Air Breathing Propulsion (Compressible Flow) Qualifier Question – January 2010

In this question we examine two aspects of compressible flow in propulsion systems.

A) An inviscid compressible fluid flows isentropically in an axisymmetric annular duct, with the duct height much smaller than the duct radius, as sketched in Figure 1. At station 1 there are axial and circumferential (θ) velocity components, and the flow direction is at an angle of 60 degrees from the x-direction. The Mach number at this location is subsonic and equal to M_1. Between station 1 and station 2, the mean radius of the duct (r in Figure 1) decreases by a factor of two and the duct height (difference between inner and outer radii) increases by a factor of two.

i) What is the ratio of the circumferential velocity at station 2 to that at station 1?
ii) Is the axial velocity at station 2 smaller, the same, or larger than that at station 1?
iii) Could the Mach number (M = $\sqrt{u_x^2 + u_\theta^2}\,/a$, where a is the speed of sound) at station 2 be subsonic? Could it be supersonic? What would determine this?

In all these questions, it is expected that the reasoning behind the answer is given.

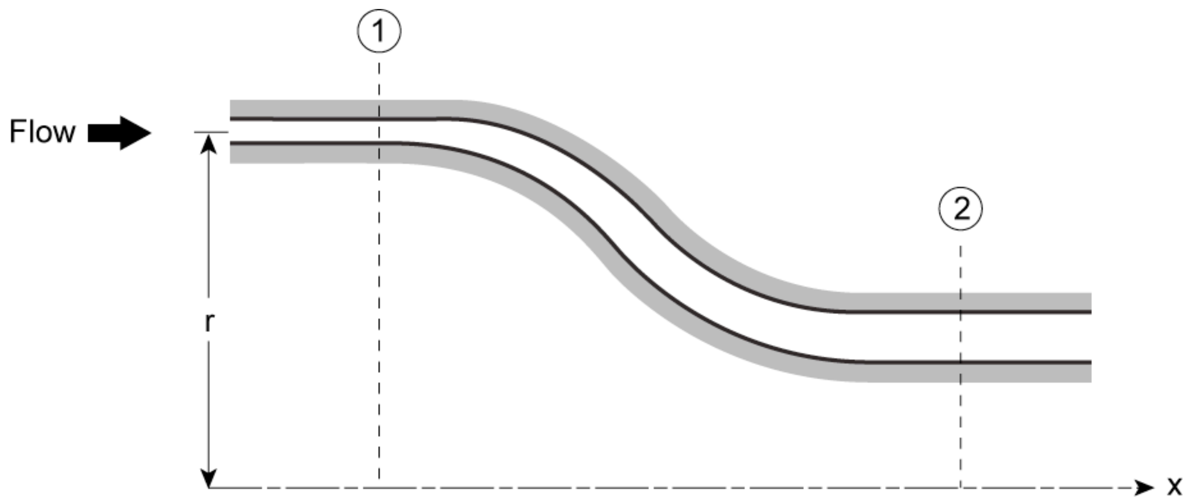

Figure 1: Axisymmetric swirling flow in an annular duct

B) In the second part of the question we consider the frictionless flow through a turbomachinery stage made up of a row of stationary blades followed by a row of moving blades. One view of these is given in Figure 2a, which shows a constant radius, constant height duct with the blade rows in the regions indicated. Figure 2b shows a radial view of the two blade rows.

The flow upstream of the stage is axial, i.e., there is no circumferential velocity component and it is subsonic. *As seen in the stationary reference frame* the first row turns the flow from axial at station 1 to 60 degrees from axial at station 2. The second row returns the flow back to axial at station 3 (again, as seen in the stationary reference frame). For questions (i) to (iii), the flow is adiabatic, for (iv) there is frictionless heat addition.

