Air Breathing Propulsion Qualifying Exam 2014

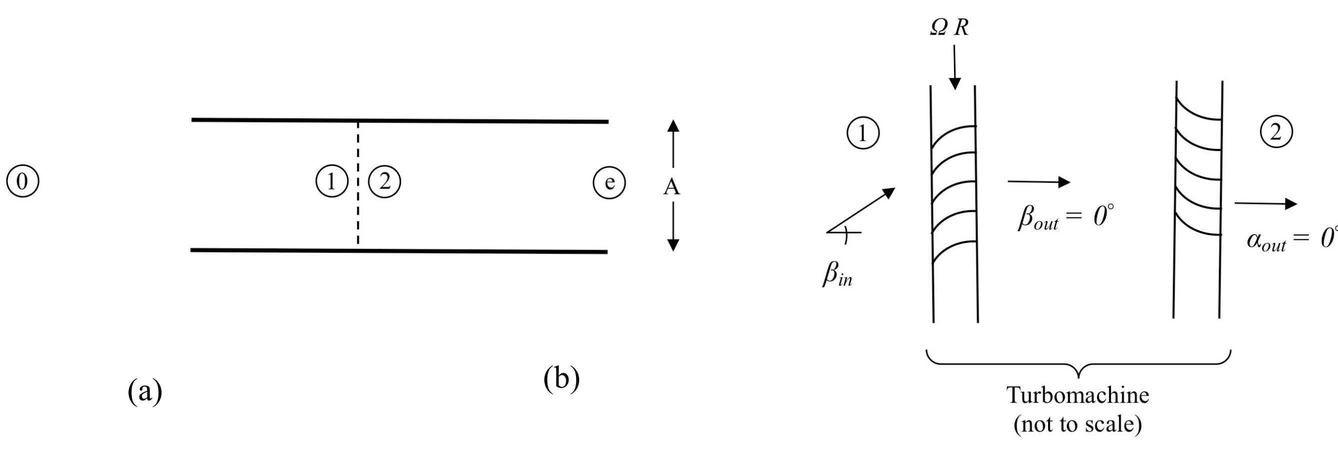
This exam consists of two problems and a total of two pages. Both problems count the same.

Question 1. Figure 1 is a schematic of a turbomachine (rotor and stator) in a constant area duct. The dashed line in Figure 1 (a) indicates the location of the turbomachine, which can be considered as a surface across which some flow properties are discontinuous. Figure 1 (b) shows the turbomachinery blading at a representative radius. The relative exit angle from the rotor is zero. This means that the relative flow (flow seen in the rotor system) exits the rotor axially, where the axial direction is along the duct. The stator exit angle for the absolute flow (flow seen in the engine fixed coordinate system) is also zero. The flow can be taken to be incompressible, inviscid, and adiabatic. The axial velocity far upstream is u_0. The angular velocity of the rotor is Ω and the duct area is A. Take the conditions at radius R to be representative of the machine as a whole.

a) Derive expressions for the following in terms of given quantities:
   i) Stagnation enthalpy change across the turbomachine;
   ii) Stagnation pressure change across the turbomachine;
   iii) Static pressure change across the turbomachine;
   iv) Static pressure change from station 2 to station *e* at the exit of the duct;
   v) Static pressure change from station 0 to station 1;
   vi) Axial force needed to hold the turbomachine in the duct;
   vii) Velocity through the duct (appropriately non-dimensionalized); and
   viii) Overall thrust produced by this propulsor (appropriately non-dimensionalized).

b) Draw the streamlines of the flow.

c) What changes (qualitatively) would you expect to the streamlines if the upstream flow had zero velocity far upstream?



(a) (b)

Figure 1: Schematic of a turbomachine in a constant area duct showing: (a) nominal location of the turbomachinery, and (b) the layout of blading. The schematic is not to scale because the rotor and stator are much closer together than shown.

Question 2. The propulsor in Question 1 made use of work addition. In contrast the propulsor in this question makes use of heat addition. Figure 2 (a) shows a duct with the region of heat addition represented by a dashed line across which fluid properties are discontinuous. Figure 2 (b) is a schematic of the region of heat addition.

The mass and momentum of the fuel can be neglected. The temperature change across the region of heat addition is such that $\rho_1/\rho_2 = 5$. The fluid can be taken to be incompressible, inviscid, and adiabatic *except* across the discontinuity. (Wall shear stresses can also be neglected in the region of heat addition.) The velocity far upstream is u_0.

a) Derive expressions for the following:
   i) Static pressure change across the region of heat addition, in terms of quantities at stations 1 and 2; does the static pressure fall or rise?
   ii) Static pressure change from station 2 to station *e*, in terms of quantities at stations 2 and *e*;
   iii) Static pressure change from station 0 to station 1 in terms of densities and velocities at stations 0 and 1;
   iv) Velocity through the duct at station 1 and at station 2 in terms of u_0; and
   v) Overall thrust produced by the propulsor (appropriately non-dimensionalized)

b) Draw the streamlines of the flow.

c) If the region of heat addition in the turbomachine of Figure 2 were instead located in an unconfined flow (no duct), would it still produce a thrust? How does this compare to a situation in which the turbomachine of Figure 1 were located in an unconfined flow?

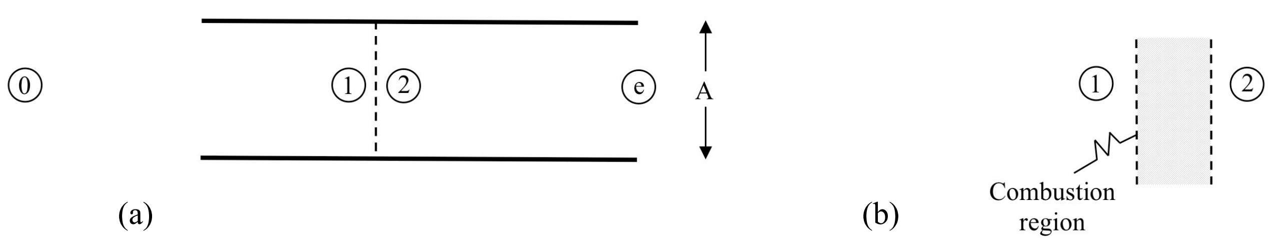


Figure 2: Schematic of a propulsor using heat addition in a constant area duct showing: (a) the nominal location of the region of heat addition, and (b) a schematic of the region of heat addition for consistency with Figure 1.