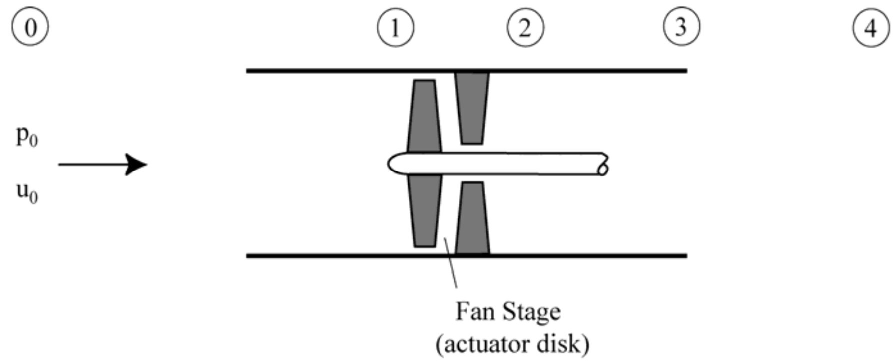
Air Breathing Propulsion January, 2011
Field Exam Z. S. Spakovszky

Consider the ducted propulsor shown below which consists of a fan stage enclosed by a cylindrical duct of constant area *A*. The flow upstream has no swirl, and downstream of the fan stage the flow leaves the stator vanes axially. The blockage due to the centerbody and blade rows can be ignored. The skin friction on the walls can be neglected and the flow can be assumed incompressible. Furthermore, the fan stage can be considered an actuator disk across which the velocity doesn't change. Far upstream the ambient pressure is p_0 and the velocity is u_0.



First, the fan stage is operated at design conditions yielding a stagnation pressure rise $\Delta p_t = p_2 - p_1 > 0$.

a) Sketch the static pressure and velocity distribution along the centerline through the ducted propulsor from station 0 to station 4.

b) Draw the streamlines into and out of the ducted propulsor.

c) What is the net thrust produced by the ducted propulsor? Indicate in your sketch where the thrust force is acting. You might want to consider appropriate control volumes.

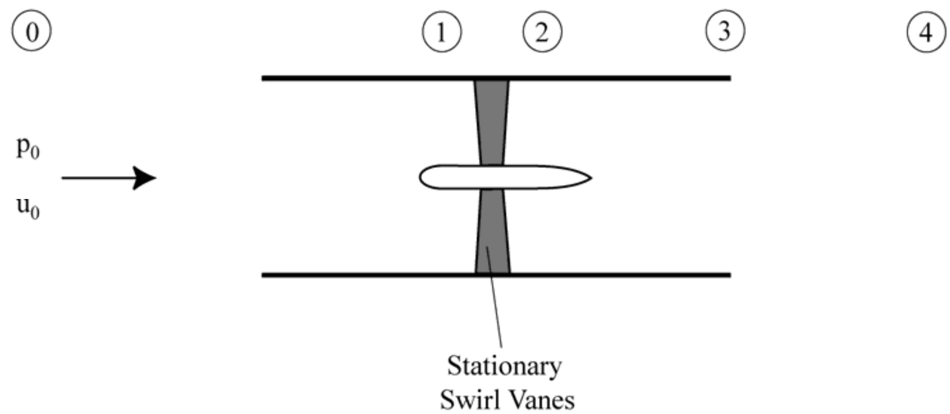
Next, assume that the fan is wind-milling and the stagnation pressure drops. Thus, the fan acts like a turbine, where $\Delta p_t = p_2 - p_1 < 0$. The upstream conditions, p_0 and u_0, are unchanged.

d) Sketch the static pressure and velocity distribution along the centerline through the ducted propulsor from station 0 to station 4.

e) Draw the streamlines into and out of the ducted propulsor.

f) What is the required non-dimensional stagnation pressure drop across the fan $\Delta p_t /(\rho u_0^2 / 2)$ for which the net force is a drag force pointing in the downstream direction? Find a relation between $\Delta p_t /(\rho u_0^2 / 2)$ and the non-dimensional velocity upstream of the fan u_1 / u_0.

For both fan operating conditions discussed above, the flow out of the duct was axial. We now turn to the situation where the flow leaving the turbomachinery stage is swirling. To do this, the fan stage is replaced by a row of inviscid, stationary swirl vanes (a non-rotating blade row). You can assume that the swirling flow at duct exit satisfies simple radial equilibrium (radial velocity is zero). The details of the radial distribution of the swirl is not important, all that is known is that the blade circulation at the hub is $\Gamma_B = 0$ and $\Gamma_B = \Gamma_o$ near the shroud.



g) Near the duct exit, station 3, sketch the approximate radial distributions of: stagnation pressure, tangential velocity, static pressure, and axial velocity.
h) The swirling motion of the out-flow yields streamwise vorticity trailing downstream from the duct. What are the origins of this streamwise vorticity (where does it come from)? In your explanation, it might be useful to draw the vortex lines.
i) Consider a contour around the swirling exhaust flow downstream of the duct (between stations 3 and 4) at a radius larger than the duct radius. What is the circulation around this contour?
j) Bonus question: can the cylindrical duct with swirling outflow generate a drag force? Why or why not?