.34
<i>Volatility</i>	0.89	0.78	0.72	0.70	0.69
<i>Autocorrelation</i>	0.86	0.93	0.93	0.93	0.94
US - 1997:I-2006:IV - Quarterly					
<i>Mean</i>	2.20	2.37	2.52	2.64	2.94
<i>Volatility</i>	1.51	1.36	1.25	1.15	0.88
<i>Autocorrelation</i>	0.95	0.97	0.97	0.97	0.96
<b>Panel B: Model</b>					
<i>Mean</i>	1.42	1.35	1.27	1.19	0.69
<i>Volatility</i>	1.86	1.92	1.98	2.05	2.45
<i>Autocorrelation</i>	0.97	0.97	0.97	0.97	0.97

**Table V**  
**Estimation Results**

This table presents the estimated values of the model's three structural parameters (risk-aversion  $\gamma$ , persistence  $\phi$ , average surplus consumption ratio  $\bar{S}$  in percentage) and the implied UIP slope coefficient  $\alpha_{implied} = \gamma(1 - \phi)/B$ . The table also reports the number of excess returns  $N$ , the minimized criterion  $J$ , and the corresponding  $p$ -value  $p = 1 - \chi^2(J, N - 3)$  testing the null hypothesis that pricing errors are zeros. In Panels A and B, test assets include foreign currency excess returns and equity excess returns. In Panel A, I consider the currency excess returns of an American investor who invests in 8 other OECD countries (Australia, Canada, France, Germany, Italy, Japan, Switzerland, and the United Kingdom). Using a constant and US interest rates as instruments, the estimation uses 16 currency excess returns. In Panel B, I consider the 8 portfolios of currency excess returns proposed in Lustig and Verdelhan (2007). These portfolios are built by sorting currencies on foreign interest rates. In Panel C, I use Fama and French (1993) equity portfolios sorted on book-to-market and size as well as CRSP value-weighted stock market returns. Data are quarterly. The sample is 1971:II-2004:IV for individual currencies (Panel A) and 1971:II-2002:IV for currency and equity portfolios (Panels B and C). Standard errors are reported between brackets.

<i>Assets</i>	<b>Panel A: Individual Currencies</b>			<b>Panel B: Currency Portfolios</b>			<b>Panel C: Equity Portfolios</b>	
	<b>8 C</b>	<b>8 C</b> <b>+6 FF +M</b>	<b>8 C</b> <b>+25 FF +M</b>	<b>8 P</b>	<b>8 P</b> <b>+6 FF +M</b>	<b>8 P</b> <b>+25 FF +M</b>	<b>6 FF +M</b>	<b>25 FF +M</b>
$N$	16	23	42	8	15	34	7	26
$J$	10.38	22.60	57.93	3.43	14.79	3.43	12.20	38.56
$p$	0.66	0.31	0.03	0.63	0.25	0.63	0.02	0.02
$\gamma$	10.06 [2.60]	6.18 [1.27]	7.23 [1.80]	7.33 [0.75]	7.20 [1.04]	7.33 [0.75]	1.79 [3.18]	8.01 [1.41]
$\phi$	0.98 [0.09]	0.95 [0.09]	0.99 [0.07]	0.81 [0.14]	0.98 [0.06]	0.81 [0.14]	0.89 [0.58]	0.97 [0.06]
$\bar{S}$	3.77 [0.42]	3.00 [0.43]	2.86 [0.05]	2.07 [0.56]	2.20 [0.25]	2.07 [0.56]	2.45 [4.45]	2.51 [0.11]
$\alpha_{implied}$	-0.12 [0.06]	-0.49 [0.11]	-0.08 [0.04]	-0.88 [0.07]	-0.07 [0.02]	-0.88 [0.07]	2.91 [2.07]	-0.13 [0.02]