Investing in Foreign Currency is like Betting on your Intertemporal Marginal Rate of Substitution.

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Abstract  

Investors earn positive excess returns on high interest rate foreign discount bonds, because these currencies appreciate on average. Lustig and Verdelhan (2005) show that investing in high interest rate foreign discount bonds exposes them to more aggregate consumption risk, while low interest rate foreign bonds provide a hedge. This paper provides a simple model that replicates these facts. Investing in foreign currency is like betting on the difference between your own intertemporal marginal rate of substitution (IMRS) and your neighbor’s IMRS. These bets are very risky if your neighbor’s IMRS is not correlated with yours, but they provide a hedge when his IMRS is highly correlated and more volatile. If the foreign neighbors that face low interest rates also have more volatile and correlated IMRS, that accounts for the spread in excess returns in the data.  

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1 Introduction  

The textbook uncovered interest rate parity (UIP) condition rules out risk premia on investments in foreign discount bonds or zero coupon bonds. This condition implies that the expected exchange rate depreciation over the duration of the bond equals the difference

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between the interest rate on foreign and domestic discount bonds. If this were actually the case, the interest rate spread ought to forecast changes in the exchanges rate over the maturity of the bond with a slope of one, but it does not (Hansen and Hodrick (1980) and Fama (1984)).

In fact, changes in exchange rates are almost impossible to predict, and, as a result, the returns from investing in foreign discount bonds are predictable by the foreign interest rate. Consider the benchmark case of a random walk in exchange rates, which is not a bad description of the data (Meese and Rogoff (1983)). The expected excess return from investing in lower-than-your-own-interest rate discount bonds is negative, while the expected excess return on investing in higher-than-your-own-interest rate discount bonds is positive. So, the absence of predictability in exchange rates imputes predictability to the excess returns on foreign discount bonds: high excess returns on high interest rate currencies, and low ones on low interest rate currencies. Turning this argument on its head, if investors want a risk premium for holding high interest rate, foreign currency discount bonds, then maybe we should not be surprised that exchange rate changes are hard to predict. In recent work, Lustig and Verdelhan (2005) show that high interest rate foreign discount bonds are indeed riskier than low interest rate ones for a domestic investor. So, a random walk in exchange rates delivers, at least roughly, the right pattern in risk premia.¹

There are two questions that need to be answered. First, in what sense are high interest rate currencies more risky? Second, why?

2 Currencies and IMRS Bets

To answer these two questions, we simply apply the basics of modern finance to currency markets.

Interest Rates and Risk Premia Why does a high interest rate today predict a high excess return on foreign discount bonds? To answer this question, Lustig and Verdelhan (2005) sort currencies into portfolios on the basis of the current foreign interest rate, from low to high interest rates, much like Fama and French (1992) and Fama and French (1993) sort stocks into bins based on size and book-to-market. This allows us to focus on what we are interested in: the spread between high and low interest rate currencies. Between

¹Actually, matters are slightly more complicated, as there is some predictability in exchange rates. More often than not, the slope coefficients in UIP regressions of changes in the exchange rate on interest rates are significantly negative: high foreign interest rates tend to predict an appreciation, but this only increases the spread in excess returns between high and low interest rate currencies.
1971 and 2002, the spread in excess returns between the first and the last bin for annually rebalanced portfolios of foreign T-bills is between 4 and 5% (in annual returns), a large number for one-year foreign discount bond investments. We obtain a similar spread if we rank the currencies on the basis of real interest rates (computed using realized inflation).

We show that, on average, the currencies in the first bin appreciate when US consumption growth is low, while the currencies in the last bin depreciate. So, investing in discount bonds of currencies in the first bin provides a hedge for US investors; discount bonds of currencies in the last bin are risky for US investors. This explains a large fraction of the variation in average excess returns across these portfolios.

We also show that the returns on foreign discount bonds in the last bin become more correlated with US consumption growth risk in bad times, while those in the first bins become less correlated. Bad times are times when the US aggregate consumption-wealth ratio is high in the language of Lettau and Ludvigson (2001), or when the housing collateral ratio is low in the model of Lustig and Nieuwerburgh (2005). From the research on domestic equity returns, we have learned that the price of aggregate consumption growth risk does increase in bad times, as one would expect. The same happens in currency markets.

Of course, we have not explained this variation in the consumption risk across different currencies. To do so, we need to think more carefully about the nature of currency risk.

**IMRS bets** As we show in Lustig and Verdelhan (2005), investing in foreign currency discount bonds is like betting on the difference between the foreign and domestic intertemporal marginal rate of substitution (IMRS).

