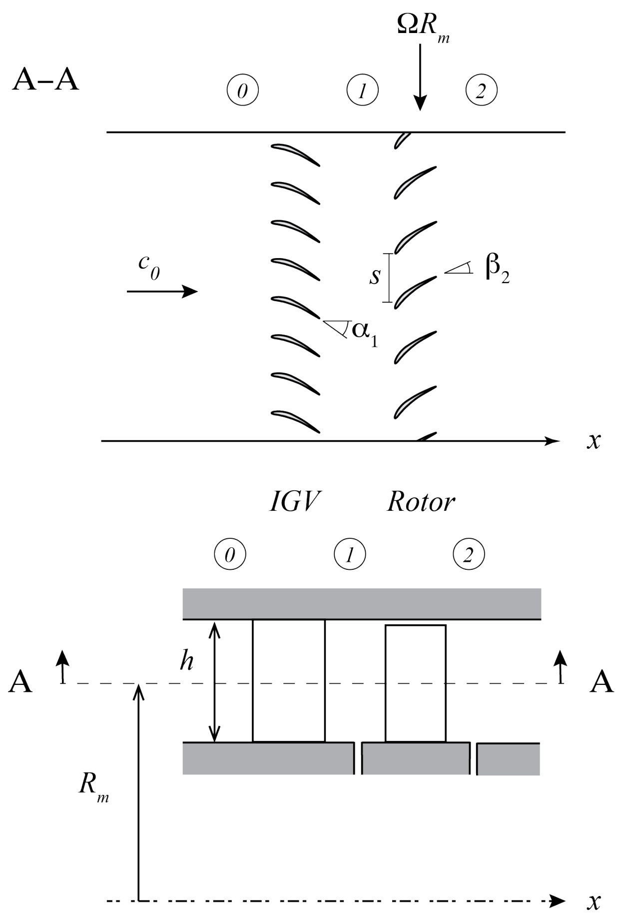
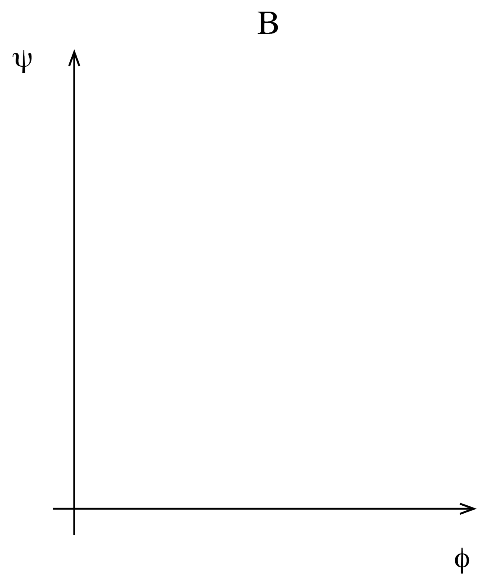
Consider a turbomachine that has a row of (stationary) inlet guide vanes (IGVs) followed by a row of (rotating) rotor blades in a constant area annular duct. The duct has a constant mean radius, R_m, and constant height, h (see the figure on the left below). The rotor rotation rate is Ω.

Upstream of the inlet guide vanes the flow is axial, i. e., the velocity is in the x-direction without any swirl, and both the velocity and the static pressure are uniform at all radii. The radial velocity can be taken as zero stations 0, 1, and 2, and the flow is incompressible and inviscid.





The problem below has three main sections. In the first section we consider the flow in the blade-to-blade (or vane-to-vane) plane at mean radius R_m, see the top left figure A-A. The *exit* flow angles relative to the blades are α_1 from the inlet guide vane and β_2 from the rotor, respectively and are the same for all flow conditions described. The inlet velocity at design conditions is c_0.

a) For the design condition, sketch the *velocity triangles* relating the absolute and relative velocities at station 1, between the inlet guide vane and the rotor, and station 2 at the rotor exit, design condition.

b) Suppose the inlet velocity c_0 is now reduced to half of its design value. Sketch the velocity triangles for this operating condition in the same diagram. The diagram does not have to be precisely to scale but it should be representative enough that the principal effects can be seen.

c) Comment, in a qualitative manner, on the changes in flow coefficient, $\phi = c_x/\Omega R_m$, and work coefficient, $\psi_h = (h_{t2} - h_{t0})/(\Omega R_m)^2$, where h_t is the stagnation enthalpy, based on the differences in velocity triangles. You do not need to compute the changes, but a physical rationale for your answer is required.

d) What is the *stagnation pressure rise coefficient*, $\psi = (p_{t2} - p_{t0})/(\rho(\Omega R_m)^2)$ as a function of flow coefficient ϕ and flow angles α_1 and β_2? (An equation is wanted.) Sketch the ψ–ϕ characteristic in the above diagram, B. [Suggestion: How is the stagnation pressure rise coefficient related to the work coefficient in Item (c)?]

In the second section of the question, we examine features of the rotor blade aerodynamics. Thus, based on the conditions at the mean radius, R_m (figure A-A):

e) What is the fluid force on the rotor blade, per unit span, in the axial direction, $F_x/(s\rho(\Omega R_m)^2)$, where s is the distance between rotor blades? Express your answer in terms of the non-dimensional parameters found earlier.

f) What is the fluid force on the rotor blade, per unit span, in the tangential direction, $F_\theta/(s\rho(\Omega R_m)^2)$? Express your answer in terms of the non-dimensional parameters found earlier.

In the third section of the question we examine the radial variations in flow properties. Suppose the IGVs are designed such that at station 1 the circulation around a circular contour centered on the axis of rotation is $\Gamma_1 = 2\pi K_1$, where K_1 is constant with radius.

g) What is the radial distribution of tangential velocity $c_\theta/(\Omega R_m)$ upstream of the rotor at station 1? Sketch this distribution in diagram C below.

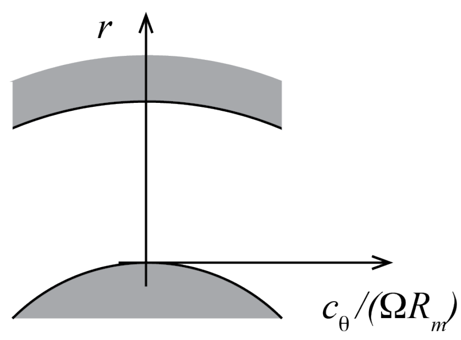
h) What is the radial distribution of static pressure $(p - p_H)/\rho$ upstream of the rotor at station 1, where p_H is the static pressure at the hub. Sketch this distribution in diagram D below.

i) Is the flow downstream of the IGV rotational or irrotational? What is the basis for your answer?

j) Upstream of the rotor at station 1, is the stagnation pressure uniform in the radial direction? Why or why not?

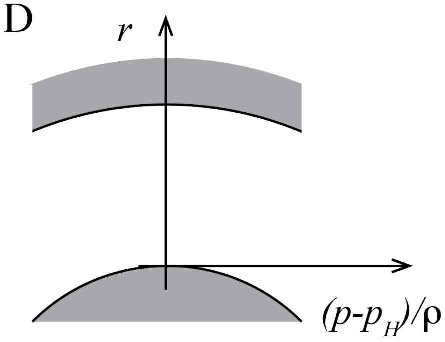
k) If there are N inlet guide vanes, what is the circulation, Γ_{IGV}, about a single vane?

C



Station 1

D



Station 1