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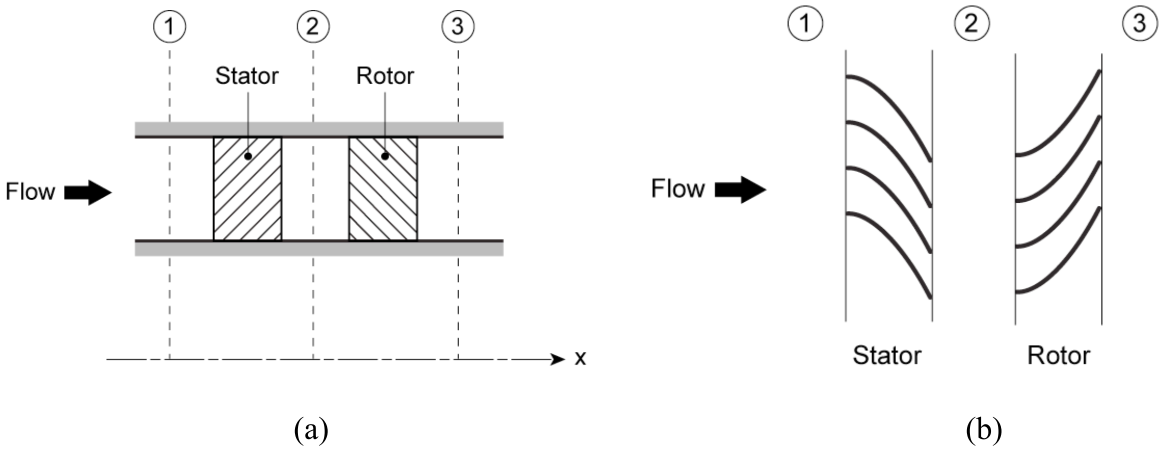

Figure 2: A turbomachinery stage. Figure 2a is a side view, Figure 2b is a view looking radially inwards

i) What direction will the rotor move in the configuration in Figure 2b? Why?

ii) If the area of the blade channel in the first row is a minimum at the blade channel exit, what can you say about the Mach number there?

iii) Sketch the "vector triangle" between the exit velocity from the stator, as seen in the stationary system, and the relative velocity into the rotor (the velocity in the rotor fixed reference frame)?

iv) Suppose the stator is choked and the stagnation temperature upstream of the stage is T_t. A small amount of heat, dq, is then added to the flow upstream of the stage. It is desired to keep the *physical mass flow* (kg/s) through the turbine, *and* the stator exit Mach number, the same as before the heat was added. What quantities upstream of the turbine need to change, and how will they change, for this to happen?

Air Breathing Propulsion (Internal Flow) Qualifier Question – January 2010

In this question we examine two aspects of swirling flow in propulsion systems.

A) An inviscid incompressible fluid flows in an axisymmetric duct. The duct is cylindrical far upstream and downstream, with radii r_1 and r_2. The downstream radius, r_2, is twice the upstream radius, r_1. There is an expanding area region between the two cylindrical sections as shown in Figure 1 below. At a "far upstream" station 1 the velocity components are

$$u_x = U = \text{constant} \qquad \text{(a)}$$
$$u_\theta = \Omega r \qquad \text{(b)}$$
$$u_r = 0 \qquad \text{(c)}$$

In expression (b), Ω is a constant.