The theory predicts large positive risk premia for the domestic investor, in the case of low correlation of the IMRS and higher volatility of the domestic IMRS, because these currencies typically depreciate when your IMRS is high. Let us consider the simplest case of no correlation between the domestic and foreign IMRS. Then you are effectively shorting a claim to your own IMRS by buying foreign currency discount bonds. Let us assume we are in a consumption-CAPM (C-CAPM) world with complete markets, and let us consider a stand-in agent with coefficient of risk aversion $\gamma$. The percentage change in the real exchange rate is $\gamma$ times domestic less foreign consumption growth: $\Delta \log q_{t+1} = \gamma (\Delta \log c_{t+1} - \Delta \log c^d_{t+1})$. If consumption growth is not correlated across countries, on average, the real exchange rate depreciates by $\gamma$% when your consumption growth drops one% below its mean, but you also experience a $\gamma$% increase in marginal utility growth.

Alternatively, the theory predicts negative risk premia, in the case of high correlation and higher volatility of the foreign IMRS. These currencies typically appreciate when your
IMRS is high and thus provide a hedge. Let us go back to the C-CAPM world, but now we assume foreign and domestic consumption growth are perfectly correlated, and foreign consumption is twice as volatile. In this case, the real exchange rate appreciates by $\gamma$ % when you experience a $\gamma$ % increase in marginal utility growth. This is a version of the Cole-Obstfeld effect; Cole and Obstfeld (1991) argued that terms of trade effects automatically insure a country against output shocks, possibly eliminating the need for international diversification.

The negative slope coefficients in the UIP regressions of changes in exchange rates on interest rate spreads tell us that conditional currency risk premia are extremely volatile (Fama (1984)). Our theory tells us that the conditional risk premium on foreign discount bonds is proportional to the difference between the standard deviation of the domestic investor’s log IMRS and the foreign one, adjusted for the correlation. From the returns in other securities markets, we know that the conditional distribution of IMRS varies dramatically over time - the Sharpe ratios on stocks certainly do-, and, hence, we expect the difference between the foreign and domestic conditional market price of risk to vary as well over time, and even to switch signs. We provide a simple model to illustrate this.

3 A New Look at Foreign Currency Investments

Following Lustig and Verdelhan (2005), we derive a closed form expression for the log of the currency risk premium in the case of log-normal returns and complete markets.

3.1 Assets Traded

We need to agree on some notation first. $e_i^t$ is the exchange rate in dollars per unit of foreign currency and $R^f_{i,t+1}$ is the risk-free one-period return in units of foreign currency $i$. $R^f_{t,t+1}$ is the gross risk-free rate in units of home consumption, and $R^f_{i,t+1}$ is the gross risk-free rate in units of foreign consumption. Finally, $R^i_{t+1}$ denotes the risky dollar return from buying a foreign one-year discount bond in country $i$, selling the payoff - one unit of foreign currency - after one year and converting the proceeds back into dollars: $R^i_{t+1} = R^f_{t+1} (e_{t+1}^i / e_t^i)$.

We use $m_{t+1}$ to denote the home investor’s IMRS. This IMRS prices payoffs in units of US consumption. In a C-CAPM world, the IMRS is given by $m_{t+1} = \beta (c_{t+1} / c_t)^{-\gamma}$, where $\gamma$ is the coefficient of relative risk aversion, $\beta$ is the discount factor and $c_t$ denotes the stand-in investor’s non-durable consumption.

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2This is the ratio of the expected excess return to its standard deviation.
In the absence of short-sale constraints or other frictions, the US investor’s Euler equation for foreign currency investments holds for each currency $i$:

$$E_t[m_{t+1} R^i_{t+1}] = 1,$$  \hspace{1cm} (1)

where $R^i_{t+1}$ denotes the random return in units of US consumption from investing in T-bills of currency $i$: $R^i_{t+1} = R^i_{t+1}^{s} (p_t/p_{t+1})$, and $p_t$ is the dollar price of a unit of the US consumption basket.

### 3.2 Euler Equation

If $\log m_{t+1}$ and $\log R^i_{t+1}$ are jointly, conditionally normal, then Lustig and Verdelhan (2005) show that the Euler equation can be restated in terms of the log of the multiplicative currency risk premium:

$$\log E_t R^i_{t+1} - \log R^f_{t+1} = -\text{Cov}_t \left( \log m_{t+1}, \log R^i_{t+1} - \Delta \log p_{t+1} \right).$$

We refer to this log currency premium as $\log(crp^i_{t+1})$. It is determined by the covariance between the log of the IMRS $m$ and the returns in units of home consumption from investing in the foreign bond.\(^3\) Substituting the definition of this return into this equation produces the following expression for the log currency risk premium:

$$\log(crp^i_{t+1}) = -[\text{Cov}_t (\log m_{t+1}, \Delta \log e^i_{t+1}) - \text{Cov}_t (\log m_{t+1}, \Delta \log p_{t+1})].$$  \hspace{1cm} (2)

The first term on the right-hand side of the equation represents pure currency risk compensation. The second term is inflation risk compensation. To simplify the analysis, we assume foreign inflation is constant in what follows.

