Air-Breathing Propulsion Qualifier Question - 2012

A gas turbine jet engine, shown schematically in Figure 1, is operated on a stationary test stand. The components of the engine are a multistage compressor, a combustion chamber (combustor), a single stage turbine, and an ideally expanded nozzle (nozzle exit pressure equal to ambient pressure). The stations referred to in the questions are depicted in the figure. At the design condition the nozzle exit velocity is $u_{4p}$.

For all questions the working fluid can be taken as air, with constant specific heats and $\gamma = 1.4$, and you can assume the only effect of the fuel flow is the heat input, i.e., assume that the fuel mass flow is negligible.

For all questions except Question (i) all components can be regarded as ideal, i.e., lossless.

The compressor performance for the engine is given in Figure 2, which shows compressor pressure ratio as a function of compressor inlet corrected flow $\dot{m}_{t_1} \sqrt{T_{t_1}} / p_{t_1}$ (see the Nomenclature list below), for three different lines of constant corrected rotor speed, $\dot{N}/\sqrt{T_{t_1}}$. The largest solid circle indicates the design condition, D. The dashed line indicates the steady-state operating line, i.e., the locus of engine operating points, as the rate of fuel into the engine is increased slowly enough such that the operating line can be considered as a sequence of steady-state points.

The questions below refer to different aspects of the engine performance and geometry:

a) What is the thrust produced by the engine? State this in terms of the symbols introduced above plus any other symbols you think appropriate (See the Nomenclature section.)

b) Given the compressor stagnation temperature ratio $\tau_C = T_{t_2}/T_{t_1}$, what is the turbine stagnation temperature ratio $\tau_T = T_{t_4}/T_{t_3}$? (Suggestion: the answer should be in terms of $\tau_C$ and a ratio of turbine inlet stagnation temperature to compressor inlet stagnation temperature, $T_{t_3}/T_{t_1}$.)

c) If the design pressure ratio, $p_{t_2}/p_{t_1}$, is 12, and the ratio of turbine inlet stagnation temperature to compressor inlet stagnation temperature, $T_{t_3}/T_{t_1}$, is 6, what is the turbine stagnation temperature ratio? (A numerical value is expected.)

d) Would you expect the flow at the nozzle exit to be subsonic or supersonic? Why?

e) If the stagnation pressure ratio and the stagnation temperature ratio across the turbine are denoted by $\pi_T$ and $\tau_T$ respectively, what is the ratio of the minimum flow area in the turbine to minimum area in the nozzle? (Suggestion: the answer should be in terms of $\pi_T$ and $\tau_T$ only.)

f) Suppose there is a step increase in the rate of fuel injected into the engine per unit time. If so, the compressor operating point may change. With reference to Figure 3, if the process of fuel injection and its combustion can be considered instantaneous, which of the four arrows will the operating point follow just after the fuel injection rate is increased?
g) What might the trajectory of the operating point look like as time goes on? Sketch the operating point motion on Figure 3. What might be the “final” operating point at the changed mass flow rate?

h) What determines the time taken to achieve the final operating point? A qualitative physical statement concerning the time evolution is desired, preferably with comments about what sets the “time constant” for this evolution.

i) Suppose the turbine deteriorates so there are substantial losses in this component. If we wished to keep the compressor and the turbine at the same inlet operating point, what changes might we have to make to the nozzle? How would this affect the thrust? Would the thrust increase, decrease, or stay the same?

For all questions, a physical justification of the answer is expected (one or two sentences, or an equation, or both, as appropriate)

Nomenclature

\[ A \] – area

\[ \dot{m} \] – mass flow rate

\[ N \] – compressor rotational speed, rpm

\[ p_t \] – stagnation pressure

\[ R \] – gas constant for air

\[ T_t \] – stagnation temperature

\[ \gamma \] – specific heat ratio

\[ \pi_c \] – compressor stagnation pressure ratio, \( p_{t2} / p_{t1} \)

\[ \pi_t \] – turbine stagnation pressure ratio, \( p_{t4} / p_{t3} \)

\[ \tau_c \] – compressor stagnation temperature ratio, \( T_{t2} / T_{t1} \)

\[ \tau_t \] – turbine stagnation temperature ratio, \( T_{t4} / T_{t3} \)

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![Figure 1: Gas turbine engine](image)
Figure 2: Engine compressor pressure ratio versus corrected flow

\[ \frac{p_{t_2}}{p_{t_1}} \]

Figure 3: Transient performance just after a step change in fuel flow rate; four proposed instantaneous operating point trajectories (denoted by arrows I through IV) are shown

\[ \frac{N_c}{\sqrt{T_{t_o}}} > \frac{N_b}{\sqrt{T_{t_o}}} > \frac{N_d}{\sqrt{T_{t_o}}} \]

\[ \dot{m} \sqrt{T_{t_o}} / p_{t_1} \]

\[ \dot{m} \sqrt{T_{t_o}} / p_{t_1} \]