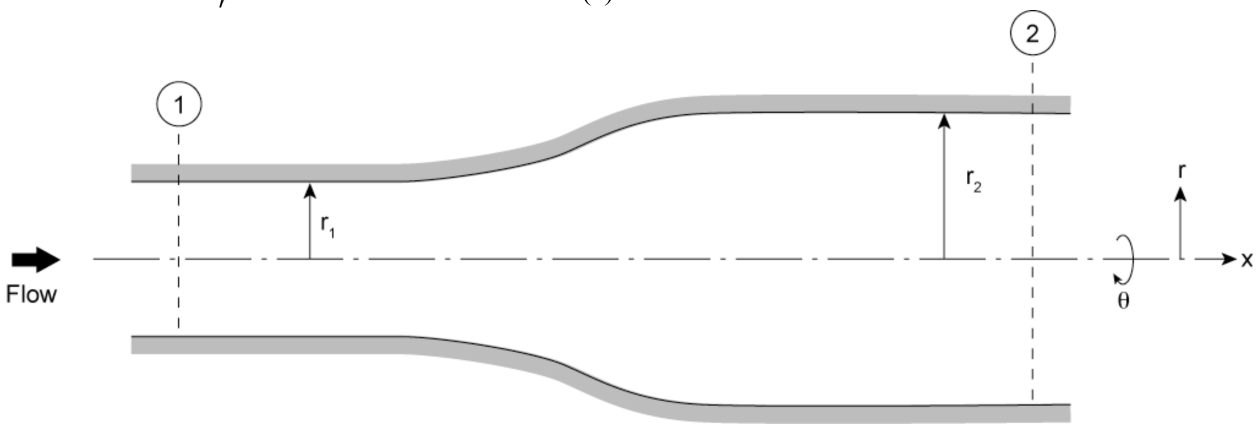

Figure 1: Swirling flow in an axisymmetric duct

iv) Sketch the vortex lines at station 1. Which components of vorticity are non-zero?
v) Consider a fluid contour at station 1 that is in a plane at constant value of x and has a radius equal to r_1. What is the circulation around this contour at station 1?
vi) What is the circulation around this contour at station 2? Why do you say this?
vii) Is there a static pressure difference between the outer radius and the centerline of the duct at station 1? If so, what is it?
viii) Would you expect the static pressure difference between the outer radius of the duct at station 2 to be larger, smaller, or the same than at station 1? Why? (Hint: You may need to make an assumption to answer this even in a qualitative manner.)
ix) Describe the velocity field at station 2. What are the differences between the velocity field at station 1 and at station 2? How would you explain these differences in physical terms?
x) Sketch the vortex lines in the expansion region and at station 2. Which components of vorticity are non-zero? Why?
xi) For a fixed radius ratio (for example, 2) and a given axial velocity at station 1, suppose we increased Ω, i.e. increased the amount of swirl at station 1. What changes would you expect in the flow? In particular, would you expect that reverse flow could occur? If so where would it occur?

In all these questions, the physical reasoning behind the answer should be presented along with the answer.

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B) In the second part of the question we consider frictionless, adiabatic, compressible flow through a turbomachinery stage made up of a row of stationary blades followed by a row of moving blades. One view is given in Figure 2a, which shows a constant radius, constant height duct with the blade rows in the regions indicated. Figure 2b shows a radial view of the two blade rows.

The flow upstream of the stage is axial, i.e., there is no circumferential velocity component and it is subsonic. *As seen in the stationary reference frame* the first row turns the flow from axial at station 1 to 60 degrees from axial at station 2. The second row returns the flow back to axial at station 3, again as seen in the stationary reference frame.

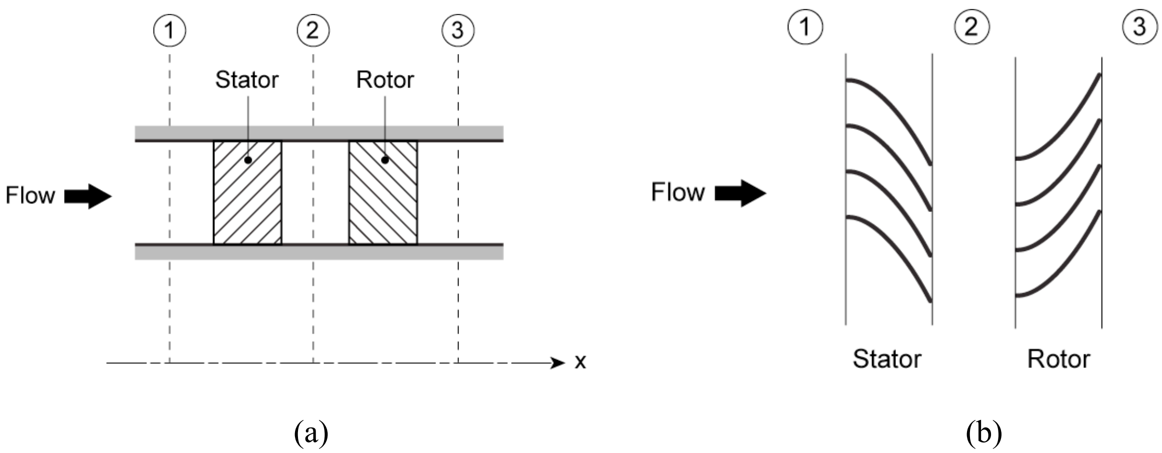

Figure 2: A turbomachinery stage. Figure 2a is a side view, Figure 2b is a view looking radially inwards

v) What direction will the rotor move in the configuration in Figure 2b? Why?
vi) If the area of the blade channel in the first row is a minimum at the blade channel exit, what can you say about the Mach number there? Why?
vii) Sketch the "vector triangle" between the exit velocity from the stator, as seen in the stationary system, and the relative velocity into the rotor (the velocity in the rotor-fixed reference frame)?
viii) Suppose the stator exit Mach number is equal to unity. If the stagnation pressure upstream of the turbine is increased, does the *physical mass flow* (kg/s) through the turbine increase, decrease or stay the same? Why?
ix) For the same turbine, the stagnation temperature upstream of the turbine is now increased. Does the *physical mass flow* (kg/s) through the turbine increase, decrease or stay the same? Why?