### 3.3 Imposing No Arbitrage and Complete Markets

Next, we use market completeness.\(^4\) In the absence of arbitrage, the percentage change in the real exchange rate $\Delta \log q^i_{t+1} = \Delta \log e^i_{t+1} - \Delta \log p_{t+1} + \Delta \log p^i_{t+1}$ equals the difference between the foreign and the domestic log IMRS: $\Delta \log q^i_{t+1} = \log m^i_{t+1} - \log m_{t+1}$, because the law of one price (in financial markets) dictates that a unit of consumption delivered in

\(^3\)Backus, Foresi and Telmer (2001) derive a similar expression for $E_t \log R^i_{t+1} - \log R^l_{t+1}$. Theirs does not have a covariance term in it, but note that they compute the expected log return: $\log E_t R^i_{t+1} = E_t \log R^i_{t+1} + .5 \text{var}_t \log R^i_{t+1}$. Our covariance term is in this last var term.

\(^4\)In the case of incomplete markets, we need to project the payoffs onto the space spanned by traded assets, but similar arguments apply (Brandt, Cochrane and Santa-Clara (2005)).
some state tomorrow has to have the same price at home and abroad: \((q_i^t / q_{i+1}^t) m_{i+1}^t = m_{t+1}\).

We focus on real risk and abstract from nominal risk by assuming that foreign inflation \(\Delta \log p_{i+1}^t\) is orthogonal to the domestic IMRS. Given all these assumption, the log currency risk premium can be stated as:

\[
\log (crp_{t+1}^i) = - \text{Cov}_t (\log m_{t+1}, \log m_{i+1}^t - \log m_{t+1})
\] (3)

So at least in the conditional sense, investing in currency is like shorting claims on one’s own IMRS \(m_{t+1}\) and going long in claims on the foreign IMRS \(m_{i+1}^t\).

Under log-normality, \(\text{std}_t \log m_{t+1} \simeq \sigma_t (m_{t+1}) / E_t (m_{t+1})\) is the maximum conditional Sharpe ratio for domestic investors, or, equivalently, the conditional market price of risk, and \(\text{std}_t \log m_{i+1}^t\) is the maximum Sharpe ratio for foreign investors. The currency risk premium expression in (3) implies that the premium is proportional to the difference between the conditional market price of risk at home and abroad, adjusted for the conditional correlation:

\[
\log (crp_{t+1}^i) = \text{std}_t \log m_{t+1} \left[ \text{std}_t \log m_{t+1} - \rho_t (\log m_{t+1}, \log m_{i+1}^t) \right] / \text{std}_t \log m_{i+1}^t.
\] (4)

Because of tractability, we define the following currency Sharpe ratio:

\[
\frac{\log (crp_{t+1}^i)}{\text{std}_t \log R_{i+1}^i} = \frac{\text{std}_t \log m_{t+1} \left[ \text{std}_t \log m_{t+1} - \rho_t (\log m_{t+1}, \log m_{i+1}^t) \right] \text{std}_t \log m_{i+1}^t}{\text{std}_t (\log m_{i+1}^t - \log m_{t+1})},
\]

instead of the standard one.\(^5\) If \(\text{std}_t \log m_{t+1} = \text{std}_t \log m_{i+1}^t\), the symmetric case, this currency Sharpe ratio is given by: \(\log (crp_{t+1}^i) / \text{std}_t \log R_{i+1}^i = \text{std}_t \log m_{i+1}^t \sqrt{(1 - \rho)/2}\).

This ratio is bounded above by the conditional market price of risk, in the case of \(\rho\) equal to minus one, the case of perfectly negatively correlated IMRS. It is bounded below by zero, in the case of \(\rho\) equal to one, the case of perfectly positively correlated IMRS.

### 3.4 Foreign Currency Sharpe Ratios Switch Signs

The expression in (4) implies that the conditional risk premium is very sensitive to the correlation and the relative standard deviation of the IMRS. Keep in mind that IMRS are extremely volatile. An annual excess return of (roughly) 8 % on stocks, and a standard

\(^5\)Note that this is not exactly the Sharpe ratio on foreign discount bonds, because we have the standard deviation of the log return in the denominator, but it is a close approximation. Also, note that \(\text{std}_t \log R_{i+1}^i = \text{std}_t (\Delta \log e_{i+1} - \Delta \log p_{i+1}) \approx \text{std}_t (\Delta \log q_{i+1}^t - \Delta \log p_{i+1}^t) \approx \text{std}_t (\Delta \log q_{i+1}^t),\) because we assumed that \(\text{std}_t (\Delta \log p_{i+1}) \approx \text{std}_t (\Delta \log p_{i+1}^t) \approx 0.\)
deviation of 16 %, implies that the maximum Sharpe ratio $std_t \log m_{t+1}$ in the US is at least 50 %.

First, we explore some simple cases, and we document that the Sharpe ratios of these foreign currency investments switch signs. Next, we calibrate the model to match some standard asset pricing moments.

We consider three different, polar cases: perfect correlation, perfect correlation and more risk abroad, and no correlation of the IMRS at home and abroad:

1. **High correlation**: Suppose the correlation is one and the conditional market price of risk is the same at home and abroad. Then the log currency risk premium is zero.\(^6\) This is the knife-edge case of UIP. In the benchmark C-CAPM, this is the case of perfectly correlated and equally volatile aggregate consumption growth at home and abroad. On average, the currency does not respond to domestic consumption growth shocks.

2. **High correlation and more risk abroad** Suppose the correlation is still one, but we double the conditional market price of risk abroad. In this case, the currency Sharpe ratio equals conditional market price of risk in absolute value:

$$\log (crp_{t+1}^i) / std_t \log R^i_{t+1} = -std_t \log m_{t+1}.$$ 

This implies a negative currency Sharpe ratio on foreign discount bonds of minus 50 %, In the C-CAPM world, this case obtains when aggregate consumption growth at home and abroad are perfectly correlated but consumption growth is twice as volatile abroad. Suppose the stand-in agent has power utility with risk aversion $\gamma$ of 5. On average, the currency appreciates by five % when consumption growth decreases by 1 % below its mean. **A perfect hedge.**

3. **Low correlation**: Suppose the correlation is zero and the conditional market price of risk is the same at home and abroad. The currency Sharpe ratio equals $1/\sqrt{2}$ or .7 times the conditional market price of risk; that is around 35 %. In this case, investing in foreign discount bonds is very risky. Why? Well, on average the domestic investor gets low returns when her IMRS is high, because the foreign currency depreciates. In the C-CAPM world, this is the case of uncorrelated consumption growth at home and abroad. Then, on average, the currency depreciates by five % when consumption

\(^6\)Of course, you could also engineer zero risk premia by lowering the correlation and increasing the volatility at the same time.
growth decreases by 1 % below its mean. This is a very risky asset.

Obviously, we can generate a huge amount of variation in the spreads, but how much do we get in a reasonably calibrated model, and how does this square with the spread in the excess returns on the portfolios we construct in the data?

4 A Toy Model

To explore our mechanism, we take the processes for the IMRS $m$ at home and abroad as given and study the implied currency risk premium.\(^7\)

4.1 Calibration

As pointed by Brandt et al. (2005), the standard deviation of changes in the real exchange rate $\Delta \log q_{t+1}$ is no higher than 15 % for most developed countries. The only way to reconcile the low volatility of the changes in the real exchange rate with the high volatility of the IMRS, is by making these highly correlated.\(^8\) We set $\rho$ equal to .95 and we fix $\text{std}_t \log m_{t+1}$ at 50 % in the benchmark calibration, is unconditional mean.\(^9\) In addition, the expected growth rate of marginal utility $E_t \log m_{t+1}$ is constant, such that we match the risk-free rate of 1 % when the standard deviation $\text{std}_t \log m_{t+1}$ is equal to 50 %. The calibration is fully symmetric for the home and foreign investor, unless otherwise stated.

In the C-CAPM, our calibration strategy is equivalent to fixing the average growth rate of consumption and allowing its standard deviation to vary. A higher standard deviation of consumption growth lowers the risk-free rate because of precautionary savings.\(^10\)

4.2 Volatile Risk Premia

There is an enormous amount of variation in the currency risk premia in response to small changes in the correlation of the IMRS across countries and their riskiness. Both the risk and the correlation effect produce the right relation between interest rates and risk premia.

\(^7\)Verdelhan (2004) provides a habit-based framework that rationalizes time-varying risk premia at home and abroad.

\(^8\)This number seems very high from the perspective of a standard C-CAPM, because consumption growth is not highly correlated across countries. Colacito and Croce (2005) argue that highly correlated long-run risk in an Epstein-Zin world can account for this high correlation.

\(^9\)Also, note that if the $\text{std}(E_t \log m_{t+1}) \to 0$, then, on average, the conditional standard deviation $\text{std}_t \log m_{t+1}$ equals the unconditional standard deviation $\text{std}_t \log m_{t+1}$. This is not a bad approximation, because the standard deviation of the risk-free rate is very low.

\(^10\)What is the evidence for this effect? Lettau and Ludvigson (2003) show that expected returns on US stocks increase when the risk-free rate decreases, while the conditional volatility of returns decreases. As a result, a drop in the riskfree rate pushes up the conditional Sharpe ratio. If the correlation between the IMRS and stock returns is constant over time, this means low risk-free rates forecast a higher conditional market price of risk $\text{std}_t \log m_{t+1}$.
**Correlation effect**  The left panel of figure 1 plots the conditional log risk premium on a one-year foreign discount bond against the correlation of the IMRS. The premium increases from minus five % when the correlation is one to five % when the correlation is .7. Small changes in the correlation have an enormous effect on the currency risk premium.

In a panel of 10 developed countries, Lustig and Verdelhan (2005) show that consumption growth between two countries does tend to be more correlated when the interest rate gap narrows, thus lending support to this correlation effect.

Figure 1: Conditional log Currency Risk premium: In the left panel, we plot the \( \log(crp_{t+1}) \) against the correlation \( \rho_t \). \( std_t \log m_{t+1} \) is .6 and \( std_t \log m_{t+1} \) is .5. In the right panel, we plot the \( \log(crp_{t+1}) \) against the volatility factor \( \mu \), where \( std_t \log m_{t+1} = \mu \times std_t \log m_{t+1} \). \( \rho_t \) is fixed at .95. \( std_t \log m_{t+1} \) is .5.

**Risk effect**  The right panel plots the risk premium against the ratio of the foreign to the domestic market price of risk. When the ratio is 1/.95, because of symmetry, the risk
premium is zero. This is the case of UIP. Increasing the ratio to 1.25 pushes the risk premium down to minus 5%. Lowering it to .75 drives the premium up to 5%. This is the risk effect. These changes are within a one-standard deviation band for the market price of risk, because the standard deviation of the conditional market price of risk is very high! Lettau and Ludvigson (2003) estimate the standard deviation of the conditional Sharpe ratio for US stocks to be around 40%. This is a reasonable estimate of the variation in the conditional market price of risk, since there is no reason to believe the conditional correlation between stock returns and log $m_{t+1}$ changes over time.

**Currency Sharpe Ratios** The currency Sharpe ratios vary even more than the risk premia. In figure 2 we consider the same two experiments, but in terms of Sharpe ratios. The currency Sharpe ratio on the foreign bond drops off to -.5 when the correlation approaches one, but it increases to .1, when the correlation decreases to .7. Similarly, varying the volatility ratio from .75 to 1.25 pushes the currency Sharpe ratio from .2 to -.2.

At the same time, the increase in foreign risk lowers the foreign riskfree rate, as shown in figure 3. This figure plots the foreign risk-free rate and the currency Sharpe ratio, as we increase $\mu$ from .5 to 1.5. The slope of this curve is quite steep when the foreign interest rate is close its average. This is exactly the relation between risk premia and interest rates Lustig and Verdelhan (2005) documented. Foreign inflation just adds noise in our model. So the same pattern would obtain if we were to plot nominal interest rates instead.

Figure 3: Conditional Currency Sharpe Ratio and Foreign Risk-free Rate: We plot the currency Sharpe ratio $\log \left( \frac{crp_{t+1}}{std_t \log R_{t+1}} \right)$ against the foreign risk-free rate. The volatility factor $\mu$ varies from .5 to 1.5, where $std_t \log m_{t+1}^f = \mu \times std_t \log m_{t+1}$. We assume $E_t(\log m^f)$ is constant at -.13 such that the (foreign) risk-free rate is 1% when $std_t \log m_{t+1}^f = .5$. $\mu_t$ is fixed at .95. $std_t \log m_{t+1}$ is .5.
5 Conclusion

As these calibrated examples show, the size and volatility of currency risk premia is not surprising, given that investors in foreign currency are placing bets on their own IMRS and the foreign IMRS.

In ongoing work, we check whether the time variation in the risk premia on foreign discount bond investments in the data is driven by the same conditioning variables that drive the variation in the conditional Sharpe ratio on stocks, like the housing collateral ratio and the consumption/wealth ratio, as the theory predicts. And, as in equity markets, we find that these conditioning variables predict excess returns in currency markets.

References


